Science World Journal Vol 12(No 3) 2017
www.scienceworldjournal.org
ISSN 1597-6343

Full Length Research Article

A REMARK ON MEASURE FUNCTIONS HAVING DOMAIN AS SIGMA ALGEBRA ON FINITE RHOTRIX SET

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ABSTRACT

Let $R_n(Z_p)$ be the set of all rhotrices of size n taking values from a field of integers Z_p, where $n = 2Z^+ + 1$. The purpose of this paper is to present some characterization of measure functions over sigma algebra on finite rhotrix set recently developed by Mohammed and Ifeanyi. The results were presented as theorems with their proof. Concrete examples were given to further reduce abstraction.

Keywords: Measure, measure function, finite rhotrix set, invertible rhotrix, sigma-algebra

INTRODUCTION

A rhotrix R of size n written as R_n, is a rhomboidal array with entries from a field F that can be expressed as coupled matrices which must necessarily be square matrices say A and C of sizes $(w \times w)$ and $(w-1)\times(w-1)$ respectively, where $w = \frac{n+1}{2}$ and $n \in 2Z^+ + 1$ is the size of the rhotrix.

The concept of rhotrix was initiated by Ajibade (2003). The measurability study of finite rhotrix set began from the work of Mohammed and Ifeanyi (2016). A finite rhotrix set, $R_n(Z_p)$, of all rhotrices of size n with entries from a finite field of integers Z_p, where p is a prime, was presented as underlying set in the study of non-commutative finite rhotrix group by Mohammed and Okon (2015). They defined $R_n(Z_p)$ as

$$R_n(Z_p) = \left\{ \left\langle \begin{array}{ccccccc} & & & a_{11} & & & \\ & & a_{12} & c_{11} & a_{12} & & \\ & - & - & - & - & - & \\ a_{t1} & - & - & - & - & - & a_{1t} \\ & - & - & - & - & - & \\ & & a_{t(t-1)} & c_{(t-1)(t-1)} & a_{(t-1)t} & & \\ & & & a_{tt} & & & \end{array} \right\rangle : a_{ij}, c_{lk} \in Z_p = \{0,1,2,\ldots,p-1\}, p \text{ is prime} \right\},$$

where $1 \le i, j \le t,\ 1 \le l, k \le t-1,\ t = \frac{n+1}{2}$, and $n \in 2Z^+ + 1$.

The various constructions of measure functions developed in (Mohammed and Ifeanyi, 2016) that can be used to determine the measure of any finite rhotrix set, form the basis for this paper. Also, using the idea of Sani (2008, 2009) for expressing rhotrix as coupled matrix and further as filled coupled matrix, each rhotrix in finite rhotrix set $R_n(Z_p)$ is written as

$$R_n(Z_p) = \left\{ \begin{bmatrix} a_{11} & \cdots & a_{12} & \cdots & a_{13} & \cdots & \cdots & \cdots & \cdots & a_{1t} \\ \cdots & c_{11} & \cdots & c_{12} & \cdots & \cdots & \cdots & \cdots & c_{1,t-1} & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & c_{t-1,1} & \cdots & c_{t-1,2} & \cdots & \cdots & \cdots & \cdots & c_{t-1,t-1} & \cdots \\ a_{t1} & \cdots & a_{t2} & \cdots & a_{t3} & \cdots & \cdots & \cdots & \cdots & a_{tt} \end{bmatrix} : a_{ij}, c_{lk} \in Z_p \right\}$$

By, filling each gap in the coupled matrix with zero, the dimension of the filled coupled matrix becomes $n \times n$, so that

$$R_n(Z_p) = \left\{ \begin{bmatrix} a_{11} & 0 & a_{12} & 0 & a_{13} & \cdots & \cdots & \cdots & 0 & a_{1t} \\ 0 & c_{11} & 0 & c_{12} & 0 & \cdots & \cdots & \cdots & c_{1,t-1} & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & c_{t-1,1} & 0 & c_{t-1,2} & 0 & \cdots & \cdots & \cdots & c_{t-1,t-1} & 0 \\ a_{t1} & 0 & a_{t2} & 0 & a_{t3} & \cdots & \cdots & \cdots & 0 & a_{tt} \end{bmatrix} : a_{ij}, c_{lj} \in Z_p \right\}$$

With the above, several measure functions M with domain as sigma algebra on $R_n(Z_p)$ to the set of positive real numbers $\Re^+$ were considered by Mohammed and Ifeanyi, (2016).

Basic definitions

The following definitions will be used frequently in the course of this paper.

Rhotrix: Rhotrix of size n written as R_n is defined as a rhomboidal method of representing array of numbers, where $n = 2Z^+ + 1$.

Non-singular (Invertible) rhotrix: A rhotrix R_n is invertible if there exists another rhotrix Q_n such that the product $(R_n)\times(Q_n) = I_n$ (an identity rhotrix). The operation is to be achieved using row column multiplication approach. By implication, determinant of non-singular rhotrix R_n in filled couple matrix form is non zero.

Triangular rhotrix: A rhotrix is triangular if all the entries to the left of the main diagonal are zero (upper triangular) or if all the entries to the right of the main diagonal are zero (lower triangular).

Non-zero rhotrix: This is a rhotrix with at least one of its entries more than zero. Analogously, a zero rhotrix is a rhotrix with every entry as zero.

i) $R_n(Z_p), \phi \in 2^{R_n(Z_p)}$

ii) If $Q_n(Z_p) \in 2^{R_n(Z_p)}$, then $(Q_n(Z_p))^c \in 2^{R_n(Z_p)}$ where $(Q_n(Z_p))^c$ is the complement of $Q_n(Z_p)$ with respect to $R_n(Z_p)$.

iii) If $(Q_n(Z_p))_1, (Q_n(Z_p))_2, (Q_n(Z_p))_3, \dots, (Q_n(Z_p))_k \in 2^{R_n(Z_p)}$ then $\bigcup_{i=1}^{k} (Q_n(Z_p))_i \in 2^{R_n(Z_p)}$

Measure: A measure $(\mathcal{M})$ is a function defined on a $\sigma-algebra$ (sigma-algebra) $\mathcal{F}$ over the set X such that $\mathcal{M}: \mathcal{F} \to \mathbb{R}^+$ satisfying the following axioms.

a) $\mathcal{M}(\mathcal{F}_i) \geq 0 \quad (\mathcal{F}_i \in \mathcal{F})$

b) $\mathcal{M}(\emptyset) = 0$ zero

Countable additivity: if E_n is countable sequence of pair-wise disjointed measurable sets in $\mathcal{F}$ then, $\mathcal{M}(\cup_{n=1}^{\infty}(E_n)) = \sum_{n=1}^{\infty} \mathcal{M}(E_n)$

Measure functions over sigma algebra on finite rhotrix set

In this section, some of the measure functions, developed in Mohammed and Ifeanyi (2016) with their extension to finite rhotrix sets in Mohammed and Ifeanyi (2017), are recorded.

1. Determinant measure function

If $R_n(Z_p)$ is a finite rhotrix set and $2^{R_n(Z_p)}$ is the collection of all subsets of $R_n(Z_p)$ and $|R_n(Z_p)|$ denotes the cardinality of $R_n(Z_p)$, then the function ${}^{d}M : 2^{R_n(Z_p)} \to \Re^+$ defined by

$$ {}^{d}M(W_n(Z_p)) = \sum_{k=1}^{|W_n(Z_p)|} \left| \det([r_{ij}])_k \right| $$

where $W_n(Z_p) \in 2^{R_n(Z_p)}$ is a measure function Mohammed and Ifeanyi (2016).

2. Trace measure function

If $R_n(Z_p)$ is a finite rhotrix set and $2^{R_n(Z_p)}$ is the collection of all subsets of $R_n(Z_p)$ and $|R_n(Z_p)|$ denotes the cardinality of $R_n(Z_p)$, then the function ${}^{t}M : 2^{R_n(Z_p)} \to \Re^+$ defined by

$$ {}^{tr}M(W_n(Z_p)) = \sum_{k=1}^{|W_n(Z_p)|} \left| tr([r_{ij}])_k \right| $$

where $W_n(Z_p) \in 2^{R_n(Z_p)}$ is a measure function Mohammed and Ifeanyi (2016).

3. Rank measure function

If $R_n(Z_p)$ is a finite rhotrix set and $2^{R_n(Z_p)}$ is the collection of all subsets of $R_n(Z_p)$ and $|R_n(Z_p)|$ denotes the cardinality of $R_n(Z_p)$, then the function ${}^{r}M : 2^{R_n(Z_p)} \to \Re^+$ defined by

$$ {}^{r}M(W_n(Z_p)) = \sum_{k=1}^{|W_n(Z_p)|} \left| rank([r_{ij}])_k \right| $$

where $W_n(Z_p) \in 2^{R_n(Z_p)}$ is a measure function, Mohammed and Ifeanyi (2016).

4. Spectral radius measure function

If $R_n(Z_p)$ is a finite rhotrix set and $2^{R_n(Z_p)}$ is the collection of all subsets of $R_n(Z_p)$ and $|R_n(Z_p)|$ denotes the cardinality of $R_n(Z_p)$, then the function ${}^{s}M : 2^{R_n(Z_p)} \to \Re^+$ defined by

$$ {}^{s}M(W_n(Z_p)) = \sum_{k=1}^{|W_n(Z_p)|} \left| spec([r_{ij}])_k \right| $$

where $W_n(Z_p) \in 2^{R_n(Z_p)}$ is a measure function, Mohammed and Ifeanyi (2016).

5. 1-norm measure function

If $R_n(Z_p)$ is a finite rhotrix set and $2^{R_n(Z_p)}$ is the collection of all subsets of $R_n(Z_p)$ and $|R_n(Z_p)|$ denotes the cardinality of $R_n(Z_p)$, then the function ${}^{1}M : 2^{R_n(Z_p)} \to \Re^+$ defined by

$$ {}^{1}M(W_n(Z_p)) = \sum_{k=1}^{|W_n(Z_p)|} \left(\left| \max_{1 \leq j \leq n} \sum_{j=1}^{n} (|r_{ij}|)_k \right| \right) $$

where $W_n(Z_p) \in 2^{R_n(Z_p)}$ is a measure function, Mohammed and Ifeanyi (2016).

6. $\infty -$ norm measure function

If $R_n(Z_p)$ is a finite rhotrix set and $2^{R_n(Z_p)}$ is the collection of all subsets of $R_n(Z_p)$ and $|R_n(Z_p)|$ denotes the cardinality of $R_n(Z_p)$, then the function ${}^{\infty}M : 2^{R_n(Z_p)} \rightarrow \Re^+$ defined by

$$ {}^{\infty}M(W_n(Z_p)) = \sum_{k=1}^{|W_n(Z_p)|} \left(\left| \max_{1 \le i \le n} \sum_{j=1}^{n} (|r_{ij}|) \right|_k \right) $$

where $W_n(Z_p) \in 2^{R_n(Z_p)}$ is a measure function, Mohammed and Ifeanyi (2016).

7. 2-norm measure function

If $R_n(Z_p)$ is a finite rhotrix set and $2^{R_n(Z_p)}$ is the collection of all subsets of $R_n(Z_p)$ and $|R_n(Z_p)|$ denotes the cardinality of $R_n(Z_p)$, then the function ${}^{2}M : 2^{R_n(Z_p)} \rightarrow \Re^+$ defined by

$$ {}^{2}M(W_n(Z_p)) = \sum_{k=1}^{|W_n Z_p|} \sqrt{\left|\max\left(eigen\left(\left([b_{ij}]_k\right)^T \left([b_{ij}]_k\right)\right)\right)\right|} $$

where $W_n(Z_p) \in 2^{R_n(Z_p)}$ is a measure function, Mohammed and Ifeanyi (2016).

8. Frobenius norm measure function

If $R_n(Z_p)$ is a finite rhotrix set and $2^{R_n(Z_p)}$ is the collection of all subsets of $R_n(Z_p)$ and $|R_n(Z_p)|$ denotes the cardinality of $R_n(Z_p)$, then the function ${}^{f}M : 2^{R_n(Z_p)} \rightarrow \Re^+$ defined by

$$ {}^{f}M(W_n(Z_p)) = \sum_{k=1}^{|W_n(Z_p)|} \left(\sqrt{\left|tr\left(\left([b_{ij}]_k\right)^T [b_{ij}]_k\right)\right|} \right) $$

where $W_n(Z_p) \in 2^{R_n(Z_p)}$ is a measure function, Mohammed and Ifeanyi (2016).

9. Euclidean norm measure function

If $R_n(Z_p)$ is a finite rhotrix set and $2^{R_n(Z_p)}$ is the collection of all subsets of $R_n(Z_p)$ and $|R_n(Z_p)|$ denotes the cardinality of $R_n(Z_p)$, then the function ${}^{e}M : 2^{R_n(Z_p)} \rightarrow \Re^+$ defined by

$$ {}^{e}M(W_n(Z_p)) = \sum_{k=1}^{|W_n(Z_p)|} \sqrt{\left(\sum_{i=1, j=1}^{n} (b_{ij})^2 \right)_k} $$

where $W_n(Z_p) \in 2^{R_n(Z_p)}$ is a measure function, Mohammed and Ifeanyi (2016).

10. Additive norm measure function

If $R_n(Z_p)$ is a finite rhotrix set and $2^{R_n(Z_p)}$ is the collection of all subsets of $R_n(Z_p)$ and $|R_n(Z_p)|$ denotes the cardinality of $R_n(Z_p)$, then the function ${}^{a}M : 2^{R_n(Z_p)} \rightarrow \Re^+$ defined by

$$ {}^{a}M(W_n(Z_p)) = \sum_{k=1}^{|W_n(Z_p)|} \left[\left(\sum_{i=1, j=1}^{n} |r_{ij}|_k \right) \right] $$

where, $W_n(Z_p) \in 2^{R_n(Z_p)}$ is a measure function Mohammed and Ifeanyi (2016).

RESULTS

Some results of the measurability study of finite rhotrix sets in Mohammed and Ifeanyi (2017) are identified in form of theorems. The set $R_3(Z_2)$ presented in Mohammed and Okon (2015) is used to demonstrate concrete examples. The evaluations of the measures are accomplished using MATLAB software.

Theorem 4.1: Let ${}^{d}M$ be a determinant measure of a finite rhotrix set $R_n(Z_p)$ and let $Q_n(Z_p)$ be a subset of $R_n(Z_p)$ consisting of singular rhotrices, then ${}^{d}M(Q_n(Z_p)) = 0$.

Proof: From definition of singular rhotrices, $Q_n(Z_p)$ contains rhotrices whose determinants are zeros.

Now, let

$Q_n(Z_p) = \{\langle q_1 \rangle, \langle q_2 \rangle, \langle q_3 \rangle, \ldots, \langle q_i \rangle\}$, where all the $q_{i's}$ are singular rhotrices and $|Q_n(Z_p)| = i$.

Then,

$$ Q_n(Z_p) = \{\langle q_1 \rangle\} \cup \{\langle q_2 \rangle\} \cup \{\langle q_3 \rangle\} \cup \ldots \cup \{\langle q_i \rangle\} $$

So that

$$ {}^{d}M(Q_n(Z_p)) = {}^{d}M(\{\langle q_1 \rangle\}) + {}^{d}M(\{\langle q_2 \rangle\}) + {}^{d}M(\{\langle q_3 \rangle\}) + \ldots + {}^{d}M(\{\langle q_i \rangle\}) $$

$$ {}^{d}M(Q_n(Z_p)) = 0 + 0 + 0 + \ldots + 0 $$

$$ {}^{d}M(Q_n(Z_p)) = 0 $$

Example 1: Consider the set

$$R_3(Z_2)=\left\{\begin{array}{l}
R_1=\left\langle\begin{array}{ccc}&1&\\1&0&1\\&1&\end{array}\right\rangle, R_2=\left\langle\begin{array}{ccc}&1&\\1&0&0\\&1&\end{array}\right\rangle, R_3=\left\langle\begin{array}{ccc}&1&\\1&0&1\\&0&\end{array}\right\rangle, R_4=\left\langle\begin{array}{ccc}&1&\\0&0&1\\&1&\end{array}\right\rangle, R_5=\left\langle\begin{array}{ccc}&1&\\0&0&0\\&0&\end{array}\right\rangle, R_6=\left\langle\begin{array}{ccc}&1&\\0&0&1\\&0&\end{array}\right\rangle, R_7=\left\langle\begin{array}{ccc}&1&\\0&0&0\\&1&\end{array}\right\rangle, R_8=\left\langle\begin{array}{ccc}&1&\\1&0&0\\&0&\end{array}\right\rangle,\\
R_9=\left\langle\begin{array}{ccc}&0&\\1&0&1\\&1&\end{array}\right\rangle, R_{10}=\left\langle\begin{array}{ccc}&0&\\1&0&0\\&1&\end{array}\right\rangle, R_{11}=\left\langle\begin{array}{ccc}&0&\\1&0&1\\&0&\end{array}\right\rangle, R_{12}=\left\langle\begin{array}{ccc}&0&\\0&0&1\\&1&\end{array}\right\rangle, R_{13}=\left\langle\begin{array}{ccc}&0&\\0&0&0\\&0&\end{array}\right\rangle, R_{14}=\left\langle\begin{array}{ccc}&0&\\0&0&1\\&0&\end{array}\right\rangle, R_{15}=\left\langle\begin{array}{ccc}&0&\\0&0&0\\&1&\end{array}\right\rangle, R_{16}=\left\langle\begin{array}{ccc}&0&\\1&0&0\\&0&\end{array}\right\rangle,\\
R_{17}=\left\langle\begin{array}{ccc}&1&\\1&1&1\\&1&\end{array}\right\rangle, R_{18}=\left\langle\begin{array}{ccc}&1&\\1&1&0\\&1&\end{array}\right\rangle, R_{19}=\left\langle\begin{array}{ccc}&1&\\1&1&1\\&0&\end{array}\right\rangle, R_{20}=\left\langle\begin{array}{ccc}&1&\\0&1&1\\&1&\end{array}\right\rangle, R_{21}=\left\langle\begin{array}{ccc}&1&\\0&1&0\\&0&\end{array}\right\rangle, R_{22}=\left\langle\begin{array}{ccc}&1&\\0&1&1\\&0&\end{array}\right\rangle, R_{23}=\left\langle\begin{array}{ccc}&1&\\0&1&0\\&1&\end{array}\right\rangle, R_{24}=\left\langle\begin{array}{ccc}&1&\\1&1&0\\&0&\end{array}\right\rangle,\\
R_{25}=\left\langle\begin{array}{ccc}&0&\\1&1&1\\&1&\end{array}\right\rangle, R_{26}=\left\langle\begin{array}{ccc}&0&\\1&1&0\\&1&\end{array}\right\rangle, R_{27}=\left\langle\begin{array}{ccc}&0&\\1&1&1\\&0&\end{array}\right\rangle, R_{28}=\left\langle\begin{array}{ccc}&0&\\0&1&1\\&1&\end{array}\right\rangle, R_{29}=\left\langle\begin{array}{ccc}&0&\\0&1&0\\&0&\end{array}\right\rangle, R_{30}=\left\langle\begin{array}{ccc}&0&\\0&1&1\\&0&\end{array}\right\rangle, R_{31}=\left\langle\begin{array}{ccc}&0&\\0&1&0\\&1&\end{array}\right\rangle, R_{32}=\left\langle\begin{array}{ccc}&0&\\1&1&0\\&0&\end{array}\right\rangle
\end{array}\right\}$$

and let the singular elements be $Q_3(Z_2)$. Then

$$Q_3(Z_2)=\left\{\begin{array}{l}
R_1=\left\langle\begin{array}{ccc}&1&\\1&0&1\\&1&\end{array}\right\rangle, R_2=\left\langle\begin{array}{ccc}&1&\\1&0&0\\&1&\end{array}\right\rangle, R_3=\left\langle\begin{array}{ccc}&1&\\1&0&1\\&0&\end{array}\right\rangle, R_4=\left\langle\begin{array}{ccc}&1&\\0&0&1\\&1&\end{array}\right\rangle, R_5=\left\langle\begin{array}{ccc}&1&\\0&0&0\\&0&\end{array}\right\rangle, R_6=\left\langle\begin{array}{ccc}&1&\\0&0&1\\&0&\end{array}\right\rangle,\\
R_7=\left\langle\begin{array}{ccc}&1&\\0&0&0\\&1&\end{array}\right\rangle, R_8=\left\langle\begin{array}{ccc}&1&\\1&0&0\\&0&\end{array}\right\rangle, R_9=\left\langle\begin{array}{ccc}&0&\\1&0&1\\&1&\end{array}\right\rangle, R_{10}=\left\langle\begin{array}{ccc}&0&\\1&0&0\\&1&\end{array}\right\rangle, R_{11}=\left\langle\begin{array}{ccc}&0&\\1&0&1\\&0&\end{array}\right\rangle, R_{12}=\left\langle\begin{array}{ccc}&0&\\0&0&1\\&1&\end{array}\right\rangle,\\
R_{13}=\left\langle\begin{array}{ccc}&0&\\0&0&0\\&0&\end{array}\right\rangle, R_{14}=\left\langle\begin{array}{ccc}&0&\\0&0&1\\&0&\end{array}\right\rangle, R_{15}=\left\langle\begin{array}{ccc}&0&\\0&0&0\\&1&\end{array}\right\rangle, R_{16}=\left\langle\begin{array}{ccc}&0&\\1&0&0\\&0&\end{array}\right\rangle, R_{17}=\left\langle\begin{array}{ccc}&1&\\1&1&1\\&1&\end{array}\right\rangle, R_{21}=\left\langle\begin{array}{ccc}&1&\\0&1&0\\&0&\end{array}\right\rangle,\\
R_{22}=\left\langle\begin{array}{ccc}&1&\\0&1&1\\&0&\end{array}\right\rangle, R_{24}=\left\langle\begin{array}{ccc}&1&\\1&1&0\\&0&\end{array}\right\rangle, R_{26}=\left\langle\begin{array}{ccc}&0&\\1&1&0\\&1&\end{array}\right\rangle, R_{28}=\left\langle\begin{array}{ccc}&0&\\0&1&1\\&1&\end{array}\right\rangle, R_{29}=\left\langle\begin{array}{ccc}&0&\\0&1&0\\&0&\end{array}\right\rangle, R_{30}=\left\langle\begin{array}{ccc}&0&\\0&1&1\\&0&\end{array}\right\rangle,\\
R_{31}=\left\langle\begin{array}{ccc}&0&\\0&1&0\\&1&\end{array}\right\rangle, R_{32}=\left\langle\begin{array}{ccc}&0&\\1&1&0\\&0&\end{array}\right\rangle
\end{array}\right\}$$

Therefore,

$$^{d}M(Q_3(Z_2))={}^{d}M(R_1)+...+{}^{d}M(R_{17})+{}^{d}M(R_{21})+{}^{d}M(R_{22})+{}^{d}M(R_{24})+{}^{d}M(R_{26})+{}^{d}M(R_{28})+...+{}^{d}M(R_{32})$$

Using Matlab the result follows thus

$$^{d}M(Q_3(Z_2))=abs(\det(R_1))+abs(\det(R_2))+...$$

$$^{d}M(Q_3(Z_2))=0$$

Theorem 4.2: Let ^{d}M be a determinant measure defined on the finite rhotrix set $R_n(Z_p)$ and let $P_n(Z_p)$ be a set of all invertible elements of $R_n(Z_p)$ then $^{d}M(P_n(Z_p))={}^{d}M(R_n(Z_p))$.

Proof: From the hypothesis,

$$P_n(Z_p)\subseteq R_n(Z_p).$$

It means that $\exists\ Q_n(Z_p)\subseteq R_n(Z_p)$ such that $Q_n(Z_p)$ contains only the singular elements of $R_n(Z_p)$.

Therefore,

$$R_n(Z_p)=P_n(Z_p)\cup Q_n(Z_p)\text{, where}$$

$$P_n(Z_p)\cap Q_n(Z_p)=\phi$$

Hence,

$$^{d}M(R_n(Z_p))={}^{d}M(P_n(Z_p))+{}^{d}M(Q_n(Z_p))$$

$$^{d}M(R_n(Z_p))={}^{d}M(P_n(Z_p))+0$$

$$^{d}M(R_n(Z_p))={}^{d}M(P_n(Z_p))$$

Example 2: Consider the finite rhotrix set $R_3(Z_2)=Q_3(Z_2)\cup P_3(Z_2)$ where $Q_3(Z_2)$ contains the singular elements of $R_3(Z_2)$ given as

$$Q_3(Z_2)=\left\{\begin{array}{l}
R_1=\left\langle\begin{array}{ccc}&1&\\1&0&1\\&1&\end{array}\right\rangle, R_2=\left\langle\begin{array}{ccc}&1&\\1&0&0\\&1&\end{array}\right\rangle, R_3=\left\langle\begin{array}{ccc}&1&\\1&0&1\\&0&\end{array}\right\rangle, R_4=\left\langle\begin{array}{ccc}&1&\\0&0&1\\&1&\end{array}\right\rangle, R_5=\left\langle\begin{array}{ccc}&1&\\0&0&0\\&0&\end{array}\right\rangle, R_6=\left\langle\begin{array}{ccc}&1&\\0&0&1\\&0&\end{array}\right\rangle,\\
R_7=\left\langle\begin{array}{ccc}&1&\\0&0&0\\&1&\end{array}\right\rangle, R_8=\left\langle\begin{array}{ccc}&1&\\1&0&0\\&0&\end{array}\right\rangle, R_9=\left\langle\begin{array}{ccc}&0&\\1&0&1\\&1&\end{array}\right\rangle, R_{10}=\left\langle\begin{array}{ccc}&0&\\1&0&0\\&1&\end{array}\right\rangle, R_{11}=\left\langle\begin{array}{ccc}&0&\\1&0&1\\&0&\end{array}\right\rangle, R_{12}=\left\langle\begin{array}{ccc}&0&\\0&0&1\\&1&\end{array}\right\rangle,\\
R_{13}=\left\langle\begin{array}{ccc}&0&\\0&0&0\\&0&\end{array}\right\rangle, R_{14}=\left\langle\begin{array}{ccc}&0&\\0&0&1\\&0&\end{array}\right\rangle, R_{15}=\left\langle\begin{array}{ccc}&0&\\0&0&0\\&1&\end{array}\right\rangle, R_{16}=\left\langle\begin{array}{ccc}&0&\\1&0&0\\&0&\end{array}\right\rangle, R_{17}=\left\langle\begin{array}{ccc}&1&\\1&1&1\\&1&\end{array}\right\rangle, R_{21}=\left\langle\begin{array}{ccc}&1&\\0&1&0\\&0&\end{array}\right\rangle,\\
R_{22}=\left\langle\begin{array}{ccc}&1&\\0&1&1\\&0&\end{array}\right\rangle, R_{24}=\left\langle\begin{array}{ccc}&1&\\1&1&0\\&0&\end{array}\right\rangle, R_{26}=\left\langle\begin{array}{ccc}&0&\\1&1&0\\&1&\end{array}\right\rangle, R_{28}=\left\langle\begin{array}{ccc}&0&\\0&1&1\\&1&\end{array}\right\rangle, R_{29}=\left\langle\begin{array}{ccc}&0&\\0&1&0\\&0&\end{array}\right\rangle, R_{30}=\left\langle\begin{array}{ccc}&0&\\0&1&1\\&0&\end{array}\right\rangle,\\
R_{31}=\left\langle\begin{array}{ccc}&0&\\0&1&0\\&1&\end{array}\right\rangle, R_{32}=\left\langle\begin{array}{ccc}&0&\\1&1&0\\&0&\end{array}\right\rangle
\end{array}\right\}$$

and $P_3(Z_2)$ contains the non-singular elements of $R_3(Z_2)$ given as;

$$P_3(Z_2)=\left\{R_{18}=\left\langle\begin{array}{ccc}&1&\\1&1&0\\&1&\end{array}\right\rangle, R_{19}=\left\langle\begin{array}{ccc}&1&\\1&1&1\\&0&\end{array}\right\rangle, R_{20}=\left\langle\begin{array}{ccc}&1&\\0&1&1\\&1&\end{array}\right\rangle, R_{23}=\left\langle\begin{array}{ccc}&1&\\0&1&0\\&1&\end{array}\right\rangle, R_{25}=\left\langle\begin{array}{ccc}&0&\\1&1&1\\&1&\end{array}\right\rangle, R_{27}=\left\langle\begin{array}{ccc}&0&\\1&1&1\\&0&\end{array}\right\rangle\right\}$$

So that

$$^{d}M(R_3(Z_2))={}^{d}M(Q_3(Z_2))+{}^{d}M(P_3(Z_2))$$

Using matlab as in example 1 above to evaluate the determinants of the sets in the right hand side it will follow that;

$$^{d}M(R_3(Z_2))=0+6$$

$$=6$$

$$={}^{d}M(P_3(Z_2))$$

Theorem 4.3: Let $Q_n(Z_p)$ be a subset of $R_n(Z_p)$ containing rhotrices having trace as zero, then $^{tr}M(Q_n(Z_p))=0$, where ^{tr}M denotes trace measure function.

Proof:

Using $^{tr}M(W_n(Z_p))=\sum_{k=1}^{|W_n(Z_p)|}\left|tr(\langle r_{ij}\rangle)_k\right|$ where $W_n(Z_p)\in 2^{R_n(Z_p)}$

Let $W_n(Z_p)=Q_n(Z_p)$ so that

$$^{tr}M(Q_n(Z_p))=\sum_{k=1}^{|Q_n(Z_p)|}\left|tr(\langle r_{ij}\rangle)_k\right|$$

Let $Q_n(Z_p)=\left\{(\langle r_{ij}\rangle)_1,(\langle r_{ij}\rangle)_2,\ldots,(\langle r_{ij}\rangle)_{|Q_n(Z_p)|}\right\}$ be rhotrices such that $tr(\langle r_{ij}\rangle)_k=0$

Science World Journal Vol 12(No 3) 2017
www.scienceworldjournal.org
ISSN 1597-6343

Clearly, $Q_n(Z_p) = \bigcup_{k=1}^{|Q_n(Z_p)|} \left(\left[r_{ij}\right]\right)_k$ and

$$ {}^{tr}M\left(Q_n(Z_p)\right) = {}^{tr}M\left(\bigcup_{k=1}^{|Q_n(Z_p)|} \left(\left[r_{ij}\right]\right)_k\right) = \sum_{k=1}^{|Q_n(Z_p)|} \left| tr\left(\left[r_{ij}\right]\right)_k \right| = 0 $$

Example 3: Given the finite rhotrix set $R_3(Z_2)$ as in example one and $Q_3(Z_2) \subseteq R_3(Z_2)$ where $Q_3(Z_2)$ contains rhotrices of trace zero. Then, $Q_3(Z_2)$ is listed as;

$$ Q_3(Z_2) = \left\{ R_{11} = \left\langle \begin{array}{ccc} & 0 & \\ 1 & 0 & 1 \\ & 0 & \end{array} \right\rangle, R_{13} = \left\langle \begin{array}{ccc} & 0 & \\ 0 & 0 & 0 \\ & 0 & \end{array} \right\rangle, R_{14} = \left\langle \begin{array}{ccc} & 0 & \\ 0 & 0 & 1 \\ & 0 & \end{array} \right\rangle, R_{16} = \left\langle \begin{array}{ccc} & 0 & \\ 1 & 0 & 0 \\ & 0 & \end{array} \right\rangle \right\} $$

$$ {}^{tr}M\left(Q_3(Z_2)\right) = {}^{tr}M\left(R_{11}\right) + {}^{tr}M\left(R_{13}\right) + {}^{tr}M\left(R_{14}\right) + {}^{tr}M\left(R_{16}\right) $$

$$ {}^{tr}M\left(Q_3(Z_2)\right) = 0 $$

Theorem 4.4: Let $P_n(Z_p)$ and $Q_n(Z_p)$ be any subsets of a finite rhotrix set $R_n(Z_p)$ of equal sizes such that $P_n(Z_p)$ and $Q_n(Z_p)$ contain upper triangular rhotrices and lower triangular rhotrices respectively, then ${}^{s}M\left(P_n(Z_p)\right) = {}^{s}M\left(Q_n(Z_p)\right)$ where, ${}^{s}M$ is the **Spectral radius measure function**.

Proof:

This proof follows since the field Z_p is the same and both $P_n(Z_p)$ and $Q_n(Z_p)$ are of equal sizes. Every lower triangular rhotrix has a reflection as upper triangular rhotrix.

Example 4

Consider $R_3(Z_2)$ as in example one, containing the sets $P_3(Z_2)$ and $Q_3(Z_2)$ as subsets with upper and lower triangular rhotrices respectively as members. We list the sets as follows;

$$ P_3(Z_2) = \left\{ \begin{array}{l} \left\langle \begin{array}{ccc} & 1 & \\ 0 & 0 & 1 \\ & 1 & \end{array} \right\rangle, \left\langle \begin{array}{ccc} & 1 & \\ 0 & 0 & 1 \\ & 0 & \end{array} \right\rangle, \left\langle \begin{array}{ccc} & 0 & \\ 0 & 0 & 1 \\ & 1 & \end{array} \right\rangle, \left\langle \begin{array}{ccc} & 0 & \\ 0 & 0 & 1 \\ & 0 & \end{array} \right\rangle, \\ \left\langle \begin{array}{ccc} & 1 & \\ 0 & 1 & 1 \\ & 1 & \end{array} \right\rangle, \left\langle \begin{array}{ccc} & 1 & \\ 0 & 1 & 1 \\ & 0 & \end{array} \right\rangle, \left\langle \begin{array}{ccc} & 0 & \\ 0 & 1 & 1 \\ & 1 & \end{array} \right\rangle, \left\langle \begin{array}{ccc} & 0 & \\ 0 & 1 & 1 \\ & 0 & \end{array} \right\rangle \end{array} \right\} $$

and

$$ Q_3(Z_2) = \left\{ \begin{array}{l} \left\langle \begin{array}{ccc} & 1 & \\ 1 & 0 & 0 \\ & 1 & \end{array} \right\rangle, \left\langle \begin{array}{ccc} & 1 & \\ 1 & 0 & 0 \\ & 0 & \end{array} \right\rangle, \left\langle \begin{array}{ccc} & 1 & \\ 0 & 0 & 0 \\ & 1 & \end{array} \right\rangle, \left\langle \begin{array}{ccc} & 0 & \\ 1 & 0 & 0 \\ & 0 & \end{array} \right\rangle, \\ \left\langle \begin{array}{ccc} & 1 & \\ 1 & 1 & 0 \\ & 1 & \end{array} \right\rangle, \left\langle \begin{array}{ccc} & 1 & \\ 1 & 1 & 0 \\ & 0 & \end{array} \right\rangle, \left\langle \begin{array}{ccc} & 0 & \\ 1 & 1 & 0 \\ & 1 & \end{array} \right\rangle, \left\langle \begin{array}{ccc} & 0 & \\ 1 & 1 & 0 \\ & 0 & \end{array} \right\rangle \end{array} \right\} $$

Using matlab tool we establish that

$$ {}^{s}M\left(P_3(Z_2)\right) = 7 $$

$$ {}^{s}M\left(Q_3(Z_2)\right) = 7 $$

Corollary 1

Let $P_n(Z_p)$ and $Q_n(Z_p)$ be any subsets of a finite rhotrix set $R_n(Z_p)$ of equal sizes such that $P_n(Z_p)$ and $Q_n(Z_p)$ contain upper triangular rhotrices and lower triangular rhotrices respectively, then ${}^{r}M\left(P_n(Z_p)\right) = {}^{r}M\left(Q_n(Z_p)\right)$ where, ${}^{r}M$ is the **rank measure function**.

Corollary 2

Let $P_n(Z_p)$ and $Q_n(Z_p)$ be any subsets of a finite rhotrix set $R_n(Z_p)$ of equal sizes such that $P_n(Z_p)$ and $Q_n(Z_p)$ contain upper triangular rhotrices and lower triangular rhotrices respectively, then ${}^{1}M\left(P_n(Z_p)\right) = {}^{1}M\left(Q_n(Z_p)\right)$ where, ${}^{1}M$ is the **1-norm measure function**.

Corollary 3

Let $P_n(Z_p)$ and $Q_n(Z_p)$ be any subsets of a finite rhotrix set $R_n(Z_p)$ of equal sizes such that $P_n(Z_p)$ and $Q_n(Z_p)$ contain upper triangular rhotrices and lower triangular rhotrices respectively, then ${}^{\infty}M\left(P_n(Z_p)\right) = {}^{\infty}M\left(Q_n(Z_p)\right)$ where, ${}^{\infty}M$ is the ∞**-norm measure function**.

Corollary 4

Let $P_n(Z_p)$ and $Q_n(Z_p)$ be any subsets of a finite rhotrix set $R_n(Z_p)$ of equal sizes such that $P_n(Z_p)$ and $Q_n(Z_p)$ contain upper triangular rhotrices and lower triangular rhotrices respectively, then ${}^{2}M\left(P_n(Z_p)\right) = {}^{2}M\left(Q_n(Z_p)\right)$ where, ${}^{2}M$ is the **2-norm measure function**.

Corollary 5
Let $P_n(Z_p)$ and $Q_n(Z_p)$ be any subsets of a finite rhotrix set $R_n(Z_p)$ of equal sizes such that $P_n(Z_p)$ and $Q_n(Z_p)$ contain upper triangular rhotrices and lower triangular rhotrices respectively, then ${}^fM(P_n(Z_p)) = {}^fM(Q_n(Z_p))$ where, ${}^fM$ is the **Frobenius norm measure function**.

Corollary 6
Let $P_n(Z_p)$ and $Q_n(Z_p)$ be any subsets of a finite rhotrix set $R_n(Z_p)$ of equal sizes such that $P_n(Z_p)$ and $Q_n(Z_p)$ contain upper triangular rhotrices and lower triangular rhotrices respectively, then ${}^eM(P_n(Z_p)) = {}^eM(Q_n(Z_p))$ where, ${}^eM$ is the **Euclidean norm measure function**.

Corollary 7
Let $P_n(Z_p)$ and $Q_n(Z_p)$ be any subsets of a finite rhotrix set $R_n(Z_p)$ of equal sizes such that $P_n(Z_p)$ and $Q_n(Z_p)$ contain upper triangular rhotrices and lower triangular rhotrices respectively, then ${}^aM(P_n(Z_p)) = {}^aM(Q_n(Z_p))$ where, ${}^aM$ is the **Additive norm measure function**.

Theorem 4.5: Let ${}^eM$ be Euclidean norm measure function on a finite rhotrix set $R_n(Z_p)$. Let $P_n(Z_p)$ be a subset $R_n(Z_p)$ containing all non zero rhotrices, then ${}^eM(P_n(Z_p)) = {}^eM(R_n(Z_p))$.

Proof: Let us set $Q_n(Z_p)$, containing zero rhotrices, as the complement of $P_n(Z_p)$ with respect to $R_n(Z_p)$ as the universal set.

$$\Rightarrow R_n(Z_p) = P_n(Z_p) \cup Q_n(Z_p)$$

Now, ${}^eM(R_n(Z_p)) = {}^eM(P_n(Z_p)) + {}^eM(Q_n(Z_p))$

Therefore,

$${}^eM(R_n(Z_p)) = {}^eM(P_i(Z_p)) + 0$$

$${}^eM(R_n(Z_p)) = {}^eM(P_n(Z_p))$$ as required.

Conclusion
In this paper we have presented some characterization of results arising from measurability of study of finite rhotrix set $R_n(Z_p)$ on the bases of measure functions developed by Mohammed and Ifeanyi (2016). Rhotrix measure theories have been extended to particular subsets of the finite rhotrix set. It is hoped that this will stimulate further studies on the measurability of finite sets whose elements are not necessarily on the real of which $R_n(Z_p)$ is one.

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