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Full Length Research Article

NUMERICAL SOLUTIONS OF SOME PARAMETRIC EFFECTS DUE TO ELECTROMAGNETIC WAVE SCATTERING BY AN INFINITE CIRCULAR CYLINDER

*[1] Suleiman A. B. and [1] Zarma S. S.

[1] Department of Physics, Federal University, Dutse P. M. B 7156, Dutse Jigawa State Nigeria.

*Corresponding author email: suleiman_abdussalam@yahoo.com , Phone: +2348039644595

ABSTRACT
The analytical solutions for the scattering of electromagnetic waves from an infinite circular cylinder and refractive index are programmed in FORTRAN (Barber and Hill, 1990). The resulting quantities include the scattering coefficients, the scattering amplitudes and the intensities. The range of variable input parameters (Aperture, size parameters, scattering intensities, efficiencies and inverse size parameters) are discussed and presented. Their respective effects were also presented. At 0o, alternate resonance was observed. The alternate resonances disappear at 90o. The resonances are also emphasized at 180o while the intensity for TM (Transverse Magnetic) polarization is continuous across the surface and that of TE (Transverse Electric) polarization is discontinuous.

Keywords: Parametric Effect, Electromagnetic Wave Scattering, Infinite Circular Cylinder, Transverse Magnetic, Transverse Electric.

INTRODUCTION

Scientific investigations in the field of electromagnetic wave scattering and radiation from objects with loaded impedance have been of greater interest in the past (Sebak, A R, 2000), (Anderson. M. G. 1965) and (Alexopoulos, N.G et al, 1974). The scattering from an infinite elliptical metallic cylinder coated by a circular dielectric has been investigated (Kakogiannos and Roumeliotis, 1990).

There are naturally occurring particles such as some viruses, asbestos fibers, collagen fibers and ice cloud which are best represented as cylinders long compared with their diameter. So, the study of the scattering of light by cylindrical particle will provide information about the effect of viruses, collagen fibers and stem of trees on the propagation of electromagnetic waves. For example, experiment show that Mie scattering from collagen fibers dominates scattering in the infrared wavelength range (Liou, 1972).

The geometry is shown in fig (1.0). The z- axis is along the axis of the cylinder, the x- axis is in the plane containing the z- axis and the direction of incidence, and the y- axis perpendicular to this plane. The coordinates r and θ are defined by:

$$x = r\cos\theta \qquad y = r\sin\theta \tag{1}$$

Boundary conditions

Within the context of electromagnetic scattering principles different types of boundary conditions are applied to characterize scattering from different objects of varying materials. In this case an electromagnetic field is required to satisfy the Maxwell equations at points where E and H are continuous. However, as one crosses the boundary, between particle and medium, there is in general a sudden change in these properties. This change occurs over a transition region with thickness of the order of atomic dimensions. From a macroscopic point of view, therefore, there is a discontinuity at the boundary. At such boundary points we impose the following conditions on the fields.

$$\left.\begin{aligned} E_{\phi^i} + E_{\phi^s} = E_{\phi^t}, \quad H_{\phi^i} + H_{\phi^s} = H_{\phi^t} \\ Z_{z^i} + Z_{z^s} = Z_{z^t}, \quad H_{z^i} + H_{z^s} = H_{z^t} \end{aligned}\right\} at\ r = a \tag{2}$$

where are electric field, magnetic field and characteristic impedance.

The boundary conditions above are the requirements that the tangential components of E and H are continuous across a boundary separating media with different properties.

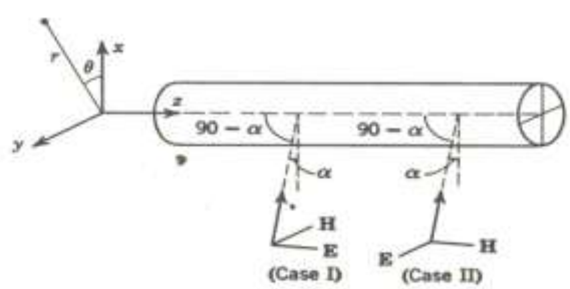

Fig. 1: Geometry of the infinite circular cylinder: showing coordinate and orientation of the vectors used in the cylinder.

Theoretical Background

For a periodic electromagnetic field with a circular frequency Maxwell equations can be expressed as:

$$\nabla^2 \vec{H} - \mu \in \frac{\delta^2 \vec{H}}{\delta t^2} - \mu\sigma \frac{\delta \vec{H}}{\delta t} = 0$$

$$\nabla^2 \vec{E} - \mu \in \frac{\delta^2 \mathrm{\vec{E}}}{\delta t^2} - \mu\sigma \frac{\delta \mathrm{\vec{E}}}{\delta t} = 0 \quad (3)$$

The electric and magnetic field vectors in a homogenous medium satisfy the following vector wave equations:

$$\nabla^2 \vec{A} + k^2 m^2 \vec{A} = 0 \quad (4)$$

where k is the wave number in vacuum and is the complex refractive index of the scattering medium

If ψ satisfies the scalar wave equation

$$\nabla^2 \psi + k^2 m^2 \psi = 0 \quad (5)$$

then the scalar wave equation (5) in cylindrical coordinate can be expressed as:

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{d\psi}{dr}\right) + \frac{1}{r^2}\frac{d^2\psi}{d\phi^2} + \frac{d^2\psi}{dz^2} = \rho^2 \quad (6)$$

where ρ is a constant.

This equation is separable by letting

$$\psi(r,\phi,z) = R(r)\Phi(\phi)Z(z) \quad (7)$$

On substitution into (6) and dividing through by

$\psi(r,\phi,z) = R(r)\Phi(\phi)Z(z)$,

we have

$$\frac{1}{rR}\frac{d}{dr}\left(r\frac{dR}{dr}\right) + \frac{1}{r^2\Phi}\frac{d^2\Phi}{d\phi^2} + \frac{1}{Z}\frac{d^2Z}{dz^2} - \rho^2 = 0 \quad (8)$$

The second to the last term depends only on z and the first and second (taken together) depends only on r and ϕ. Taking the separation constant to be a_z^2 we have

$$\frac{1}{Z}\frac{d^2Z}{dz^2} = a_z^2 \quad (9)$$

This follows that equation (2.16) becomes

$$\frac{1}{rR}\frac{d}{dr}\left(r\frac{dR}{dr}\right) + \frac{1}{\Phi r^2}\frac{d^2\Phi}{d\phi^2} + a_z^2 - \rho^2 = 0 \quad (10)$$

Multiplying equation (8) by r^2 we obtain

$$\frac{r}{R}\frac{d}{dr}\left(r\frac{dR}{dr}\right) + \frac{1}{\Phi}\frac{d^2\Phi}{d\phi^2} + \left(a_z^2 - \rho^2\right)r^2 = 0 \quad (11)$$

Observe that the second term depends only on Φ and the other terms depends only on r. Taking the second separation constant to be $-\gamma^2$, we find

$$\frac{1}{\Phi}\frac{d^2\Phi}{d\phi^2} = -\gamma^2 \quad (12)$$

where γ is a separation constant.

So that

$$\Phi(\phi) = C\cos m\phi + D\sin m\phi$$

Replacing the second term by $-m^2$ and multiply by R we get

$$\frac{r}{dr}\frac{d}{dr}\left(r\frac{dR}{dr}\right) + \left[\left(a_z^2 - \rho^2\right)r^2 - \gamma^2\right]R = 0 \quad (13)$$

This is a Bessel equation whose solutions are the cylindrical Bessel function (Riley and Horbson, 2006).

$$z_n\left(r\sqrt{\gamma^2\rho^2 - h^2}\right)e^{-ihz} \quad (14)$$

Since within a homogenous, isotropic medium every electromagnetic field may be represented by a linear combination of wave functions, then solution of eqn. (14) can be written in the form

$$\psi_n = e^{in\phi} Z_n[r\sqrt{(\gamma^2\rho^2 - h^2)}]\ e^{-ihz+i\omega t}, \quad (15)$$

where h is an arbitrary constant, n is an integer, and Z_n is any Bessel function of order n.

The components of the vectors $\vec{M}$ and $\vec{N}$ for the solution of this form are:

$$\mathrm{M_r} = -\frac{\mathrm{in}}{\mathrm{r}}\psi, \quad \gamma\rho \mathrm{N_r} = -\mathrm{ih}\frac{\partial\psi}{\partial \mathrm{r}},$$

$$\mathrm{M}_\phi = -\frac{\partial\psi}{\partial \mathrm{r}}, \quad \gamma\rho \mathrm{N}_\phi = -\frac{\mathrm{nh}}{\mathrm{r}}\psi, \quad (16)$$

$$\mathrm{M_z} = 0, \quad \gamma\rho \mathrm{N_z} = (\gamma^2\rho^2 - \mathrm{h}^2)\psi.$$

Suppose that u and v are two orthogonal solutions of equation (13). Then Maxwell eqns. (16) are satisfied by the field vectors defined by

$$\left.\begin{aligned}\vec{E} &= \vec{M}_v + i\vec{N}_u \\ \vec{H} &= \gamma\left(-\vec{M}_u + i\vec{N}_v\right)\end{aligned}\right\} \quad (17)$$

A plane wave traveling in vacuum in the direction as indicated in Fig. 1.0 can be represented by the scalar wave function

$$\psi_o = e^{i\omega t - i\rho(x\cos\alpha + z\sin\alpha)}$$

Or

$$\psi_o = e^{i\omega t - ilx - ihz} \quad (18)$$

where $h = k\sin\alpha$ and $l = k\cos\alpha$. The corresponding vectors M and N are derived from the curl in rectangular coordinate. Hence,

The components of $\vec{\mathrm{M}}_\psi = \vec{\nabla}\mathrm{x}(\hat{\mathrm{a}}_\mathrm{z}\psi)$ and $\gamma\rho\vec{N}_\psi = \vec{\nabla}\times\vec{M}_\psi$ in rectangular coordinates can be written as

$$\mathrm{M_x} = 0, \quad \rho\mathrm{N_x} = -hl\psi_o,$$

$$\mathrm{M_y} = il\psi_o, \quad \rho\mathrm{N_y} = 0, \quad (19)$$

$$M_z = 0 \quad \rho N_z = l^2\psi_o$$

This solution may be used to construct a plane electromagnetic wave traveling in the same direction.

Polarization States

As for the problem of polarization, we shall consider two cases separately. One is when the electric vector $\vec{E}$ vibrates parallel to the plane containing the direction of the incident ray and the z

axis (case I), while the other is when the electric vector $\vec{E}$ vibrates perpendicular to this plane (case II). At Perpendicular incidence $\alpha = 0$ .This follows that, $l = \rho$ and $h = 0$.

Also at boundary conditions the function u and v are no longer mixed.

Thus the assumption that $v = 0$ for the incident wave in case I entails that it is 0 also for the scattered wave.

Writing

$$F_n = e^{in\theta + i\omega l}(-1)^n \qquad (20)$$

and combining at once the incident and scattered waves in one formula, we have (Van de Hulst, 1969)

Case I, $v = 0 \ (E // axis)$

$$\left.\begin{aligned} &(r > a) \quad u = \sum_{n=-\infty}^{\infty} F_n\left(J_n(\rho r) - b_n H_n(\rho r)\right), \\ &(r < a) \quad u = \sum_{n=-\infty}^{\infty} F_n d_n J_n(\gamma \rho r), \end{aligned}\right\} \qquad (21)$$

$$(\mathrm{r} = \mathrm{a}) \ \ \mathrm{mu \ and \ m}\frac{\partial \mathrm{u}}{\partial \mathrm{r}} \ \ \text{are continous.}$$

Case II, $u = 0. \ (H // axis)$

$$\left.\begin{aligned} &(r > a) \quad v = \sum_{n=-\infty}^{\infty} F_n\{J_n(\rho r) - a_n H_n(\rho r)\}, \\ &(r < a) \quad v = \sum_{n=-\infty}^{\infty} F_n c_n J_n(\gamma \rho r), \end{aligned}\right\} \qquad (22)$$

$$(\mathrm{r} = \mathrm{a}) \ \ \mathrm{m^2 v \ and} \ \frac{\partial \mathrm{u}}{\partial \mathrm{r}} \ \ \text{continous.}$$

Solutions for the coefficients give:

$$\left.\begin{aligned} &Case \ \ I: \ \ b_n = \frac{mJ_n'(y)J_n(x) - J_n(y)J_n'(x)}{mJ_n'(y)H_n(x) - J_n(y)H_n'(x)}, \\ &Case \ \ II: \ \ a_n = \frac{J_n'(y)J_n(x) - mJ_n(y)J_n'(x)}{J_n'(y)H_n(x) - mJ_n(y)H_n'(x)}, \end{aligned}\right\} \qquad (23)$$

where primes denotes derivatives and $x = \rho a, \ y = \gamma \rho a.$

Computational procedures

The calculations performed in this work were done using a FORTRAN program developed by Barber and Hill, (1990). The Mie functions were computed using the improved algorithm of Cai and Shen (2005) in collaboration with the program mentioned above. The program consists of a MAIN routine and two subroutines. Subroutine CYLDR calculates the scattering coefficients and subroutine BESH calculates Bessel function of the first and second kind.

Main Routine

(a) Since an e^{-iwt} time variation is used in the derivation of the scattering solution, the complex index of refraction is given by $m = m' + im''$. Program inputs are m' and m'' .Therefore m'' should be entered as a positive number for absorption.

(b) The intensity against scattering angle calculation requires that the increment in scattering angle, dlt, be specified as an input. This should be selected so that the ratio $180/dlt$ is an integer because of the way in which the number of scattering angles is calculated. For example, dlt can be 0.1, 0.25, or 2.0, but not .7, or 92.

(c) The intensity against size parameter and intensity against size parameter calculations can be made for increments in size parameter or increments in inverse size parameter. In the second case, the calculations are made in order of increasing inverse size parameter.

CYLDR

(a) The index of the algorithm derivative, Bessel functions, and scattering coefficients are incremented by one to avoid the need for zero subscripts in the program. For example, amat (Van de Hulst, 1969) in the program is the logarithmic derivative A_0 . Comparison of the expression for amat (n) in the do 20 loop with (21) should clarify the relationship.

(b) The downward recursion of the logarithmic derivative is done in two steps to avoid having to store functions for $n > n_c$. These higher index functions are not used in subsequent calculations.

(c) The scattering coefficient calculation obtains b_n and a_n from (23) depending on which polarization is selected.

(d) Calculation of the zeroth mode scattering coefficient requires Bessel and Hankel functions of order -1. These are obtained by using

$$Z_{-1} = -Z_1,$$

where Z represents either of the functions

BESH

(a) Bessel functions of the first kind and second kind are generated using downward recursion and upward recursion, respectively, using the basic recursion relationship, (Gupta, 2010).

$$J_{n-1}(x) + J_{n+1}(x) = \frac{2n}{x} J_n(x) \cdot \qquad (24)$$

(b) The downward recursion for Bessel functions of the first kind begins with two successive functions equal to 0.0 and 10^{-35} and produces functions proportional to the Bessel functions rather than the actual Bessel functions. The constant of proportionality between the two sets of functions is obtained by enforcing the relationship

$$J_e \div 2\sum_{m=1}^{\infty} J_{2m} = 1. \qquad (25)$$

Bessel functions of the second kind are calculated using upward recursion starting with a function of order zero obtained from a series solution and a function of order one obtained from the Wronskian relating Bessel functions of the first and second kinds for orders zero and one (Bohren and Huffman, 2004).

The accuracy of the calculated Bessel functions directly affects the validity of the final calculated scattering results

RESULTS AND DISCUSSION

Effect of Aperture Variation

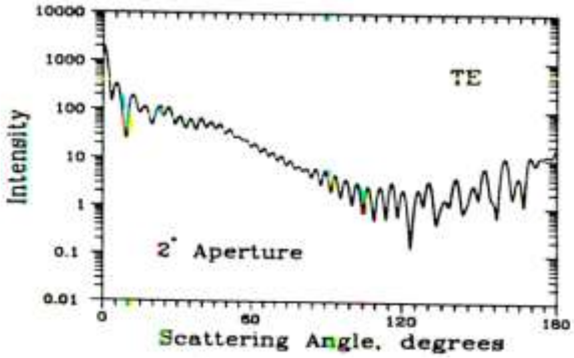

Fig. 2: Angular scattered intensity for a circular cylinder with a size parameter of 50 and an index of refraction of 1.5 for TE polarization and a 2^0 aperture**.**

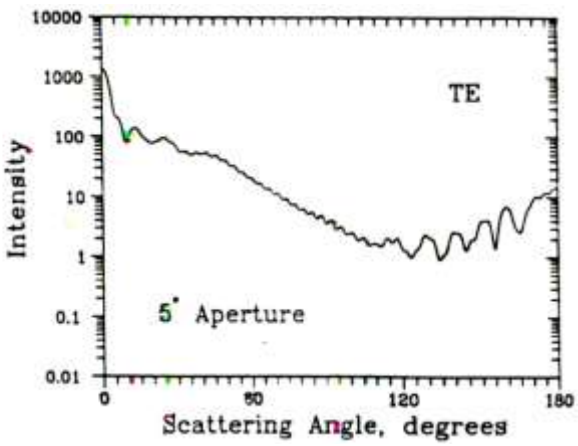

Fig. 3: Angular scattered intensity for a circular cylinder with a size parameter of 50 and an index of refraction of 1.5 for TE polarization and a 5^0 aperture.

Effect of size parameter

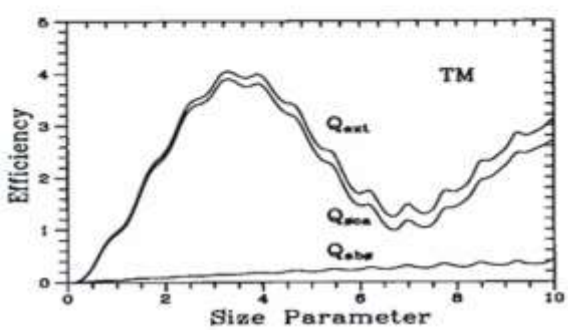

Fig. 4 Extinction Scattering and Absorption Efficiency as a function of size parameter for a circular cylinder with a complex index of refraction m = $1.5 + i0.01$ for TM polarization.

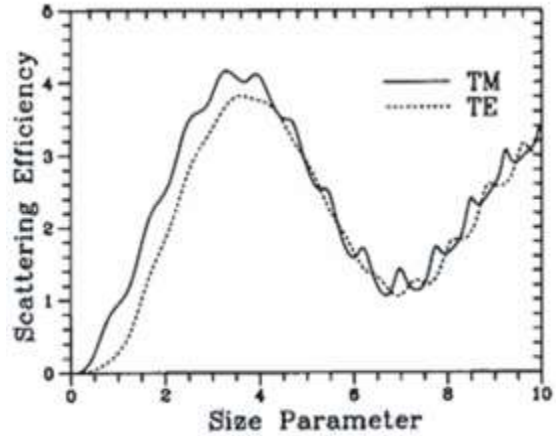

Fig. 5: Scattering efficiency as a function of size parameter for a circular cylinder with an index of refraction of 1.5 for TM and TE Polarization.

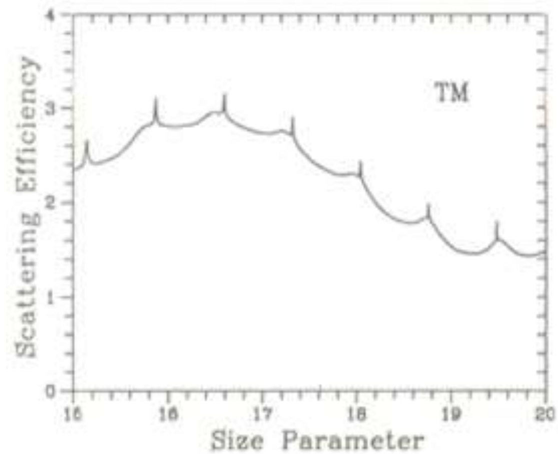

Fig. 6: Scattering efficiency as a function of size parameter for a cylinder with an index of refraction of 1.5 for TM polarization.

Variation of intensity with size parameter

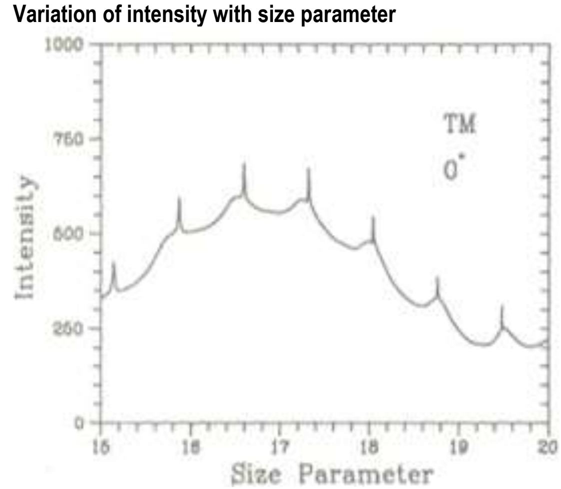

Fig. 7: Scattered intensity at 0^0 as a function of size parameter for a circular cylinder with an index of refraction of 1.5 for TM polarization.

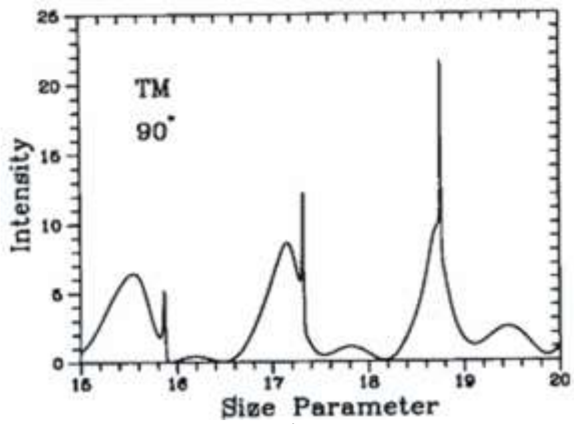

Fig. 8: Scattered intensity at 90^0 as a function of size parameter for a circular cylinder with an index of refraction of 1.5 for TM polarization.

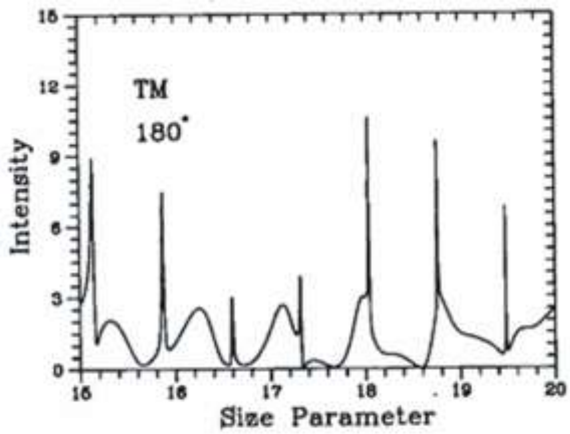

Fig. 9: Scattered intensity at 180^0 as a function of size parameter for circular cylinder with an index of refraction of 1.5 for TM polarization.

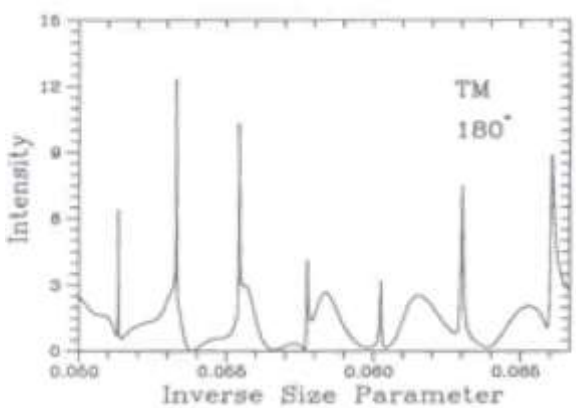

Fig. 10: Scattered intensity at 180^0 as a function of size parameter for a circular cylinder with an index refraction of 1.5 for TM .

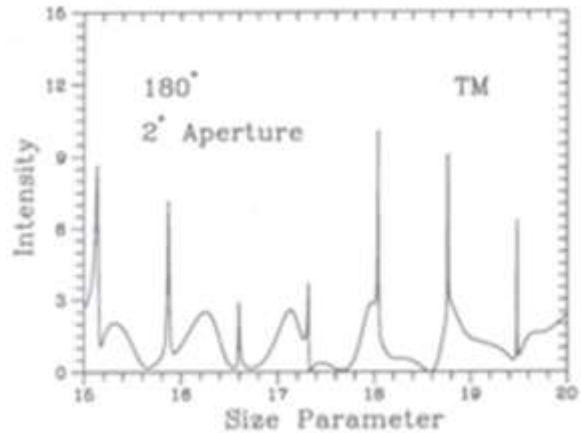

Fig. 11: Scattered intensity at180^0 function of size parameter for a circular cylinder with an index of refraction of m = 1.5 for TM polarization and a 2^o aperture.

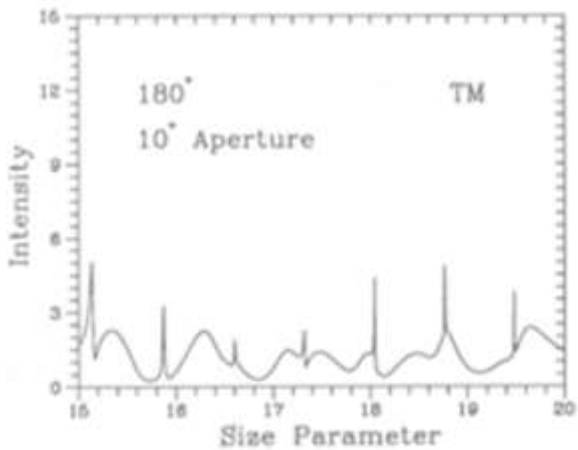

Fig. 12: Scattered intensity at 180^0 as function of size parameter for a circular cylinder with an index of refraction of m = 1.5 for TM polarization and a 10^0 aperture.

CONCLUSION

In this work it was found that the extinction and scattering efficiency factors increase rapidly as the size parameter increases, reaching a peak and then slowly oscillating. The absorbing efficiency rises slowly from zero. The variation of scattered intensities at different angles and degree aperture were calculated, it was found that at 0^0 alternate resonances was observed and disappears at right angles and further emphasized at 180^0. In a nutshell according to the result obtained the detection aperture should be small enough to follow the rapid oscillation of the angular variation and also the detection aperture should be even smaller for cylinders of larger size parameters for which the oscillation of the scattered intensity with scattering angle becomes rapid.

The result generated in this work could be important, as they can be used as benchmarks to validate similar results obtained using other approximate or numerical methods and also to have a better perception on the effects of some quantities related to an infinitely long cylinders undergoing electromagnetic wave scattering.

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