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A COMPARATIVE STUDY OF SOME ROBUST RIDGE AND LIU ESTIMATORS

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ABSTRACT
In multiple linear regression analysis, multicollinearity and outliers are two main problems. When multicollinearity exists, biased estimation techniques such as Ridge and Liu Estimators are preferable to Ordinary Least Square. On the other hand, when outliers exist in the data, robust estimators like M, MM, LTS and S Estimators, are preferred. To handle these two problems jointly, the study combines the Ridge and Liu Estimators with Robust Estimators to provide Robust Ridge and Robust Liu estimators respectively. The Mean Square Error (MSE) criterion was used to compare the performance of the estimators. Application to the proposed estimators to three (3) real life data set with multicollinearity and outliers problems reveals that the M-Liu and LTS-Liu Estimator are generally most efficient..

Keywords: Ordinary Least Squares, Ridge Regression Estimator, Liu Estimator, Robust Estimator, Robust Ridge Regression Estimator, Robust Liu Estimator

1.0. INTRODUCTION

Regression analysis is used to study the relationship between a single variable Y, called the response variable, and one or more explanatory variable(s), $X_1,\ldots,X_p$ by a linear model. The method of Ordinary Least Squares (OLS) estimator of model parameters is best linear unbiased estimator (BLUE) and most efficient under certain assumptions (Gujarati, 2003).
One of the assumptions of Linear Regression model is that of independence between the explanatory variables (i.e. no multicollinearity). Violation of this assumption arises most often in regression analysis. Among methods used in detecting the presence of Multicollinearity is variance inflation factor (VIF). The performance of OLS estimator is inefficient if this assumption is not valid and the regression coefficients have large standard errors and sometimes have wrong sign (Gujarati, 2003). In this situation, many estimators have been proposed to combat this problem among which are: Stein Estimator by Stein (1956), Liu Estimator by Liu (1993) and Ridge Estimator proposed by Hoerl and Kennard (1970).
Other problems in regression analysis include the problem of outlier and leverage points. An outlier is an observation that is distant from other observations. Leverage points are points that appear to be outlying in the regressors. Methods such as studentized deleted residual and Mahalanobis distance are used to detect the presence of outliers and leverage point respectively. Cooks D and DFFITS are often used to determine if either the outliers or leverage points influences the regression coefficients. Robust regression estimator is commonly used to circumvent the problem of outlier. Examples of these estimators are M-estimator proposed by Huber (1964), Least Trimmed Mean (LTS) by Rousseeuw and Van Driessen (1998), S estimator proposed by Rousseeuw and Yohai (1984) and MM estimator by Yohai (1987) among others.
These two problems may jointly exist in regression analysis. This has attracted the attention of some researchers. Holland (1973) proposed robust M-estimator for ridge regression to handle the problem of multicollinearity and outliers. Askin and Montgomery (1980) proposed ridge regression based on the M-estimator. Walker (1984) modified Askin and Montgomery's approach to allow the use of Generalized M estimators instead of M estimators. Simpson and Montgomery (1996) proposed a biased-robust estimator that uses a multistage Generalized M estimator with fully iterated ridge regression. Lukman *et al*. (2014) proposed and applied some Robust Ridge Regression Estimators to the Hussein and Abdalla (2012) data. Alpu and Samkar (2010) applied Liu estimator based on M estimator to a VO2 data.
The aim of this study is to combine Ridge and Liu estimators with some robust estimators to jointly handle the problem of Multicollinearity and outliers. Also, to compare the performances of these combined estimators with their individual counterparts

2.0. MATERIALS AND METHOD

2.1. Ordinary Least Square Estimator

Consider the standard regression model:

$$Y = X\beta + \varepsilon \quad (1)$$

where X is an $n \times p$ matrix with full rank, Y is a $n \times 1$ vector of dependent variable, β is a $p \times 1$ vector of unknown parameters, and ε is the error term such that $E(\varepsilon)=0$ and $E(\varepsilon\varepsilon')=\sigma^2 I$.
Provided $X'X$ is invertible, the OLS estimator is given by

$$\hat{\beta} = (X'X)^{-1}X'y \quad (2)$$

The regression model in equation (1) can be written in canonical form

$$y = Z\alpha + \varepsilon \quad (3)$$

where $Z = XQ$, $\alpha = Q'\beta$ and Q is the orthogonal matrix with columns that constitute the eigenvectors of $X'X$. Then

$$Z'Z = Q'X'XQ = \Lambda = diag(\lambda_1, \lambda_2, ..., \lambda_p),$$

where $\lambda_1 \le \lambda_2 \le ... \le \lambda_p$ are ordered eigenvalues of X'X.

Thus, the Ordinary Least Square Estimator of α is

$$\hat{\alpha}_{OLS} = \Lambda^{-1} Z' y \qquad (4)$$

2.2. Ridge Regression Estimator

If $Z'Z$ matrix is ill-conditioned, (especially when there is a near-linear dependency among the explanatory variables), the OLS estimator of α tends to have a large variance. Ridge parameter is added to the $Z'Z$ matrix to reduce the collinearity effect. Hoerl and Kennard (1970) defined the ridge regression estimator of α as:

$$\hat{\alpha}(k) = (Z'Z + kI)^{-1} Z'y \qquad (5)$$

where k is the ridge parameter. The value of k used in this study is the one proposed by Lukman (2015).

$$K_{AL} = \frac{1}{\sqrt{\frac{\hat{\sigma}^2}{\max(\hat{\alpha}_i^2)}}} \qquad (6)$$

where $\hat{\sigma}^2$ is the estimated MSE calculated as $\hat{\sigma}^2 = \frac{\sum_{i=1}^{n} e_i^2}{n-p}$ and $\hat{\alpha}$ is the estimated regression coefficient.

2.3. Liu Estimator

To overcome multicollinearity, Liu (1993) proposed the Liu Estimator by combining the Stein estimator with Ridge estimator to form

$$\hat{\beta}_L = (X'X + I)^{-1}(X'X + dI)\hat{\beta}_{OLS} \qquad (7)$$

where d is the Biasing parameter and can be computed according to Liu by

$$d_L = 1 - \hat{\sigma}^2 \left[\frac{\sum_{i=1}^{p} \frac{1}{\lambda_i(\lambda_i + 1)}}{\sum_{i=1}^{p} \frac{\hat{\alpha}_i^2}{(\lambda_i + 1)^2}} \right] \qquad (8)$$

where $\hat{\alpha} = Q'\hat{\beta}_{OLS}$ and $\hat{\sigma}^2$ are Ordinary Least Square estimators of α and σ^2 respectively, and Q is the matrix of eigenvectors corresponding to the eigenvalues o the matrix X'X.

Now equation (7) can be written in canonical form:

$$\hat{\alpha}_L = (\Lambda + I_p)^{-1}(\Lambda + d_L I_p)\hat{\alpha}_{OLS} \qquad (9)$$

where I_P is the p-identity matrix

2.4. Robust Regression

2.4.1. M-Estimator

The most common method of robust regression is M-estimation, introduced by Huber (1964). It is nearly as efficient as OLS. Rather than minimizing the sum of squared errors, M-estimate chooses β to minimize

$$\sum_{i=1}^{n} \rho\left(\frac{y_i - x_i^T\hat{\beta}}{\sigma}\right) \qquad (10)$$

Possible choices of ρ are:

$\rho(x) = x^2$ is just least squares and $\rho(x) = |x|$ is called the least absolute deviations regression (LAD).

$$\rho(x) = \begin{cases} x^2/2, & if\ |x| \le c \\ c|x| - \frac{c^2}{2}, & otherwise \end{cases} \qquad (11)$$

Differentiating the M-estimate criterion with respect to β_j and setting to zero, we get;

$$\sum_{i=1}^{n} \rho\left(\frac{y_i - \sum_{j=1}^{p} x_{ij}\beta_j}{\sigma}\right) x_{ij} = 0,\ j = 1, \dots, p \qquad (12)$$

Now let $u_i = y_i - \sum_{j=1}^{p} x_{ij}\beta_j$ to get

$$\sum_{i=1}^{n} \frac{\rho'(u_i)}{u_i} x_{ij}\left(y_i - \sum_{j=1}^{p} x_{ij}\beta_j\right) = 0 \qquad (13)$$

making the identification of

$w(u) = \frac{\rho'(u)}{u}$ and find $w(u)$ for choices of ρ above:

1. LS: $w(u)$ is constant.
2. LAD: $w(u) = 1/|u|$
3. Huber: $w(u) = \begin{cases} 1, & if\ |u| \le c \\ \frac{c}{|u|}, & othrwise \end{cases}$

2.4.2. MM-Estimator

It was first introduced by Yohai (1987). It has become increasingly popular and perhaps one of the most commonly employed robust regression technique. The "MM" in the name refers to the fact that more than one M-estimation procedure is used to calculate the final estimates. Following from the M-estimation case, iteratively reweighted least squares (IRLS) is employed to find estimates. The procedure is as follows:

1. Initial estimates of the coefficients $\hat{\beta}^{(1)}$ and corresponding residuals $e_i^{(1)}$ are taken from a highly resistant regression (i.e., a regression with a breakdown point of 50%). Although the estimator must be consistent, it is not necessary that it be efficient. As a result, S-estimation with Huber or bisquare weights (which can be seen as a form of M-estimation) is typically employed at this stage.
2. The residuals $e_i^{(1)}$ from the initial estimation at Stage 1 are used to compute an M-estimation of the scale of the residuals, $\hat{\sigma}_e$.
3. The initial estimates of the residuals from Stage 1 and of the residual scale $\hat{\sigma}_e$ from Stage 2 are used in the first iteration of weighted least squares to determine the M-estimates of the regression coefficients

$$\sum_{i=1}^{n} w_i \left(e_i^{(1)}/\hat{\sigma}_e\right) x_i = 0 \qquad (14)$$

where the w_i are typically Huber or bisquare weights.

4. New weights are calculated, $w_i^{(1)}$, using the residuals from the initial WLS (Step 3).
5. Keeping constant the measure of the scale of the residuals from Step 2, Steps 3 and 4 are continually reiterated until convergence.

2.4.3. S-Estimator

In response to the low breakdown point of M-estimators. Rousseeuw and Yohai (1984) proposed S-estimates by considering the scale of the residuals. S-estimates are the solution that finds the smallest possible dispersion of the residuals $min\hat{\sigma}\left(e_i(\hat{\beta}), \ldots, e_n(\hat{\beta})\right)$. Rather than minimizing the variance of the residuals, robust S-estimation minimizes a robust M-estimate of the residual scale

$$\frac{1}{n}\sum_{i=1}^{n} \rho\left(\frac{e_i}{\hat{\sigma}_e}\right) = b, \qquad (15)$$

where b is a constant defined as $b = E_{\phi}[\rho(e)]$and ϕ represents the standard normal distribution. Differentiating Equation 18 and solving results in

$$\frac{1}{n}\sum_{i=1}^{n} \psi\left(\frac{e_i}{\hat{\sigma}_e}\right), \qquad (16)$$

where ψ is replaced with an appropriate weight function. As with most M-estimation procedures, either the Huber weight function or the biweight function is usually employed.

2.4.4. Least Trimmed Squares (LTS) Estimator

Another method developed by Rousseeuw (1998) is least trimmed squares (LTS) regression. Extending from the trimmed mean, LTS regression minimizes the sum of the trimmed squared residuals. The LTS estimator is found by;

$$min \sum_{i=1}^{q} e_{(i)}^2,$$

where $q = [n(1-\alpha)+1]$ is the number of observations included in the calculation of the estimator, and ***α*** is the proportion of trimming that is performed. Using $q = \left(\frac{n}{2}\right) + 1$ ensures that the estimator has a breakdownpoint of 50%.

2.5. Robust Ridge Regression (RRR)

To solve the problems of multicollinearity and outlier simultaneously, the ridge estimator was combined with some robust estimators (M, MM, LTS and S estimators) to form robust ridge estimator (RRE) which are M-Ridge, MM-Ridge, LTS-Ridge and S-Ridge estimators (Lukman *et al*., 2014). These Robust ridge estimators can be computed as:

$$\hat{\alpha}_{RR} = \left(Z'Z + k_{ALrobust} I_p\right)^{-1} Z'Y \qquad (17)$$

where $k_{ALrobust}$ is the robust ridge parameter. It is obtained from the robust regression methods instead of the OLS estimation, and can be computed as given below;

$$k_{ALrobust} = \frac{1}{\sqrt{\dfrac{\hat{\sigma}_{Robust}^2}{\max(\hat{\alpha}_{Robust})}}}$$

2.6. Robust Liu Estimator (Proposed)

To solve the problems of multicollinearity and outlier simultaneously, Liu estimator combined with some robust estimators (M, MM, LTS and S estimators) to provide robust Liu estimator (RLE) which are M-Liu, MM-Liu, LTS-Liu and S-Liu estimators. These Robust Liu estimators can be computed as:

$$\hat{\alpha}_{RLE} = (\Lambda + I_p)^{-1}(\Lambda + d_R I_p)\hat{\alpha}_R \qquad (19)$$

where d_R is the biasing parameter obtained from robust estimators, computed as:

$$d_R = 1 - \hat{\sigma}_{Robust}^2 \left[\frac{\sum_{i=1}^{p} \frac{1}{\lambda_i(\lambda_i + 1)}}{\sum_{i=1}^{p} \frac{\hat{\alpha}_{iRobust}^2}{(\lambda_i + 1)^2}} \right] \qquad (20)$$

2.7. Data Description

Three datasets are used in this study to examine the performance of the estimators. The datasets are given in details below.

2.7.1. Longley Data

Longley Data is a macroeconomic dataset which provides a well-known example for a highly collinear regression. A data frame with seven economic variables observed yearly from 1947 to 1962. The variables are: Employment, Prices, Unemployed, Military, GNP, Population Size, Year. GNP is the Gross National Product, Employment is the number of people employed, Unemployed is the number of unemployed, Military is the number of people in the armed forces, Population size is the non-institutionalized population of persons at age ≥14 years, Price is the GNP implicit price deflator and year is the time.
Longley data have been diagnosed to suffer both problems of multicollinearity and outlier (Cook, 1977; Besley *et al*., 1980; and Jahufer, 2013).

2.7.2. Portland cement data

Portland dataset traceable to Woods *et al.* (1932) has been widely analysed by Kaciranlar *et al.* (1999). The dataset contains four explanatory varaiables which are tricalcium aluminate (X_1), tricalcium silicate (X_2), tetracalcium aluminoferrite (X_3) and β-dicalcium silicate (X_4). The heat evolved after 180 days of curing is the dependent variable (Y). The dataset suffers multicollinearity since variance inflation factors are greater than 10. Mahalanobis distances of observations 3 and 10 revealed that the observations are leverage. With this it is obvious that there is outlier in the x-direction and no outlier in the y-direction. As a result, it is observed that multicollinearity and leverage point exists jointly in the dataset.

2.7.3. Hussein and Abdalla data

This dataset was used by Hussein and Abdalla (2012) and it covered the products in the manufacturing sector of Iraq in the period of 1960 to 1990. The variables used are the product value in the manufacturing sector**(Y),** value of imported intermediate (X_1), imported capital commodities (X_2) and value of imported raw materials (X_3). Hussein and Abdalla (2012) showed that the dataset suffers the problem of multicollinearity since VIF > 10. Lukman *et al.* (2014) identified case number: 12, 14, 15, 16, 17, 18, 19, 20 and 21 as outliers in the y-direction and also identified case number 12, 14 and 15 as leverages. Therefore, outliers exist in the y and x direction.

2.8. Criterion for investigation

To investigate whether the robust ridge estimator is better than the OLS estimator, the MSE was calculated as follows:

$$MSE(\hat{\alpha}_{Ridge}) = \hat{\sigma}^2_{OLS} \sum_{i=1}^{p} \frac{\lambda_i}{(\lambda_i + k_{OLS})^2} + k_{OLS} \sum_{i=1}^{p} \frac{\alpha^2_{i,OLS}}{(\lambda_i + k_{OLS})^2} \quad (20)$$

$$MSE(\hat{\alpha}_{Robust\ Ridge}) = \hat{\sigma}^2_{robust} \sum_{i=1}^{p} \frac{\lambda_i}{(\lambda_i + k_R)^2} + k_R^2 \sum_{i=1}^{p} \frac{\alpha^2_{i,robust}}{(\lambda_i + k_R)^2} \quad (21)$$

$$MSE(\hat{\alpha}_{Liu}) = \hat{\sigma}^2 \sum_{i=1}^{P} \frac{(\lambda_i + d)^2}{(\lambda_i + 1)^2} + (d-1)^2 \sum_{i=1}^{P} \frac{\hat{\alpha}_i^2}{(\lambda_i + 1)^2} \quad (22)$$

$$MSE(\hat{\alpha}_{Robust\ Liu}) = \hat{\sigma}^2_{robust} \sum_{i=1}^{P} \frac{(\lambda_i + d_R)^2}{(\lambda_i + 1)^2} + (d_R - 1)^2 \sum_{i=1}^{P} \frac{\hat{\alpha}^2_{i,robust}}{(\lambda_i + 1)^2} \quad (23)$$

$$MSE(\hat{\alpha}_{OLS}) = \hat{\sigma}^2 \sum_{i=1}^{p} \frac{1}{\lambda_i} \quad (24)$$

where $\lambda_i, (i = 1,2, \dots, p)$ are the eigenvalues of $X'X$, k is the ridge parameter obtained from OLS and Robust estimates, α_i $(i = 1,2, \dots, p)$ is the ith element of the vector $\alpha = Q'\beta$.

3.0. RESULTS AND DISCUSSION

3.1. Result for Longley Data

Table 1: Summary of OLS and Robust Estimates

COEFFICIENT	OLS	M	MM	LTS	S	VIF
$\hat{\alpha}_1$	0.1548	0.1547	0.1547	0.1547	0.1549	135.5358
$\hat{\alpha}_2$	-0.5494	-0.5496	-0.5495	-0.5458	-0.5448	1788.498
$\hat{\alpha}_3$	0.8455	0.8156	0.8351	0.7562	0.7092	399.1466
$\hat{\alpha}_4$	1.0138	0.9347	0.9784	0.9897	1.0413	33.6188
$\hat{\alpha}_5$	42.6115	37.3772	40.4466	19.1757	13.2611	3.5889
$\hat{\alpha}_6$	-57.7536	-25.012	-42.7171	-81.9413	-72.3343	758.9658
k_{AL}	0.008042	0.0142985	0.008224	0.0036817	0.000885	
d	0.0004587	0.0001718	0.000589	0.0001601	0.001709	
MSE($\hat{\alpha}$)	17095.175	7397.8642	15060.94	7562.0395	15047.43	

Table 2: Summary of the Influential and Leverage Points

Observation	Residual	Leverage	Studentized Deleted Residuals	Dffit
1	267.34	0.36204	1.18111	1.014*
2	-94.014	0.50248	-0.4463	-0.509
3	46.2872	0.29957	0.17959	0.135
4	-410.11	0.30973	-1.9417	-1.495*
5	309.715	0.55301	1.84403	2.333*
6	-249.31	0.30707	-1.0339	-0.792
7	-164.05	0.42903	-0.7351	-0.723
8	-13.18	0.44216	-0.0579	-0.058
9	14.3048	0.39462	0.06006	0.055
10	455.394	0.26812	2.16945	1.525*
11	-17.269	0.29738	-0.0668	-0.05
12	-39.055	0.42062	-0.1683	-0.163
13	-155.55	0.31181	-0.6227	-0.482
14	-85.671	0.16588	-0.3034	-0.165
15	341.932	0.31037	1.51479	1.168*
16	-206.76	0.62611	-1.2534	-1.864*

Table 3: Estimates of Ridge, Liu, Robust Ridge and Robust Liu Estimators

	COEFFICIENTS						
Estimators	$\hat{\alpha}_1$	$\hat{\alpha}_2$	$\hat{\alpha}_3$	$\hat{\alpha}_4$	$\hat{\alpha}_5$	$\hat{\alpha}_6$	MSE($\hat{\alpha}$)
ORR	0.15354	-0.5450	0.83879	1.0057	42.272	-57.293	17074.69
Liu	0.1548	-0.5494	0.8455	1.0138	42.587	-53.7192	14818.97
M-Ridge	0.1526	-0.54164	0.83361	0.9995	42.011	-56.939	7382.117
M-Liu	0.1547	-0.5496	0.8156	0.9347	37.3557	-23.267	**6412.16**
MM-Ridge	0.1535	-0.5449	0.8386	1.0055	42.264	-57.282	15042.48
MM-Liu	0.1547	-0.5496	0.8225	0.9481	38.3467	-28.151	13054.71
LTS-Ridge	0.15421	-0.54737	0.84243	1.0101	42.455	-57.542	7557.889
LTS-Liu	0.1547	-0.5458	0.752	0.9897	19.1647	-76.2155	6582.04
S-Ridge	0.15391	-0.54632	0.84081	1.0081	42.374	-57.431	15045.45
S-Liu	0.1547	-0.5496	0.8157	0.9348	37.3698	-23.3325	13058.909

From Table 1, it is obvious that the data suffers a severe problem of multicollinearity since the Variance Inflation Factors (VIF) are greater than 10 except for X_5. Also from Table 2, the data has outliers in the y-direction, hence the data suffers both problem of multicollinearity and outliers. It follows from Table 3 that the two problems have been circumvented with the use of Robust Liu and Robust Ridge Regression Estimators and it is found that the Robust Liu is the most efficient in term of MSE.

3.2. Results for Portland Cement Data

Table 4: Estimates of OLS and Robust estimators

Coefficient	OLS	M	MM	LTS	S	VIF
$\hat{\alpha}_1$	1.6373	1.6371	1.6371	1.6377	1.6388	38.496
$\hat{\alpha}_2$	-0.2097	-0.2032	-0.2027	-0.1806	-0.1831	254.423
$\hat{\alpha}_3$	0.9160	0.8905	0.8889	0.8205	0.8255	46.868
$\hat{\alpha}_4$	-1.8405	-1.8672	-1.8693	-1.9697	-1.9623	282.513
k_{AL}	0.0104	0.1124	0.7739	0.3407	0.8144	
D	4.1604	0.2934	0.2928	0.2639	0.2659	
$\hat{\sigma}^2$	5.8454	3.2671	5.8342	1.5561	5.8057	
MSE($\hat{\alpha}$)	0.0638	0.0356	0.0637	0.0170	0.0633	

Table 5: Estimates of Ridge, Liu, Robust Ridge and Robust Liu estimators

	Coefficients				
Estimators	$\hat{\alpha}_1$	$\hat{\alpha}_2$	$\hat{\alpha}_3$	$\hat{\alpha}_4$	MSE($\hat{\alpha}$)
ORR	0.9296	-0.1191	0.5201	-0.0540	0.0691
LIU	1.6374	-0.2099	0.9196	-1.8952	0.0674
M-Ridge	1.4190	-0.1886	0.8235	-1.6545	0.0359
M-Liu	1.6367	-0.2032	0.8897	-1.8548	0.0354
MMridge	0.9230	-0.1182	0.5164	-1.0396	0.0691
MM-Liu	1.6371	-0.2027	0.8881	-1.8569	0.0631
LTS-ridge	1.4302	-0.1832	0.8002	-1.6077	0.0174
LTS-Liu	1.6377	-0.1806	0.8198	-1.9561	**0.0170**
S-Ridge	0.9024	-0.1156	0.5049	-1.0144	0.0691
S-Liu	1.6388	-0.1831	0.8247	-1.9488	0.0628

From Table 4 and 5, it can be seen that in terms of MSE criterion of the estimators, the LTS-Liu, LTS-Ridge in this order, are more efficient than the OLS. Thus, LTS-Liu is most efficient.

3.3. Result for Hussein and Abdalla Data

Table 6: Estimates of OLS and Robust estimator

Coefficient	OLS	M	MM	LTS	S
$\hat{\alpha}_1$	1.3143	1.3948	1.3821	1.3803	1.3807
$\hat{\alpha}_2$	-1.5151	-1.8513	-4.9978	-5.7278	-5.8198
$\hat{\alpha}_3$	2.0164	1.7145	-3.6142	-4.9724	-5.2153
k_{AL}	0.0104	0.0209	0.0685	0.0918	0.0800
D	0.4395	0.3396	0.0758	0.0403	0.0367
$\hat{\sigma}^2$	37736	7851.32	5316.35	4017.17	5297.70
MSE($\hat{\alpha}$)	4.7230	0.9827	0.6654	0.5028	0.6631

Table 7: Estimates of Ridge, Liu, Robust Ridge and Robust Liu estimators

COEFFICIENTS				
Estimators	$\hat{\alpha}_1$	$\hat{\alpha}_2$	$\hat{\alpha}_3$	$MSE(\hat{\alpha}_i)$
ORR	1.3007	-1.4996	1.9956	4.723
LIU	1.3143	-1.5151	2.0162	4.7225
M-Ridge	1.2874	-1.4841	1.9751	0.9826
M-Liu	1.3948	-1.8513	1.7144	0.9825
MM-ridge	1.23	-1.418	1.8871	0.6653
MM-Liu	1.3821	-4.9977	-3.6138	0.6653
LTS-ridge	1.2038	-1.3895	1.8464	0.5028
LTS-Liu	1.3803	-5.7277	-4.9719	**0.5027**
S-Ridge	1.2169	-1.4029	1.867	0.6630
S-Liu	1.3807	-5.8197	-5.2147	0.6629

The result in Table 7 shows that robust Liu (LTS-Liu) and robust Ridge (LTS-Ridge) regression estimators have least mean square error.

4.0. Conclusion

Ordinary Least Square (OLS), Liu Regression and Ordinary Ridge Regression (ORR) estimators could not perform well in term of their Mean Squared Error (MSE) in the presence of multicollinearity and outlier but ORR and Liu estimator performs better than that of Ordinary Least Square (OLS) Estimator. It is observed that Robust Ridge Estimators (RRE) and Robust Liu estimators perform better than the ORR, LRE and OLS estimators when both problems exist. Finally, M-Liu and M-Ridge perform most in this order when the outliers are in y-direction, while LTS-Ridge and LTS-Liu perform better when the outliers are in x-direction (Leverage).

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