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A MATHEMATICAL FRAMEWORK FOR ANALYZING INCENTIVES IN PEER-TO-PEER NETWORKS

Mohammed Onimisi Yahaya
Department of Mathematical Sciences, Abubakar Tafawa Balewa University, Bauchi, Nigeria.

email: mdonimisi@atbu.edu.ng

ABSTRACT
The existence and performance of peer-to-peer systems depend on the contribution of resources from interacting peers. One of the challenges of resource sharing in peer-to-peer systems is free riding. A situation users attempt to exploit the system by utilizing the resources of others without contributing. We view this from rationality perspective that every peer in the network will attempt to maximize their utility of the system. In this paper, we approach the problem of free riders mitigation from utility optimization point of view, by modeling each peer's interest as Utility Maximization Problem (UTP). We propose analytical model for the whole network as a mixed integer linear programming model. The super peers in the network are given the responsibility of maximizing the utility of all peers connected to them. This is to ensure fairness among the interacting peers and the stability of the entire system. This technique allows peers to either upload or download resources based on their best strategy and interest.

Keywords: Free rider, Utility, Peer-to-Peer, Incentives, Maximization, Resources

INTRODUCTION
The ever increasing demand for exchange and sharing of resources amongst millions of users the world over led to the upsurge in the use of P2P systems. A system is said to be P2P if it exhibits the following characteristics; self-organization, distributed control and symmetric communication (Roussopoulos *et al*., 2004). Also, the inherent distributed nature of users, systems and resources as well as the inability of client-server model to support numerous exchanges of resources made P2P networks a viable alternative. But, the democratic or anarchic nature of P2P systems where peers can be equal, anonymous and autonomous raises a lot of challenges to the management and performance of such systems. One of such challenges is the problem of free riding. The existence and survival of any P2P resource sharing network depends on resources contribution from each participating peer. Based on general rationality assumption, most peers would always attempt to maximize their utility of the system. Resource sharing in P2P systems has been likened to private provisioning of public goods in which free riding will occur (Krishnan et al.,, 2008). In such cases, peers may tend not to contribute resources so as not to incur cost, but would attempt to use the resources contributed by others. This phenomenon is refers to as free riding (Adar et al., 2000; Belmonte et al., 2012, and Azzedin *et al*., 2014).

Several studies e.g (Adar *et al*., 2000; Sariou *et al*., 2002; Asvanund *et al*., 2004 and Silverston *et al*., 2008) have confirmed the existence of free riding in P2P systems. Free riding if left uncontrolled may cause serious degradation to the performance of the system (Ksai *et al*., 2012). It increases network stress (Belmonte et al., 2012), decreases scalability and content availability (Zhang *et al*., 2012). Hence, there is a need to encourage cooperation among peers to be able to deliver a robust P2P system.

Despite the prevalence of P2P systems such as BitTorrent, KaZaa, emule etc. and their efforts to tackle free riding, the problem still persists. Every solution proffered to counteract their effects always meet with challenges of conflicting requirements. There are always trade-offs amongst design and performance considerations such as overhead, ease of use, centralization/ decentralization and efficiency. There are several approaches proposed in the literature to combat free riding. These approaches may be broadly categorized as follows; micropayment-based schemes, Trust/reputation-based approaches, reciprocity/barter-based approaches, game theory and utility based system. Each of these approaches has their advantages and shortcomings.

In this paper, we approach the problem of free riding in P2P system from a different perspective. We view sharing in P2P systems as an optimization problem. We consider the network as a system in which every peer tries to maximize their utility. The model is based on the general idea that each peer should get a satisfactory service from the system based on their behaviour and still have a stable system. This will serve as incentive for peers to contribute and discourage free riding behaviour.

The remaining parts of this paper are organized as follows: section 2 presents a review of relevant literature. Section 3 detailed the problem formulation and motivation. We present the mathematical model and illustrative examples in section 4, 5 and 6. In section 7, we discuss designing utility functions and section 8 presents possible model implementation in real systems. Discussion and conclusion is presented in section 9.

LITERATURE REVIEW
Recently, the popularity of P2P as a de facto content sharing network brought about upsurge in research effort to address some of it challenges. One of such challenges is the problem of free riding, a phenomenon in which some peers in a P2P environment use the resources of others without contributing to them (Adar et al 2000, Belmonte *et al*., 2012, Azzedin *et al*., 2014). Several analytical methods have been proposed in the literature in order to understand resource availability, peers' interactions and stability of the network. Modeling peers interactions using game theory has been investigated by several researchers. For example see (Golle et al., 2001; Krishnan *et al*., 2004; Zhao *et al*., 2012; Hua *et al*. 2012 and Azzedin *et al*., 2014).

Azzedin and Yahaya (2014) modeled the sharing interactions in BitTorrent (Cohen, 2003) as games, in an attempt to understand different sharing characteristics of peers in a BitTorrent network, one of the most popular P2P systems. The authors analyzed the Nash equilibrium of each game and identified the best strategy for each player in order to inhibit the activities of free riders.

Zhao et al., (2012) presented a mathematical framework for incentive analysis in P2P based on game strategy. The entire P2P network is described as a discrete time system, in which every time, each peer decides to share or not depending on the gain and the cost of sharing. Two learning algorithms were proposed and investigated in collaboration with the game. The authors analyzed their analytical model and concluded that in order to have a robust P2P systems, every peer must be made to contribute by designing efficient free riders mitigation techniques.

Hua *et al*., (2012) used game theory to model the interaction among peers in an unstructured P2P networks. They grouped the peers in their networks into three categories, namely; Altruist- peers that always give resources to others irrespective of their gain, free riders- peers that do not contribute to others and In-between are peers that are somewhat between altruism and free riding. The authors incorporated reputation to monitor peers' sharing behavior and designed mechanism to inhibit free riding.

Gupta and Somani (2005) proposed a simple game model as a tool to understand and predict peers' sharing attitude in a P2P file sharing system. They classified the strategies of the peer into two shares or not share. The authors analyzed the mixed and pure strategy of the game and concluded that an equilibrium in which all peers decide not to share will lead to total collapse of P2P networks.

In this work, we deviated from using game theory by modeling the peer sharing process as an optimization problem.

PROBLEM FORMULATION

The existence of P2P systems depend on peers interactions through sharing or exchange of resources. Every rational peer in the system wants to maximize their utility. Hence, incentive protocol is crucial to the performance of the entire system. In this study, we propose a general analytical framework for both the entire network utility and peers' utility in a P2P system. We approach the whole problem as Utility Maximization Problem (UTP) and formulate it as a Mixed Integer Linear Programming Model. We assume that the decisions are made at the tracker's level. The objective of the tracker would be to maximize the total sum of the utilities of all the peers in the system. The tracker would take the best decision for the community based on the number of files available to be shared by uploaders and the number of requests from downloaders. It determines the best assignment of uploaders to downloaders by maximizing the sum of all utilities. This will encourage peers participation in the improvement of the performance of the network.

Table 1: The sets notations of the proposed MLP model

Symbols	Type	Descriptions
F	Set	Set of Files
O	Set	Set of uploaders
D	Set	Set of downloaders
F_R	Element of a set	File R from F
O_k	Element of a set	Uploader k
D_j	Element of a set	Downloader j

The proposed model has two main views, namely; the network view and the peers' view.

(a) The network view: In this view of the model, we assume that the decision is made at the tracker's level. The objective of the tracker would be to maximize the total sum of the utilities of all the peers in the system. The tracker would take the best decision for the community based on the number of files available to be shared by uploaders and the number of requests from downloaders. It determines the best assignment of uploaders to downloaders by maximizing the sum of all utilities. This will encourage peers participation in the improvement of the performance of the network.

(b) The peers' view: This view is further divided into downloaders' view and uploaders' view. The general assumption here is that the tracker maximizes the utility of every peer in the system. Each peer will tries to maximize his utility. We assume in this model, that the tracker would maximize the utility of each peer on their behalf.
In the downloaders' view, the tracker maximizes the utility of each downloader on their behalf and in the uploaders' view, the tracker maximizes the utility of each uploader in the system.

LINEAR PROGRAMMING MODELS FOR P2P SYSTEM

Consider a P2P file sharing system where a peer wants to download a file FR, partitioned into equal pieces. A piece i is a chunk of file FR, which is also referred to as a block in the literature. Throughout the description of this model, we will use the word piece to represent part of a file. Each piece i is of size siR. We assume that all piece i are not overlapping. We use o_k^i to represent the kth owner of a piece i. We define mi as the number of peers owning piece i. We define $X_{jk}^i \epsilon \{0;1\}$ s a real variable that takes the value between 0 and 1. This means that downloader D_j^i may get non, part or the whole piece i from uploader $k\,(O_k^i)$. The utility derived by Dj for downloading i from its kth owner is denoted by U_{jk}^i;(i = 1,2,. . ., l; k = 1,2,...,mi). Where i is the index of the file, l is total number of pieces available for file R, and k is the index of uploaders. The maximum capacity of uploader k (Ok) is denoted by Tk and q denotes the number of downloaders served in sequence by Ok before the current j. The model can be formulated as a mixed integer linear program. Table 1 and Table 2 details the sets and notations to be used in the formulation of our model.

Table 2: The notations of the proposed MLP model

Symbol	Type	Description
i	Index	Piece's index
j	Index	Downloaders' index
k	Index	Uploaders' index
l	Integer	The total number of pieces
n_i	Integer	The total number of downloaders of piece i
m_i	Integer	The total number of uploaders of piece i
T_k	Real	Capacity of uploader k
X_{jk}^i	Real variable	$X_{jk}^i \epsilon \{0;1\}$

U^i_{jk}	Real parameter	The utility derived by D_j for downloading Piece i from its k^{th} owner. (i = 1, 2; ...; l; k = 1, 2, ...,m_j)
V^i_{jk}	Real parameter	The utility derived by O_k for uploading piece i to its j^{th} requester. (i = 1,2, ... l ; j = 1, 2,...,n_i)

CONSTRAINTS

We now describe the constraints to be considered in our model. To ensure that each downloader Dj gets the piece i from exactly one uploader Oi, we formulate the following constraint:

$$\sum_{k=1}^{m_i} X^i_{jk} = 1 \qquad \forall j \quad and \ \forall i \qquad (1)$$

The second constraint is on the service capacity of each uploader. It is formulated as follows:

$$\sum_{k=1}^{l}\sum_{j=1}^{n_i} X^i_{jk} \le \quad T_k \qquad \forall k \qquad (2)$$

Where Tk is the capacity of uploader k.

OBJECTIVE FUNCTION

The objective function to maximize corresponds to the total sum of all utilities of all downloaders and uploaders.

$$Maximize \quad \sum_{l=1}^{l}\sum_{j=1}^{n_i}\sum_{k=1}^{m_i} U^i_{jk} X^i_{jk} + \sum_{l=1}^{l}\sum_{j=1}^{n_i}\sum_{k=1}^{m_i} V^i_{jk} X^i_{jk} \qquad (3)$$

THE NETWORK VIEW MODEL

The complete mixed integer linear programming model is described as follows:

$$Maximize \quad \sum_{l=1}^{l}\sum_{j=1}^{n_i}\sum_{k=1}^{m_i} U^i_{jk} X^i_{jk} + \sum_{l=1}^{l}\sum_{j=1}^{n_i}\sum_{k=1}^{m_i} V^i_{jk} X^i_{jk} \qquad (4)$$

Subject to

$$\sum_{k=1}^{m_i} X^i_{jk} = 1 \qquad \forall j \quad and \ \forall i \qquad (5)$$

$$\sum_{k=1}^{l}\sum_{j=1}^{n_i} X^i_{jk} \le 1 \qquad \forall k \qquad (6)$$

$$0 \le X^i_{jk} \le 1$$

Table 3: Utility Table for piece 1

jnk	$O1$	$O2$	$O3$
D1	(2,7)	(3, 2)	(-, -)
D2	(-, -)	(4,1)	(-, -)
D3	(-,-)	(-,-)	(6, 3)
D4	(5,1)	(-,-)	(3,1)

Table 3: Utility Table for piece

jnk	$O1$	$O2$	$O3$
D1	(1,4)	(4,3)	(-,-)
D2	(-,-)	(3,1)	(-,-)
D3	(6,5)	(-,-)	(7, 6)
D4	(1, 2)	(-,-)	(1, 2)

ILLUSTRATIVE EXAMPLE FOR NETWORK VIEW MODEL

We now illustrate the proposed MLP model with a simple example. We consider a file sharing system with seven peers {p1,...,p7}. There are three uploading peers {O1, O2, O3} and four downloading peers {D1, D2, D3, D4}. There are two peers owning at least one of the two or both pieces. The strategy of the tracker is to maximizes the total utility derived from these peer interactions. Let us assume the values of the utility of each piece are as shown in Tables 3 and Table 4. Initially for the sake of simplicity of the example, we use (-) to denote dont care values. The tracker will maximize:

$$Maximize \qquad 2X_{11} + 3X_{12} + 4X_{22} + \ldots + 2X_{43} \qquad (7)$$

Subject to

$$\sum_{k=1}^{3} X^i_{jk} = 1 \qquad j = 1,2,3,4 \quad and \ i = 1,2 \qquad (8)$$

$$\sum_{k=1}^{2}\sum_{j=1}^{4} X^i_{jk} \le T_k \qquad (9)$$

$$0 \le X^i_{jk} \le 1$$

We solve the above mixed integer linear programming problems using Microsoft Excel Solver.

Table 5: Utility of each downloaders

Downloader	Piece 1	Piece 2	Sub total
D_1	2	4	6
D_2	4	3	7
D_3	6	7	13
D_4	5	1	6

Table 6: Utility of each uploaders

Downloader	Piece 1	Piece 2	Sub total
O_1	(7+1)	-	8
O_2	1	(3 +1)	5
O_3	3	(6+2)	11

The total utility of all the peers in the system is 56 units. The variables $X_{jk}s$ now has values as shown in Tables 7 and Table 8. The total utility of 56 units is distributed amongst all the downloaders and uploaders as shown in Table 5 and Table 6.

These results are interpreted as follows; to maximize the sum of total utilities of all peers in the network, it is better for the community if Downloader D_1 gets piece 1 from O_1 and piece 2 from O_2. Downloader 2 (D_2) should downloads piece 1 and 2 from uploader 2 (O_2). Also, for downloader 3, it is better to download piece 1 and 2 from uploader 3. Finally, downloader 4 should get piece 1 from O_1 and piece 2 from O_3.

Table 7: Variable X_{1j}^{k} Table for piece 1

jnk	O_1	O_2	O_3
D1	1	0	0
D2	0	1	0
D3	0	0	1
D4	1	0	0

Table 8: Variable X_{2j}^{k} Table for piece 2

jnk	O_1	O_2	O_3
D1	0	1	0
D2	0	1	0
D3	0	0	1
D4	0	0	1

THE DOWNLOADERS' VIEW MODEL

In this view of the model, the tracker maximizes on behalf of each downloader irrespective of other peers in the system. In this model, we used the variables, parameters and notations from the previous model.

CONSTRAINTS

The constraints to be considered in this model are described as follows: To ensure that each downloader D_j do not download an overlapping pieces from multiple uploaders O_k. We formulate the following constraint.

$$\sum_{k=1}^{m_i} X_{jk}^{i} = 1 \qquad \forall i \tag{10}$$

The second constraint is on the service capacity of each uploader. Which is given by the following equation

$$\sum_{l=1}^{l}\sum_{j=1}^{n_i} X_{jk}^{i} \leq T_k - \left(\sum_{k=1}^{m_i}\sum_{q<j} X_{jk}^{i}\right), \tag{11}$$

$$X_{jk}^{i} \epsilon \{0,1\}.$$

Where T_k is the capacity of uploader k can serve and q is the number of downloaders' uploader O_k served before current j.

OBJECTIVE FUNCTION

The following equation corresponds to the utility of each downloader, which is the Objective function to be maximized on their behalf by the tracker.

$$Maximize \quad \sum_{k=1}^{m_i} U_{jk}^{i} X_{jk}^{i} \tag{12}$$

THE COMPLETE DOWNLOADERS VIEW MODEL

The complete linear model based on downloaders' view is written as follows:

$$Maximize \quad \sum_{k=1}^{m_i} U_{jk}^{i} X_{jk}^{i} \tag{13}$$

Subject to

$$\sum_{k=1}^{m_i} X_{jk}^{i} = 1 \qquad \forall i \qquad (14)$$

$$\sum_{l=1}^{l}\sum_{j=1}^{n_i} X_{jk}^{i} \leq T_k - \left(\sum_{k=1}^{m_i}\sum_{q<j} X_{qk}^{i}\right), \qquad (15)$$

$$X_{jk}^{i} \in \{0,1\}.$$

Where Tk is the capacity of uploader k can serve and q is the number of downloaders uploader Ok served before current j.

ILLUSTRATIVE EXAMPLE FOR DOWNLOADERS' VIEW MODEL

To illustrate this model, we will consider the same file sharing system we used in section 4.

The strategy of the tracker is to maximize the utility derived by each downloaders D_j s from these peer interactions without considering the utilities of other peers in the system. Let us assume the values of downloaders' utility for each piece are as shown in Tables 9 and 10. The tracker will maximize the following objective function on behalf of D_1:

$$Maximize \qquad 2X_{11} + 3X_{12} + X_{11} + 4X_{12} \qquad (16)$$

Subject to

$$\sum_{k=1}^{3} X_{jk}^{i} = 1 \qquad j = 1, \quad and \ i = 1,2 \qquad (17)$$

$$\sum_{i=1}^{2}\sum_{j=1}^{1} X_{jk}^{i} \leq T_k \qquad (18)$$

$$X_{jk}^{i} \in \{0,1\}$$

Table 9: Downloaders' utilities table for piece 1

jnk	O_1	O_2	O_3
D1	(2,-)	(3,-)	(-,-)
D2	(-,-)	(4, -)	(-,-)
D3	(-,-)	(-,-)	(6, -)
D4	(5, -)	(-,-)	(3, -)

Table 10: Downloaders' utilities table for piece 2

jnk	O_1	O_2	O_3
D1	0	1	0
D2	0	1	0
D3	0	0	1
D4	1	0	0

The tracker will do same for D_2, D_3, and D_4.

We solve the above mixed integer linear programming problems in sequence. The maximum utility for each of the downloaders in the system are as shown in Table 11. The variables X_{jk}^{i}s now has values as shown in Tables 12 and Table 13. This result translates to it is best for the downloader D_1 to download piece 1 from uploader 2 (O_2) chunk 2 from uploader O_2 with total utility 7. It is better for Downloader D_2 to download piece 1 from uploader 2 and piece 2 from uploader 3. Also, it is in the best interest of D_3 and D_4 to get piece 1 and 2 from uploader 3 and uploader 1 respectively. We observed that by the time each downloader maximized their utility one after the other, the total utility is 48 as compared to 56 when the tracker maximized for all peers in the system. Furthermore,

Table 11: Utility of each downloaders

Downloader	Utility
D1	7
D2	4
D3	13
D4	6

Table 12: Variable X_{jk}^{1} table for piece 1

jnk	O_1	O_2	O_3
D1	(1,-)	(4,-)	(-,-)
D2	(-,-)	(3,-)	(-,-)
D3	(6,-)	(-,-)	(7,-)
D4	(1,-)	(-,-)	(1,-)

Table 13: Variable X_{jk}^{2} table for piece 2

jnk	O_1	O_2	O_3
D1	0	1	0
D2	0	0	1
D3	0	0	1
D4	1	0	0

the capacity of uploader 1 (O_1) remaining 2, while uploader 2 and 3 used up their maximum capacity of 3 as shown in Table 14.

THE UPLOADERS' VIEW MODEL

In this model, the tracker maximizes on behalf of the uploaders. Using the same constraints as in downloaders' view in the illustrative example for downloaders' view model.

OBJECTIVE FUNCTION

The objective function of each uploader to be maximized on their behalf by the tracker is given by the following equation

$$Maximize \sum_{j=1}^{n_i} V_{jk}^{i} X_{jk}^{i} \qquad (19)$$

Table 14: Uploaders' Capacity

			T_k
O1	2	≤	4
O2	3	≤	3
O3	3	≤	3

Table 15: Utility Table for piece 1

Uploader	utility
O1	18
O2	2
O3	4

THE COMPLETE UPLOADERS' VIEW MODEL

The total utility of each uploader to be maximized by the tracker corresponds to the following objective function.

$$Maximize \sum_{j=1}^{n_i} V_{jk}^{i} X_{jk}^{i} \qquad (20)$$

Subject to

$$\sum_{l=1}^{n_i} X_{jk}^{i} = 1 \qquad \forall i \qquad (21)$$

$$\sum_{l=1}^{l}\sum_{j=1}^{n_i} X_{jk}^{i} \le T_k - \left(\sum_{k=1}^{m_i}\sum_{q<j} X_{qk}^{i}\right), \qquad (22)$$

$$X_{jk}^{i} \epsilon \{0,1\}.$$

ILLUSTRATIVE EXAMPLE FOR UPLOADERS' VIEW MODEL

We will consider the same file sharing system we used in Section 5 to illustrate this model.

The strategy of the tracker in this case is to maximize the utility derived by each uploader (O_k) from these peers interactions without considering the utilities of other peers in the system. Let us assume the values of uploaders' utility for each piece are as shown in Tables 15 and Table 16.

Table 16: Utility Table for piece 2

jnk	O_1	O_2	O_3
D1	(-,4)	(-,3)	(-,-)
D2	(-,-)	(-,1)	(-,-)
D3	(-,5)	(-,-)	(-, 6)
D4	(-, 2)	(-,-)	(-, 2)

Table 17: Utility of each Uploader

jnk	O_1	O_2	O_3
D1	(-,7)	(-, 2)	(-,-)
D2	(-,-)	(-,1)	(-,-)
D3	(-,-)	(-,-)	(-, 3)
D4	(-,1)	(-,-)	(-,1)

The tracker will maximize the following objective function on behalf of O_1

$$Maximize 7X_{11} + X_{14} + 4X_{11} + 5X_{13} + 2X_{14} \quad (23)$$

Subject to

$$\sum_{k=1}^{3} X_{jk}^{i} = 1 \qquad j = 1, \quad and\ i = 1,2 \quad (24)$$

$$\sum_{i=1}^{2}\sum_{j=1}^{1} X_{jk}^{i} \leq T_k \quad (25)$$

$$X_{jk}^{i} \in \{0,1\}$$

The tracker will repeat same for O_2, O_3 and O_4.

We solved the above mixed integer linear programming problems in sequence. The maximum utility for each of the uploaders in the system are as shown in Table 17. The variables X_{jk}^{i}s now has values as shown in Tables 18 and Table 19. This result is translated as, it is the best for the uploader O1 to upload piece 1 and piece 2 to downloader 1, 3 and 4 with total utility of 18 units. Uploader 2 (O2) should upload piece 1 and 2 to downloader 2. Also, it is in the best interest of O3 to give piece 1 to downloader 3 and 4. We observed that by the time each uploader maximized their utility one after the other, the total utility of the system is 50 as compared to 56 when the tracker maximized for all peers and 48 when only downloaders utilities were maximized in the system. Furthermore, the capacity of uploader 1 (O1) is completely utilized, while uploader 2 and 3 capacity were remaining 1 each as shown in Table 20.

Table 18: Variable X_{jk}^{1} Table for piece 1

jnk	O_1	O_2	O_3
D_1	1	0	0
D_2	0	1	0
D_3	0	0	1
D_4	0	0	1

Table 19: Variable X_{jk}^{2} Table for piece 2

jnk	O_1	O_2	O_3
D_1	1	0	0
D_2	0	1	0
D_3	1	0	0
D_4	1	0	0

Table 20: Uploaders' Capacity

			T_k
O_1	4	≤	4
O_2	2	≤	3
O_3	2	≤	3

DESIGNING THE UTILITY FUNCTION

One of the fundamental issue we observe in the study of P2P interactions using game theory utility notion is the formulation and modeling of the utility functions. In this section, we discuss the detail components of the functions. Let f1(:) be the benefit function and g1(:) be the cost function. The utility function Uik is the utility derived by Dj for downloading piece i from it kth owner. This is the net gain, that is the difference between the benefit and cost function f1(:) and g1(:). The utility function could depends on resource and uploading peer's attributes. The benefit function f1(:) parameters that depend on resource attributes are file size (FS) or size of a chunk, file popularity (FP), and file quality (FQ). While those attributes that depend on peers, are download speed (DS), peer reliability (PR), trustworthiness (PT) and geographical distance (GD).

The cost function g1(:) is a monotonically increasing functions that may depend on Bandwidth (B) and Disk space Used (DS). For all the benefit parameters except for geographical distance the higher the better. But for the cost function, the lower the better for all peers. These values may vary in real time depending on number of peers interacting, their capacity, their behaviour, network and time. In reality, some of these parameters are subjective. In this work, we model Ui as a monotonically increasing function of any of the afore-mentioned parameters. This is motivated by other P2P studies that model utility function such as [Hua *et al,* 2012; Zhao *et al*, 2012].

THE CHALLENGES IN DESIGNING THE UTILITY FUNCTIONS

Game theory modeling is appealing to researchers of incentives and interactions in P2P networks due to the fact that the difference of cost and incentives are natural net benefit that can easily be represented as utility function. Also, the rationality assumption of game theory that every players try to maximize their utility tends to fit exactly the situations in P2P system. However, the system dynamics and interaction mode in real P2P systems affect this modeling. For instance, in storage sharing, CPU sharing networks, each peers contribution level affects the global resource availability. Hence, if total contribution is optimal, further contribution of the same resource would yield less utility. Generally, Utility functions may vary with time, due to variation in cost and benefit (Wang et al, 2004). Furthermore, the presence of Altruists in the system also affect the rationality assumptions.

Altruists are peers that contribute to the common good irrespective of their gain from the system. All these factors such as cost and benefit variation, peer variational behaviour and decisions under uncertainties poses challenges to the design of utility functions.

MODEL IMPLEMENTATIONS

Despite increase popularity in the use of P2P systems, there are only few general definition of P2P in the literature (Stoica *et al,* 2003) This is due to the fact that P2P networks are more of a concept (Roussopoulos *et al*, 2004). Currently, all P2P can be classified based on structure as structured and unstructured P2P systems. Structured systems are mostly DHTs based (Distributed Hash Tables), where the keys and locations of resources to be shared are predetermined and indexed into a known location. Searching for resources in this type of network is not difficult. A good example is chord (Stoica *et al,* 2003). But unstructured P2P system, which is the most common is further divided into centralized, hybrid and pure unstructured. In centralized unstructured like Napster, the server is used for indexing. In hybrid unstructured, there exist overlay nodes. Examples are KaZaA and BitTorrent. While pure unstructured is without any centralized or overlay nodes. Searching for resources in this type of systems is through flooding. Example is Gnutella network.

All existing P2P systems are based on any of this architecture. Our proposed analytical model can be implemented on a centralized or hybrid based system.

CONCLUSION

In this paper, we propose an analytical framework for incentive in P2P systems. Second we illustrate the models with the aid of examples and solved the examples with Microsoft Excel Solver. The results prove to be promising in the analysis of resources optimization in P2P networks. Furthermore, as future work, we plan to investigate the suitability of the proposed model in tackling the problem of free riding in P2P systems.

REFERENCES

Adar, E and Huberman, B.A. (2000) “Free Riding on Gnutella," First Monday, pp. 1-22.

Asvanund, A, Karen, K. Ramayya, C .and Micheal, D. S. (2004) “An Empirical Analysis of Network externalities in Peer-to-Peer Music Sharing networks," Information System Research, vol. 15, no. 2, pp. 155-174.

Azzedin, F. and Yahaya, M. O. (2014) “Analyzing and modeling BitTorrent: A game theory approach. High Performance Computing & Simulation (HPCS)”, 2014 International Conference on.

Belmonte, M.-V., ManuelDaz, J.-L. and Cruz A. (2013). "COINS: COalitions and INcentiveS for Effective Peer-to-Peer Downloads." J. Netw. Comput. Appl. 36(1): 484-497. Online]. Available: http://dx.doi.org/10.1016/j.jnca.2012.04.

Cohen, B. (2003) “Incentives Build Robustness in BitTorrent," in P2PECON Workshop Berkeley.

Golle, P., Leyton-brown, K. , Mironov, I. and Lillibridge, M. (2001) “Incentives for Sharing in Peer-to-Peer Networks," Electronic Commerce,,

Gupta, R. and Somani, A. K. (2005) “Game theory as a tool to strategize as well as predict nodes behavior in peer-to-peer networks," in Proceedings of the 11th International Conference on Parallel and Distributed Systems - Volume 01, ser. ICPADS '05. Washington, DC, USA: IEEE Computer Society, pp. 244-249. [Online]. Available: http://dx.doi.org/10.1109/ICPADS.2005.157

Hua, J. S., Huang, D. C. Yen, S. M. and Chena, C. W. (2012) “A dynamic game theory approach to solve the free riding problem in the peer-to-peer networks," Journal of Simulation, vol. 6, pp. 43-55.

Krishnan, R and Smith, M. D. (2008) “The Virtual Commons: Why FreeRiding Can Be Tolerated in File Sharing Networks," In International Conference on Information Systems,.

Roussopoulos, M., Baker, and Rosenthal D. S. H. (2004). "2 P2P or Not 2 P2P ?" IPTPS'04 Proceedings of the Third international conference on Peer-to-Peer Systems: 1-6.

Saroiu, S., Gummadi, K. P., Dunn, R.J., Gribble, S. D., Levy, H.M, (2002). "An analysis of internet content delivery systems." Proceedings of the 5th symposium on Operating systems design and implementation - OSDI '02: 315. New York, USA: ACM Press, 2002, p. 315. [Online]. Available: http://portal.acm.org/citation.cfm?doid=1060289.1060319

Silverston, T. and Fourmaux, O. (2008). "Towards an incentive mechanism for peer-to-peer multimedia live streaming systems." 8th International Conference on Peer-to-Peer Computing,: 6-9. [Online]. Available: http://ieeexplore. ieee.org/xpls/absn all.jsp?arnumber=4627269

Stoica, I., Morris, Karger, R. D. Kaashoek, M. F and Balakrishnan, H. (2003). "Chord : A Scalable Peer-to-peer Lookup Protocol for Internet Applications." IEEE Transactions on Networking 11: 1-14.

Tsai, K. Ho, K. and Weimin, H. (2012) “Tolerant Tit-for-Tat and Fibonacci Transmission Scheme," in Recent Advances in Computer Science and Information Engineering, Q. Zhihong, C. Lei, and H. Y. Weilian Su, Tingkai Wang, Eds. Springer-Verlag Berlin Heidelberg 2012, vol. 4.

Wang, W. and Li, B (2003). "To Play or to Control: a Game-based Control-theoretic Approach to Peer-to-Peer Incentive Engineering." In International Workshop on Quality of Service: 174-192.

Zhang, K. and Antonopoulos, N. (2013). "A novel bartering exchange ring based incentive mechanism for peer-to-peer systems." Future Gener. Comput. Syst. 29(1): 361-369 [Online]. Available: http: //dx.doi.org/10.1016/j.future.2011.06.005

Zhao, B. Q., Lui J. C. S. and Chiu, D.M. (2012). "A mathematical framework for analyzing adaptive incentive protocols in P2P networks." IEEE/ACM Trans. Netw. 20(2): 367-380[Online]. Available: http://dx.doi.org/10.1109/TNET.2011.2161770.