Science World Journal Vol 9 (No 3) 2014
www.scienceworldjournal.org
ISSN 1597-6343

Full Length Research Article

APPLICATION OF SECOND VERTICAL DERIVATIVE ANALYTICAL METHOD TO BOUGUER DATA FOR THE PURPOSE OF DELINEATION OF LITHOLOGICAL BOUNDARIES

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ABSTRACT
Formulation by means of which the second vertical derivative (SVD) of gravity field may be computed at any point of gravity observation was carried out. In order to calculate the second vertical derivative of the gravity data, Fast Fourier Transform (FFT) was applied to the gravity data of Gusau area,North West, Nigeria. The second vertical derivative was computed by multiplying the Bouguer gravity value of each station by its weighting coefficient and using the upward continuation technique as a noise-filter to reduce the noise in the data. The resultant SVD gravity map forms elongated zero contours which correspond to the edges of local geologically anomalous density distributions features. The zero milliGal per metre square (0 mgal/m^2) quantity coincides remarkably well with most lithological boundaries when compared with the major geologic contacts. Thus a properly designed second vertical derivative map, in addition to enhancing weaker local anomalies, can be a supplement to geologic mapping in the identification of lithological units.

KEYWORDS: Formulation, gravity, second vertical derivative, filter, analytical separation, gusau

INTRODUCTION
The second vertical derivative (SVD) technique is one of several methods of removing the regional trend. Some gravity anomalies may be distinct on examination of the Bouquer map, while other weak anomalies arising from sources that are shallow and limited in depth and lateral extent may be obscured by the presence of stronger gravity effects associated with deeper features of larger dimensions. The main function of the second vertical derivative map is to accentuate shallow features at the expense of deep features. The application of the second vertical derivative in gravity interpretation to enhance localized small and weak near-surface features (i.e. improving the resolving power of the gravity map) has long been established (Baranov, 1975; Gupta and Ramani, 1982). The gravity measurements can directly be transformed to vertical gravity gradients. The basic concept of this process of anomaly separation is similar to filtering process in optical or electrical systems. However, it is often more realistic to consider the desired residual as random noise or frequencies. Thus the passband is not well defined except in the simplest cases and there is inevitably some bias involved in the process of removing the regional effect. Gusau was selected as a study area because of availability of gravity data in order to demonstrate the capabilities of the SVD method in boundary detection. The area covered approximately 7,200 km^2, has relatively good road networks and lies between longitudes 6^o 15' and 7^o 00' E and latitudes 12^o 00' and 12^o 40' N.

The detailed geological map of the study area is shown in Figure 1. The area is entirely underlain by rocks of the Nigerian basement complex. Fairly extensive geological work has been carried out in the northwestern part of Nigeria which covers the area of study (Ogezi, 1988; Fitches et al. 1985; Turner, 1983 and Grant, 1978 among others). It is represented centrally by the Bungudu migmatites and Bunji migmatite granite-gneiss and is bounded to the east and northeast by the Wonaka - Chafe schist belt, and to the west by the Maru schist belt. Between 30 - 40% of the survey area is underlain by Older Granite plutons, mainly minor bodies less than 1km across (Fitches et al., 1985).

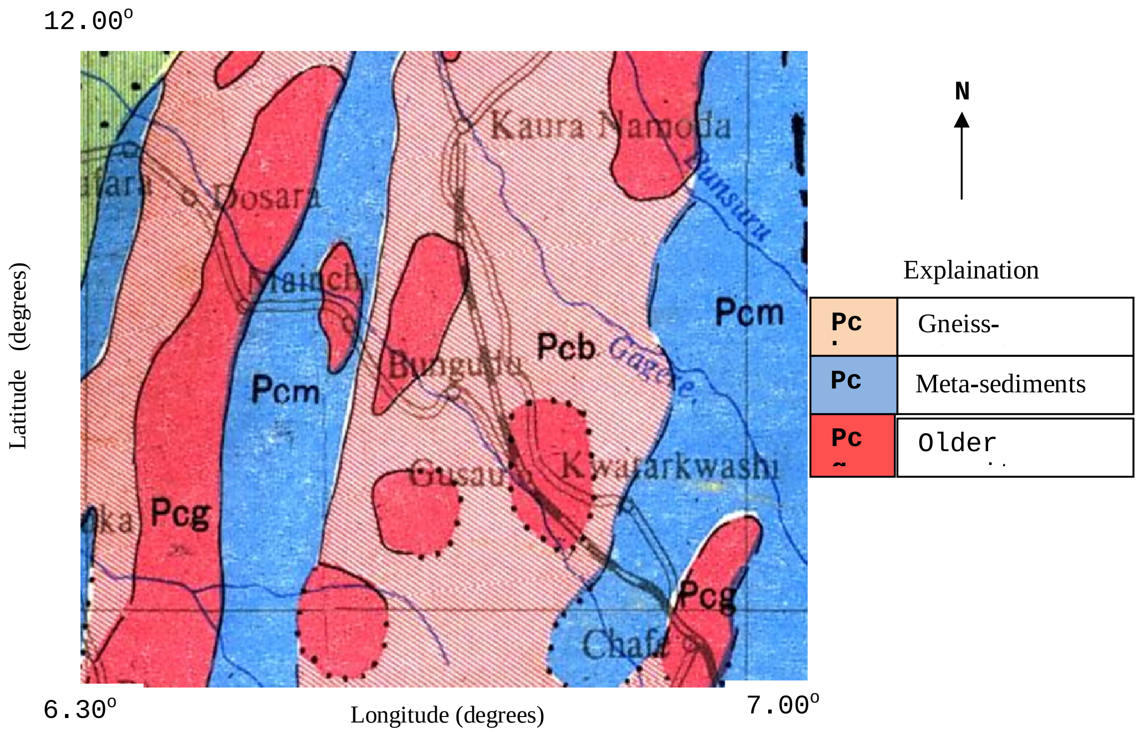

Fig.1 Geology Map of Study Area **(GSN, 1994)**

SVD FORMULATION

The second vertical derivatives enhances near-surface effects than it does the deeper sources. Therefore, there should be a relationship between a second derivative map and a residual map. Second derivatives are a measure of curvature; large curvatures are associated with shallow or residual anomalies. The regional trend which is considered to be the average value of gravity in the vicinity of the gravity station is obtained by averaging observed gravity values on the circumference of a circle of radius r centred on the station . Mathematically the average value of gravity is geven by

$$\bar{g}(r) = \frac{1}{2\pi}\int_0^{2\pi} g(r,\theta)d\theta \qquad (1)$$

Consider a function g(x,y,z) which is harmonic, that is it has continous second derivatives and satisfies Laplace equation. Let the plane z = 0 be the datum plane of the gravity map. Then equation (1) becomes

$$\bar{g}(r) = \frac{1}{2\pi}\int_0^{2\pi} g(rcos\theta, rsin\theta)d\theta \qquad (2)$$

The integral in equation (2) can be replaced by summation over discrete points on a circle radius r whose centre is the gravity station with gravity value g_s . The circle is drawn on plane z = 0 and equation (2) can be rewritten as a power series which converges about r = 0. That is

$$\bar{g}(r) = a_0 + a_2 r^2 + a_4 r^4 + \ldots\ldots \qquad (3)$$

Note that the series is also harmonic, the oodd powers of r are absent because in equation (2) we would have terms of the form $sin^m\theta cos^n\theta$ multiplied by r^{m+n} and $\int_0^{2\pi} sin^m\theta cos^n\theta d\theta = 0$ if $m + n$ is odd.

Laplace equation can be written as

$$\frac{\partial^2 g}{\partial z^2} = -\left(\frac{\partial^2 g}{\partial x^2} + \frac{\partial^2 g}{\partial y^2}\right)$$

$$\therefore \int_0^{2\pi}\frac{\partial^2 g}{\partial x^2}d\theta = -\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}\right)\int_0^{2\pi} g d\theta - \frac{1}{r^2}\int_0^{2\pi}\frac{\partial^2 g}{\partial \theta^2}d\theta$$

But $\frac{1}{r^2}\int_0^{2\pi}\frac{\partial^2 g}{\partial\theta^2}d\theta = \left[\frac{1}{r^2}\left(\frac{\partial g}{\partial\theta}\right)\right]_0^{2\pi} = 0$ because derivative is equal to series having terms in θ of the form $sin^m\theta cos^n\theta$, where m and n are non-zero integers.

$$\therefore \int_0^{2\pi}\frac{\partial^2 g}{\partial z^2}d\theta = -\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}\right)\int_0^{2\pi} g d\theta \qquad (4)$$

From equation (2), $\int_0^{2\pi}\frac{\partial^2 g}{\partial z^2}d\theta = -2\pi\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}\right)\bar{g}(r)$

i.e $\frac{1}{2\pi}\int_0^{2\pi}\frac{g(rcos\theta,\ rsin\theta,\ z)}{\partial z^2}d\theta = -\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}\right)\bar{g}(r) \qquad (5)$

$$\frac{1}{2\pi}\int_0^{2\pi}\left[\frac{g(rcos\theta,\ rsin\theta,\ z)}{\partial z^2}\right]_{\substack{r=0\\z=0}} d\theta$$

$$= \lim_{r\to 0}\frac{\partial^2}{\partial z^2}\left[\frac{1}{2\pi}\int_0^{2\pi} g(rcos\theta, rsin\theta)d\theta\ \right]_{z=0}$$

$$= \lim_{r\to 0}\left(\frac{\partial^2 g}{\partial z^2}\right)_{z=0} = \left(\frac{\partial^2 g}{\partial z^2}\right)_{\substack{r=0\\z=0}}$$

$$= \left[-\left\{\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}\right)\right\}(a_0 + a_2 r^2 + a_4 r^4 + \ldots)\right]_{r=0}$$

$$= -4\,a_2$$

i.e. $\frac{1}{2\pi}\int_0^{2\pi}\left[\frac{g(rcos\theta,\ rsin\theta,\ z)}{\partial z^2}\right]_{\substack{r=0\\z=0}} d\theta = -4\,a_2 \qquad (6)$

Now if we differentiate equation (3) with respect to r_2 in the vicinity of r = 0, we get

$$\frac{\partial \bar{g}}{\partial (r^2)} = a_2 \qquad (7)$$

Equation (7) suggests a graphical method of obtaining $\frac{\partial^2 g}{\partial z^2}$ for the station at $r = z = 0$.

$g(r)$ can be determined for several radii and the values plotted against r^2. A more accurate determination of $\frac{\partial^2 g}{\partial z^2}$ can be obtained by using several concentric circles (generally three circles) of different radii. Thus

$$\frac{\partial^2 g}{\partial z^2} = \frac{K}{S^2}(K_0 g_0 + K_1 \bar{g}_1 + K_2 \bar{g}_2 + \ldots) \qquad (8)$$

where g_0 is the station gravity (at the centre of the circle), $\bar{g}_1, \bar{g}_2, \ldots$, $\bar{g}_n$ are average gravity values on successive circles, $K_0, K_1, K_2, \ldots, K_n$ are weighty coefficients such that $\sum_{i=0}^{n} K_i = 0$

K is a numerical factor , S is a distance, usually the station spacing in the grid. Convinient radii are, S, $\sqrt{2S}$, $\sqrt{5S}$ etc. with $r_0 = 0$

A major setback in using this method is that the coefficients used in equation (9) vary from one individual to another, thus introducing some bias (Baranov, 1975).

DATA AND SECOND VERTICAL DERIVATIVES COMPUTATION

The data employed in this work were acquired using a network of 4 base stations that enabled corrections made for the drifts of instrument. A density reduction of 2.67 g/cm3 was used for the Bouguer correction and all the data were tied to the first order gravity network base station Number 990476 at Gusau with a value of 978102.755 mGals (Osazuwa, 1985 and Aku, 1996). Bouquer anomalies were computed using the Geodatic Reference System 1967 (GRS 67). The Bouguer map is shown in Fig. 2

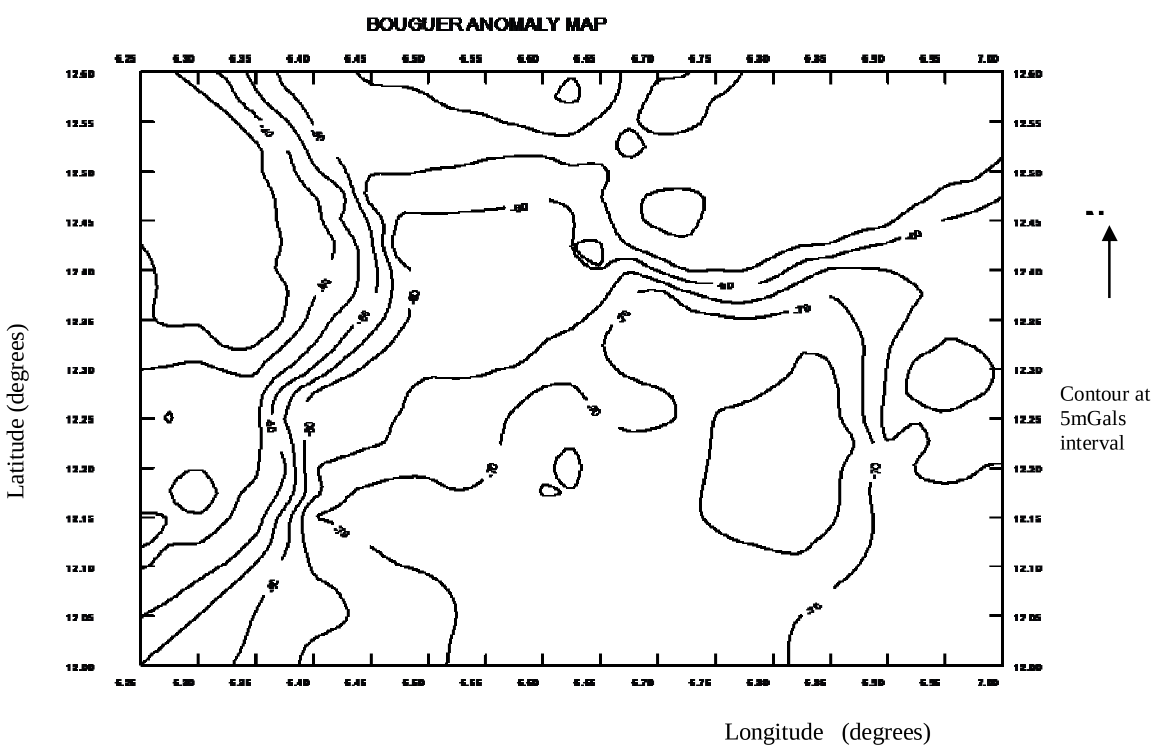

Fig. 2: Bouguer anomaly map of study area

SVD COMPUTATION

The second vertical derivative method of analytical regional – residual separation was applied to the available reduced data of the study area in order to calculate the second vertical derivative using Fast Fourier Transform (FFT). The second vertical derivative was computed by multiplying the Bouguer gravity value of each station by its weighty coefficient (Bhattacharyya, 1969). The derivatives were obtained with a computer program (Matlab & Simulink), which employs the method that carries out using equations 1 to 8 in the formulation above. The upward continuation technique was used as a noise-filter to reduce the noise. The second derivative technique utilizes Wiener theory to design filters from an analysis of the power spectrum to suit the degree of noise in the gravity data (Pennington, 1965).

The various gravity gradient components are computed by taking the horizontal x- and y-derivatives and vertical z-derivative of each of the three gravity components by Laplace's equation. Much interest was aimed to achieve a derivative map that contains the correct balance between the enhancement of signal and noise.

RESULT AND DISCUSSION

The second vertical derivatives of gravity data calculated using the Fast Fourier Transform (FFT) resulted in the second vertical derivative gravity map shown in Fig.3. It is an enhanced anomaly or residual map related to the "curvature" of the input gravity data. Contouring and surface mapping software, surfer 8 was used to produce the SVD gravity map.

The quantity zero milliGals per metres square (0 mgal/m²) should indicate the edges of local geological features and so the zero contours represent generated lithological boundaries.

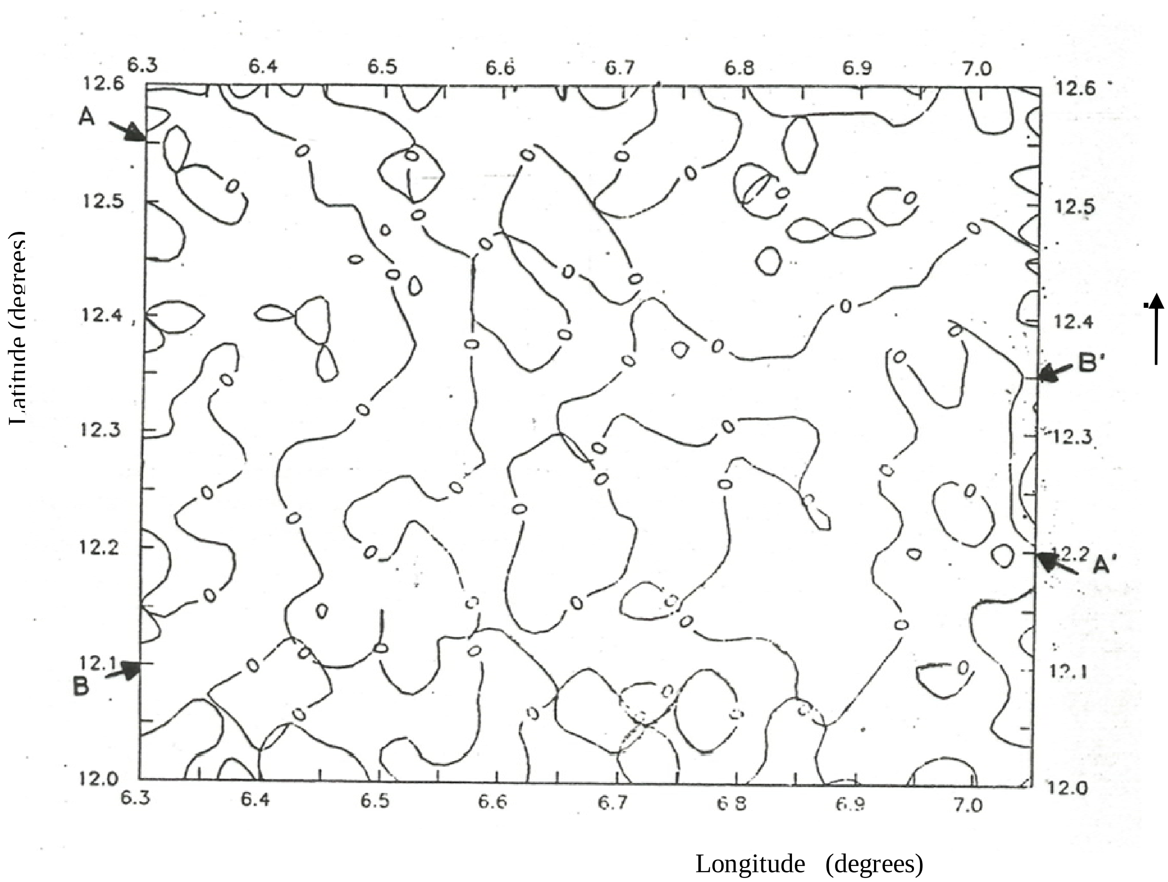

Fig. 3: Second Vertical Derivative (SVD) Gravity Map of Study Area

The resultant (SVD) gravity map corresponds remarkably well when compared with the major geologic contacts; the elongated zero contours coinciding with most lithological boundaries (Fig. 4). In particular, the Older Granite boundaries with the schist belts and granite-gneiss which represent a large mean density contrast are outlined by the zero contours.

The absence of large number closures in the central region of the SVD map could be supportive evidence that the granite plutons between the Chafe –Wonaka schist belts coalesce at depth into a single body (Aku, 1996).

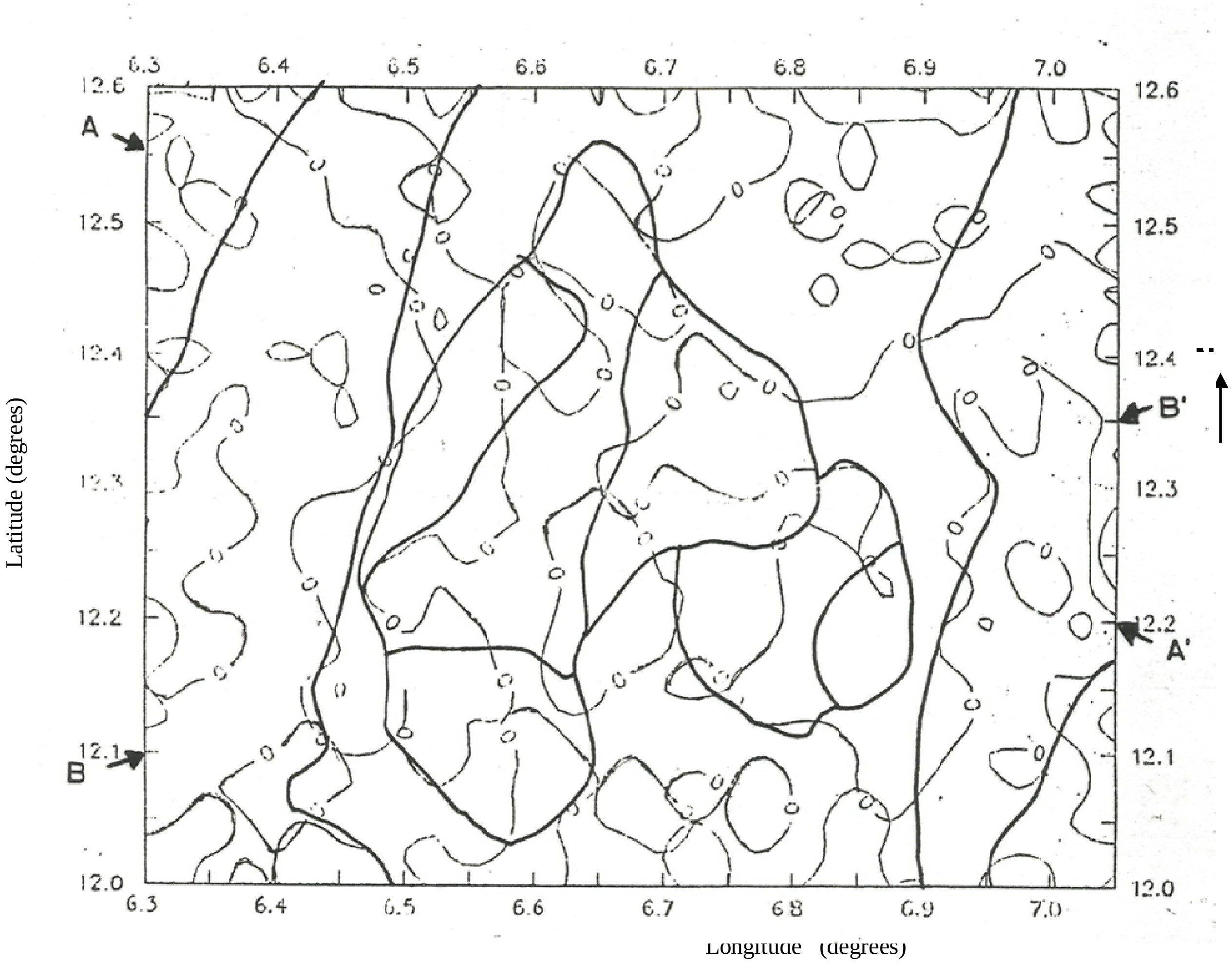

Fig. 4: SVD Map Superimposed on Geology Boundaries of Study Area

The SVD map tends to emphasize local anomalies and isolate them from the regional background. The SVD enhances nearsurface effects at the expense of deeper anomalies. Many transformations were applied in order to minimize the noise and enhance the gravity data

SUMMARY AND CONCLUSION

Formulation by means of which the second vertical derivative (SVD) of gravity field may be computed at any point of observation was carried out. In order to calculate the second vertical derivative of the gravity data, Fast Fourier Transform (FFT) was applied to the gravity data. The second vertical derivative was computed by multiplying the Bouguer gravity value of each station by its weighty coefficient. The upward continuation technique was used as a noise-filter to reduce noise in the data. With the aid of a simple computer program, the formulation was demonstrated on gravity data of Gusau area to produce a second vertical derivative gravity map of the area using contouring and mapping software, Surfer.

Applying the SVD analytic method in this study shows that it is useful and a complementary tool in the analysis of complex geological structures. The analytic SVD method defines the edges of geologically anomalous density distributions. Thus a properly designed second vertical derivative map, in addition to enhancing weaker local anomalies, can be a supplement to geologic mapping in the identification of lithological units. It is easy to understand the complex pattern of gravity gradients if one considers the fact that they are derivable from the simple gravitational potential, being the directional second derivatives of the potential.

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