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EFFECT OF CHEMICAL REACTION ON UNSTEADY MHD FREE CONVECTIVE TWO IMMISCIBLE FLUIDS FLOW

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ABSTRACT
The effect of chemical reaction on unsteady MHD free convective two immiscible fluids flow has been studied. Approximate analytical solutions to the governing equations are found for the coupled and linear differential equations using regular perturbation method. Graphs depicting the effect of chemical reaction parameter K_c and others flow parameters on velocity, temperature and concentration profiles are obtained and discussed accordingly. The effect of flow parameters on the coefficient of skin friction, Nusselt number and Sherwood number are also tabulated and discussed appropriately. It was observed that the increase in chemical reaction coefficient/parameter K_c suppresses both velocity and concentration profiles.

Keywords: Chemical Reaction, MHD, Convective, Immiscible, Unsteady

INTRODUCTION

Chemical Reaction is a process that involves rearrangement of the molecular or ionic structure of a substance, as distinct from a change in physical form or a nuclear reaction. There are two types of such reactions namely homogeneous reaction which occurs uniformly throughout a given phase of a flow and heterogeneous reaction which takes place in a particular region or within the boundary of a phase, (Umavathi, 2014).

Satya *et al* (2015) said, "The study of heat transfer with chemical reaction is of most realistic significance to engineers and scientists because of its universal incidence in many branches of science and engineering. This phenomenon plays a significant role in chemical industry, power and cooling industry for dying, evaporation, energy transfer in a cooling tower and the flow in a desert cooler, etc." According to umavathi (2014), the ever increasing industrial application of combined heat and mass transfer of fluid flow with chemical reaction, such as polymer production and manufacturing of ceramics among others has vested a great deal of importance on this area.

Satya *et al* (2015) investigated the effect of chemical reaction and heat source on MHD oscillatory flow in an irregular channel. However the flow is through an irregular channel and zero convection was assumed. He obtained among other results, a decrease in concentration and velocity profiles with increase in chemical reaction, Sherwood number increase with increasing chemical reaction parameter. This shows that the motion of the fluid have negative acceleration due to chemical reaction.

Krishnamurthy *et al* (2015) studied the effect of chemical reaction on MHD boundary layer flow & melting heat transfer of Williamson nanofluid in porous medium. He found among other results that an increase in the value of chemical reaction parameter decreases the concentration of species in the boundary layer, whereas the velocity and temperature of the fluid are not affected with the rise of chemical reaction parameter. This is due to the fact that chemical reaction in the study leads to consumption of chemical and thus results to decrease in the concentration profile. The flow is in two dimensions.

Srinivasacharya and Reddy (2014) studied chemical reaction and radiation effects on mixed convection heat and mass transfer over a vertical plate in power-law fluid saturated porous medium. The fluid is not MHD and the flow is two dimensional. The following result among others were found. The velocity and concentration of the fluid increases and suppresses respectively with increase in the value of the chemical reaction parameter.

It is worthwhile to note that all the literatures above are limited to single phase flow and some considered non MHD fluid, but problems in the petroleum sector, magneto fluid dynamics etc are multiphase and MHD flow is a good way to transport fluids that are weak conductors in a microscale system (Dragisa *et al,* 2011).

Prathap-Kumar *et al* (2014) investigated chemical reaction effect on mixed convection flow of two immiscible viscous fluids in a vertical channel. The governing equations were solved both numerically by finite difference method and analytically by perturbation. It was found among other results that the Grashof number for mass and heat transfer affect the flow in the two regions with and without chemical reaction, the flow was suppressed by the first order reaction in both regions and viscous dissipation, viscosity ratio, width ratio, conductivity ratio enhance the flow. The two methods used were also found to agree very well for small value of perturbation parameter. It is important to state that although the flow is two phase as well the fluids under consideration are non MHD, viscous dissipation was considered and transport properties of the fluids were assumed to be constant.

Umavathi *et al* (2010) studied unsteady flow and heat transfer of porous media sandwiched between viscous fluids through a horizontal channel with isothermal wall temperature. Brinkman equation was used to model the flow in the porous region. The governing equations in the three regions were transformed to ordinary differential equation by perturbation where the period and non - periodic terms where collected separately. Boundary conditions were applied to the resulting ordinary differential equations for both region and a closed form solutions were obtained. The effect of physical parameters such as prandtl number, viscosity ratio, conductivity ratio, etc on the flow were computed numerically and presented graphically. It was found among other results that both the, Prandtl number, viscosity ratio & porous parameter have a negative effect on velocity and temperature profiles in the porous region as well as in the clear regions. The flow under consideration is three phase & non MHD and mass transfer is not considered in the study.

In line to the above literatures, this paper focuses on the effect of chemical reaction on unsteady MHD free convective two immiscible fluids flow.

Formulation of Problem

We consider two immiscible fluids flow in a horizontal channel with the assumptions that the upper channel is porous and the lower non porous. The fluid is bounded by two infinite horizontal parallel plates X and Z. There are two regions $y[0, h]$ and $y[-h, 0]$ depicted as Region I and Region II on the geometry as depicted below (Umavathi *et al* (2010)) Chemical Reaction is a process that involves rearrangement of the molecular or ionic structure of a substance, as distinct from a change in physical form or a nuclear reaction. There are two types of such reactions namely homogeneous reaction which occurs uniformly throughout a given phase of a flow and heterogeneous reaction which takes place in a particular region or within the boundary of a phase, (Umavathi, 2014).

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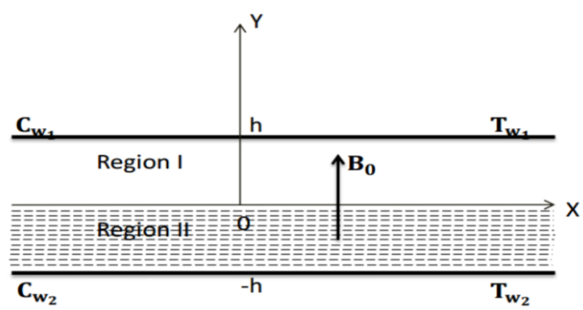

Figure 1: Geometric configuration of the system

The concentration and temperature at the walls are Cw_1, Cw_2 and Tw_1 and Tw_2. The fluid in Region I is having density ρ_1, dynamic viscosity μ_1, thermal conductivity k_1, thermal diffusivity D_1, while Region II is having density ρ_2, dynamic viscosity μ_2, thermal conductivity k_2 , thermal diffusivity D_2.

The heat transfer is due to the difference in temperature of the plates where Tw_1 is greater than Tw_2 and the density decreases in the direction of gravitational force.

It is also assumed that the flow is fully developed and all variables are functions of y' and t' only since the binding surface is infinite. The magnetic field is assumed very small. Therefore, the governing equations for the flow are given as follows:

Region I (Porous Region)

$$\frac{\partial v_1'}{\partial y'} = 0 \tag{1}$$

$$\rho_1\left(\frac{\partial u_1'}{\partial t'} + V_1'\frac{\partial u_1'}{\partial y'}\right) = -\frac{\partial P}{\partial x} + \mu_1\frac{\partial^2 u_1'}{\partial y'^2} - \sigma B_0 u_1' - \mu_1 u_1'/K' + \rho_1\beta_{f1}g(T_1' - T_{w1}') + \rho_1\beta_{c1}g(C_1' - C_{w1}') \tag{2}$$

$$\rho_1 c_p\left(\frac{\partial T_1'}{\partial t'} + V_1'\frac{\partial T_1'}{\partial y'}\right) = k_1\frac{\partial^2 T_1'}{\partial y'^2} - \frac{\partial q_r}{\partial y} \tag{3}$$

$$\frac{\partial C_1'}{\partial t'} + V_1'\frac{\partial C_1'}{\partial y'} = D_1\frac{\partial^2 C_1'}{\partial y'^2} - K_1(C_1' - C_{w1}') \tag{4}$$

Region II (Clear Region)

$$\frac{\partial v_2'}{\partial y'} = 0 \tag{5}$$

$$\rho_2\left(\frac{\partial u_2'}{\partial t'} + V_2'\frac{\partial u_2'}{\partial y'}\right) = -\frac{\partial P}{\partial x} + \mu_2\frac{\partial^2 u_2'}{\partial y'^2} - \sigma B_0 u_2' + \rho_1\beta_{f2}g(T_2' - T_{w1}') + \rho_2\beta_{c2}g(C_2' - C_{w1}') \tag{6}$$

$$\rho_2 c_p\left(\frac{\partial T_2'}{\partial t'} + V_1'\frac{\partial T_2'}{\partial y'}\right) = k_2\frac{\partial^2 T_2'}{\partial y'^2} - \frac{\partial q_r}{\partial y} \tag{7}$$

$$\frac{\partial C_2'}{\partial t'} + V_2'\frac{\partial C_2'}{\partial y'} = D_2\frac{\partial^2 C_2'}{\partial y'^2} - K_2(C_2' - C_{w1}') \tag{8}$$

Assuming the boundary and interface conditions on velocity are no slip.
Thus, the boundary and interface conditions on velocity for both fluids are:

$$U_1'(h) = 0$$
$$U_2'(-h) = 0$$
$$U_1'(0) = U_2'(0) \tag{9}$$
$$\mu_1\frac{\partial U_1'}{\partial y} = \mu_2\frac{\partial U_2'}{\partial y} \; at \; y' = 0$$

The boundary and interface conditions on temperature for both fluids are:

$$T_1'(h) = T_{w1}'$$
$$U_2'(-h) = T_{w2}'$$
$$T_1'(0) = T_2'(0) \tag{10}$$
$$k_1\frac{\partial T_1'}{\partial y} = k_2\frac{\partial T_2'}{\partial y} \; at \; y' = 0$$

The boundary and interface conditions on temperature for both fluids are:

$$C_1'(h) = C_{w1}'$$
$$C_2'(-h) = C_{w2}'$$
$$C_1'(0) = C_2'(0) \tag{11}$$
$$D_1\frac{\partial T_1'}{\partial y} = D_2\frac{\partial T_2'}{\partial y} \; at \; y' = 0$$

From equation (1) and (5), it is clear that the V_1' and V_2' do not vary with **y'**, they are function of time only. Thus, assuming $V_1' = V_2' = V'$, we can write the cross velocity as:
$V_1' = V_0(1 + \varepsilon A e^{iwt})$.
Where ω is the frequency parameter, ε is a small positive constant and A is a real positive constant such that εA<<1. It is assumed that the transpiration velocity varies periodically with time about a non-zero constant mean (Sturat, 1955). When εA=0, the constant transpiration is recovered.

Applying the following dimensionless quantities:

$$U_i = \frac{U_i'}{u},\; y = \frac{y'}{h},\; t = \frac{t'\upsilon_1}{h^2},\; V = \frac{h}{\upsilon_1}V' = \frac{v}{v_0},\; P\frac{-h^2}{\mu_1 u}\left(\frac{\partial P'}{\partial x'}\right),\; \theta_i = \frac{T_i' - T_{w1}'}{T_{w2}' - T_{w1}'},$$

$$\alpha_1 = \frac{\mu_2}{\mu_1},\; \gamma = \frac{D_2}{D_1},\; Pr = \frac{\mu_1 c_p}{k_1},\; \tau_1 = \frac{\rho_2}{\rho_1},\; \beta_1 = \frac{k_2}{k_1},\; \eta_1 = \frac{\beta_{f2}}{\beta_{f1}},\; \varphi_1 = \frac{\beta_{c2}}{\beta_{c1}},$$

$$K^2 = \frac{h^2}{K'},\; Sc = \frac{\upsilon_1}{D_1},\; M^2 = \frac{\sigma h^2 B_0^2}{\mu_1}, C_i = \frac{C_i' - C_{w1}'}{C_{w2}' - C_{w1}'},\; F = \frac{4I'h_1^2}{k_1},$$

$$\frac{\partial q_r}{\partial y} = 4(T_i' - T_{w1}')I',\; Gr = \frac{\rho_1 g h^2 \beta_{f1}(T_{w2}' - T_{w1}')}{\mu_1 u},\; Gc = \frac{\rho_1 g h^2 \beta_{c1}(C_{w2}' - C_{w1}')}{\mu_1 u},$$

$$\xi = \frac{\rho_1}{\rho_2} = \frac{1}{\tau_1},\; K_{c1} = \frac{K_1 h^2}{D_1},\; K_{c2} = \frac{K_2 h^2}{D_2}$$

To transform the governing equations to dimensionless form

REGION I

Dividing equation (2) by ρ_1 we get

$$\left(\frac{\partial u_1'}{\partial t'} + V_1'\frac{\partial u_1'}{\partial y'}\right) = \frac{\mu_1}{\rho_1}\frac{\partial^2 u_1'}{\partial y'^2} - \frac{1}{\rho_1}\frac{\partial P'}{\partial x'} - \frac{\sigma B_0 u_1'}{\rho_1} - \frac{\mu_1 u_1'/K'}{\rho_1} + \beta_{f1}g(T_1' - T_{w1}') + \beta_{c1}g(C_1' - C_{w1}') \tag{12}$$

$$y' = yh,\; T_1' = \theta_1(T_{w2}' - T_{w1}') + T_{w1}',\; t' = \frac{th^2}{\upsilon},\; \frac{\partial P'}{\partial x'} = -\frac{P\mu_1 u}{h^2},\; U_1' = U_1 u$$

Introducing the above dimensionless quantities into equation (12), we get

$$\frac{\partial U_1}{\partial t} + (1 + \varepsilon e^{i\omega t})\frac{\partial U_1}{\partial y} = \frac{\partial^2 U_1}{\partial y^2} + P - (M^2 - K^2)U_1 + G_r\theta_1 + G_c C_1 \tag{13}$$

Similarly, by inserting the appropriate dimensionless quantities into equations (3) and (4), we get

$$\frac{\partial \theta_1}{\partial t} + (1 + \varepsilon e^{i\omega t})\frac{\partial \theta_1}{\partial y} = \frac{1}{P_r}\frac{\partial^2 \theta_1}{\partial y^2} - \frac{F\theta_1}{P_r} \tag{14}$$

$$\frac{\partial C_1}{\partial t} + (1 + \varepsilon e^{i\omega t})\frac{\partial C_1}{\partial y} = \frac{1}{S_c}\frac{\partial^2 C_1}{\partial y^2} - \frac{K_{c1}C_1}{S_c} \tag{15}$$

REGION II

In similar manner as in Region I by inserting the appropriate dimensionless quantities into equation (6), (7) and (8) we get

$$\frac{\partial U_2}{\partial t} + (1 + \varepsilon e^{i\omega t})\frac{\partial U_2}{\partial y} = \alpha\xi\frac{\partial^2 U_2}{\partial y^2} + \xi P - \xi M^2 U_1 + G_r m\theta_2 + G_c\eta C_2 \tag{16}$$

$$\frac{\partial \theta_2}{\partial t} + (1 + \varepsilon e^{i\omega t})\frac{\partial \theta_2}{\partial y} = \frac{\xi\beta}{P_r}\frac{\partial^2 \theta_2}{\partial y^2} - \frac{\xi F\theta_2}{P_r} \tag{17}$$

The boundary and interface conditions for velocity are:

$$\frac{\partial C_2}{\partial t}+(1+\varepsilon e^{i\omega t})\frac{\partial C_2}{\partial y}=\frac{\gamma}{S_c}\frac{\partial^2 C_2}{\partial y^2}-\frac{\gamma K_{c2}C_2}{S_c} \quad (18)$$

$$U_1(1)=0$$
$$U_2(-1)=0$$
$$U_1(0)=U_2(0) \quad (19)$$
$$\frac{\partial U_1}{\partial y}=\alpha\frac{\partial C_2}{\partial y} \quad at \ \ y=0$$

The boundary and interface conditions for Temperature are:

$$\theta_1(1)=0$$
$$\theta_2(-1)=1$$
$$\theta_1(0)=\theta_2(0) \quad (20)$$
$$\frac{\partial\theta_1}{\partial y}=\beta\frac{\partial\theta_2}{\partial y} \quad at \ \ y=0$$

The boundary and interface conditions for Concentration are:

$$C_1(1)=0$$
$$C_2(-1)=1$$
$$C_1(0)=C_2(0) \quad (21)$$
$$\frac{\partial C_1}{\partial y}=\gamma\frac{\partial C_2}{\partial y} \quad at \ \ y=0$$

Solution of the Problem

To solve equation (12) to (18), we expanding the solutions in the perturbation term ε to separate the periodic and non – periodic parts. Assuming ε<<1.

$$U_1(y,t)=U_{10}(y)+\varepsilon e^{i\omega t}U_{11}(y)$$
$$\theta_1(y,t)=\theta_{10}(y)+\varepsilon e^{i\omega t}\theta_{11}(y)$$
$$C_1(y,t)=C_{10}(y)+\varepsilon e^{i\omega t}C_{11}(y)$$

$$U_2(y,t)=U_{20}(y)+\varepsilon e^{i\omega t}U_{21}(y)$$
$$\theta_2(y,t)=\theta_{20}(y)+\varepsilon e^{i\omega t}\theta_{21}(y)$$
$$C_2(y,t)=C_{20}(y)+\varepsilon e^{i\omega t}C_{21}(y)$$

We get:

Region I

The period and non-periodic parts for the velocity:

$$\varepsilon^0: U_{10}^{''}-U_{10}^{'}-(M^2+K^2)U_{10}=-P-G_r\theta_{10}-G_cC_{10} \quad (22)$$
$$\varepsilon^1: U_{11}^{''}-U_{11}^{'}-(M^2+K^2+i\omega)U_{11}=U_{10}^{'}-G_r\theta_{11}-G_cC_{11} \quad (23)$$

The period and non-periodic parts for the Temperature:

$$\varepsilon^0:\theta_{10}^{''}-\Pr\theta_{10}^{'}-F\theta_{10}=0 \quad (24)$$
$$\varepsilon^1:\theta_{11}^{''}-\Pr\theta_{11}^{'}-(i\omega\Pr+F)\theta_{11}=\Pr\theta_{10}^{'} \quad (25)$$

The period and non-periodic parts for the Concentration:

$$\varepsilon^0:C_{10}^{''}-ScC_{10}^{'}-K_cC_{10}=0 \quad (26)$$
$$\varepsilon^1:C_{11}^{''}-ScC_{11}^{'}-(i\omega Sc+K_c)C_{11}=ScC_{10}^{'} \quad (27)$$

Region II

The period and non-periodic parts for the velocity:

$$\varepsilon^0: U_{20}^{''}-\frac{U_{20}^{'}}{\alpha\xi}-\frac{U_{10}}{\alpha}=-\frac{P}{\alpha}-\frac{G_rm\theta_{20}}{\alpha\xi}-\frac{G_c\eta C_{20}}{\alpha\xi} \quad (28)$$
$$\varepsilon^1: U_{21}^{''}-\frac{U_{21}^{'}}{\alpha\xi}-(\frac{M^2\xi+i\omega}{\alpha\xi})U_{21}=\frac{U_{20}^{'}}{\alpha\xi}-\frac{G_rm\theta_{21}}{\alpha\xi}-\frac{G_c\eta C_{11}}{\alpha\xi} \quad (29)$$

The period and non periodic parts for the temperature:

$$\varepsilon^0:\theta_{20}^{''}-\frac{\Pr\theta_{20}^{'}}{\beta\xi}-\frac{F\theta_{20}}{\beta}=0 \quad (30)$$
$$\varepsilon^1:\theta_{21}^{''}-\frac{\Pr\theta_{21}^{'}}{\beta\xi}-(\frac{i\omega\Pr+F\xi}{\beta\xi})\theta_{21}=\frac{\Pr\theta_{20}^{'}}{\beta\xi} \quad (31)$$

The period and non periodic parts for the concentration:

$$\varepsilon^0:C_{20}^{''}-\frac{ScC_{20}}{\gamma}-K_{c2}C_{20}=0 \quad (32)$$
$$\varepsilon^1:C_{21}^{''}-\frac{ScC_{21}^{'}}{\gamma}-(\frac{i\omega Sc}{\gamma}+K_{c2})C_{21}=\frac{ScC_{20}^{'}}{\gamma} \quad (33)$$

The corresponding boundary and interface conditions become:

Region I:

Non Periodic Part

$$U_{10}(1)=0$$
$$U_{20}(-1)=0$$
$$U_{10}(0)=U_{20}(0) \quad (34)$$
$$\frac{\partial U_{10}}{\partial y}=\alpha\frac{\partial C_{20}}{\partial y} \quad at \ \ y=0$$

$$\theta_{10}(1)=0$$
$$\theta_{20}(-1)=1$$
$$\theta_{10}(0)=\theta_{20}(0) \quad (35)$$
$$\frac{\partial\theta_{10}}{\partial y}=\beta\frac{\partial\theta_{20}}{\partial y} \quad at \ \ y=0$$

$$C_{10}(1)=0$$
$$C_{20}(-1)=1$$
$$C_{10}(0)=C_{20}(0) \quad (36)$$
$$\frac{\partial C_{10}}{\partial y}=\gamma\frac{\partial C_{20}}{\partial y} \quad at \ \ y=0$$

Periodic Part:

$$U_{11}(1)=0$$
$$U_{21}(-1)=0$$
$$U_{11}(0)=U_{21}(0) \quad (37)$$
$$\frac{\partial U_{11}}{\partial y}=\alpha\frac{\partial C_{21}}{\partial y} \quad at \ \ y=0$$
$$\theta_{11}(1)=0$$
$$\theta_{21}(-1)=1$$
$$\theta_{11}(0)=\theta_{21}(0) \quad (38)$$
$$\frac{\partial\theta_{11}}{\partial y}=\beta\frac{\partial\theta_{21}}{\partial y} \quad at \ \ y=0$$

$$C_{11}(1)=0$$
$$C_{21}(-1)=1$$
$$C_{11}(0)=C_{21}(0) \quad (39)$$
$$\frac{\partial C_{11}}{\partial y}=\gamma\frac{\partial C_{21}}{\partial y} \quad at \ \ y=0$$

Consider equation (24)

$$\theta_{10}^{"}-\Pr\theta_{10}^{'}-F\theta_{10}=0$$
$$m^2-P_r m-F=0$$
$$\Rightarrow m_1=\frac{Pr+\sqrt{Pr^2+4F}}{2}, \ \& \ m_2=\frac{Pr+\sqrt{Pr^2+4F}}{2}$$

Thus

$$\theta_{10}(y)=C_1e^{m_1y}+C_2e^{m_2y} \quad (46)$$

Consider (26)

$$C_{10}^{"}-ScC_{10}^{'}-K_cC_{10}=0$$
$$\Rightarrow m^2-Scm-K_{c1}=0$$
$$\Rightarrow m_3=\frac{Sc+\sqrt{Sc^2+4Kc_1}}{2}, \quad \& \ m_4=\frac{Sc-\sqrt{Sc^2+4Kc_1}}{2}$$

,

$$C_{10}=C_3e^{m_3y}+C_4e^{m_4y} \quad (47)$$

Consider (23)

$$U_{10}^{"}-U_{10}^{'}-(M^2+K^2)U_{10}=-P-G_r\theta_{10}-G_cC_{10}$$
$$\Rightarrow m^2-m-V_3=-P-Gr(C_1e^{m_1y}+C_2e^{m_2y})-Gc(C_3e^{m_3y}+C_4e^{m_4y})$$
$$\Rightarrow U_{10_{Complementary}}=C_5e^{m_5y}+C_6e^{m_6y} \quad (48)$$

For particular solution we assume

$$U_{10_{Particular}}=K_1+K_2e^{m_1y}+K_3e^{m_2y}+K_4e^{m_3y}+K_5e^{m_4y}$$

Differentiating and substituting into the original equation to solve for K's, we get the following general solution

$$U_{10}=C_5e^{m_5y}+C_6e^{m_6y}+K_1+K_2e^{m_1y}+K_3e^{m_2y}+K_4e^{m_3y}+K_5e^{m_4y} \quad (49)$$

Consider (25)

$$\theta_{11}^{"}-\Pr\theta_{11}^{'}-(i\omega\Pr+F)\theta_{11}=\Pr\theta_{10}^{'}$$
$$\Rightarrow m^2-m\Pr-V_1=\Pr\theta_{10}^{'} \quad (50)$$

Substituting equation (46) into (50) and solving the homogeneous part

$$\theta_{11_{Complementary}}=C_7e^{m_7y}+C_8e^{m_8y} \quad (51)$$

Now let

$$\theta_{11_{Particular}}=K_6e^{m_1y}+K_7e^{m_2y} \quad (52)$$

Differentiating and substituting into the original equation to solve for unknown expressions, we get the following general solution

$$\theta_{11}=C_7e^{m_7y}+C_8e^{m_8y}+K_6e^{m_1y}+K_7e^{m_2y} \quad (52)$$

Nusselt number: The rates of heat transfer in the upper and lower plates are given as follows:

$$Nu(U)=\left[\frac{\partial\theta_{10}}{\partial y}\right]_{y=1}+\varepsilon e^{iwt}\left[\frac{\partial\theta_{10}}{\partial y}\right]_{y=1}$$
$$=c_1m_1e^{m1}+c_2m_2e^{m2}\varepsilon e^{iwt}(c_7m_7e^{m7}+c_8m_8e^{m8}+k_6m_1e^{m1}+k_7m_2e^{m2})$$

$$Nu(L)=\left[\frac{\partial\theta_{20}}{\partial y}\right]_{y=-1}+\varepsilon e^{iwt}\left[\frac{\partial\theta_{21}}{\partial y}\right]_{y=-1}$$
$$=c_{13}m_{13}e^{-m13}+c_{14}m_{14}e^{-m14}\varepsilon e^{iwt}(c_{19}m_{19}e^{-m19}+c_{20}m_{20}e^{-m20}+k_{25}m_{13}e^{-m13}+k_{26}m_{14}e^{-m14})$$

Sherwood number: The ratio of convective to diffusive mass transfer at the upper and lower plates is given as follows:

$$Sh(U)=\left[\frac{\partial C_{10}}{\partial y}\right]_{y=1}+\varepsilon e^{iwt}\left[\frac{\partial C_{11}}{\partial y}\right]_{y=1}$$
$$=c_3m_3e^{m3}+c_4m_4e^{m4}\varepsilon e^{iwt}(c_9m_9e^{m9}+c_{10}m_{10}e^{m10}+k_8m_3e^{m3}+k_9m_4e^{m4})$$

$$Sh(L)=\left[\frac{\partial C_{20}}{\partial y}\right]_{y=-1}+\varepsilon e^{iwt}\left[\frac{\partial C_{21}}{\partial y}\right]_{y=-1}$$
$$=c_{15}m_{15}e^{-m15}+c_{16}m_{16}e^{-m16}\varepsilon e^{iwt}(c_{21}m_{21}e^{-m21}+c_{22}m_{22}e^{-m22}+k_{27}m_{15}e^{-m15}+k_{28}m_{16}e^{-m16})$$

DATA PRESENTATION AND DISCUSSION OF RESULTS

Numerical evaluation of the analytical solutions obtained by regular perturbation method has been carried out where epsilon is taking as the perturbation parameter, and the graphical representation of the result obtained has been computed as depicted on figure 2 – 17. This is to explore the important features of the governing parameters in the velocity profile, temperature profile, and concentration profile. Throughout the computation, we employ constant values to the following governing parameters Gr=5, Gc=5, Pr=1, α=1, β=1, γ=1, ξ=1, η=1, Sc=0.78, F=3, K=2, M=1, P=1, Kc=0, except the varying ones in the respective figures.

In section 4.1, graphs and tables depicting relationships between various flow parameters and the flow are presented and discussed.

Table 1 demonstrates the effect of flow parameters in Sherwood number. It is observed that increase in Schmidt number leads to a decrease in Sherwood number at the upper plate and a decrease at the lower plate. An opposite behavior is observed due to increase in Chemical reaction parameter, while increase in conductivity ratio leads to an increase in Sherwood number at both plates.

Table 2 depicts the effect of flow parameters on Nusselt number at both plates. Increase in Prandtl number and conductivity ratio all cause decrease in Nusselt number at the upper plate with opposite behavior noticed at the lower plate. The Radiation parameter (F) has no effect on the Nusselt number at both plates.

Table 3 shows the effect of flow parameter on the Coefficient of skin friction. Increase in Grashof number for heat and mass transfer, Prandtl number, and Schmidt number all yield a decrease in the coefficient of skin friction at the upper plate and a corresponding increase at the lower plate. It is also observed that increase in Radiation Parameter, Hartman number, chemical reaction parameter and viscosity ratio results in an increase in the coefficient of skin friction at the upper plate and a corresponding decrease at the lower plate, and increase in porosity parameter increase the coefficient of skin friction at the upper plate only, while increase in conductivity ratio and diffusivity ratio increase both and decrease both the upper and lower plates respectively.

The effect of first order chemical reaction on concentration and velocity profile is depicted on figure 2 and 3 respectively. These figures show that an increase in the chemical reaction parameter (Kc) leads to a decrease in both the concentration and velocity profiles. This result is similar to what Prathap, Umavathi and Shreedevi obtained for mixed convective flow of two immiscible viscous fluids in a vertical channel.

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Ideally, taking the coefficient of chemical reaction to zero, the effect of all other flow parameter should be the same as in the original work. However, the porosity term in the original work was taking to Region II instead of Region I and although velocity no slip is assumed, the boundary conditions for temperature and concentration was as well interchange, these problems translated to the result originally obtained.
The effect of Grashof number for heat, and mass transfer on the velocity profiles are depicted on the figure 4 and 5 respectively. In both figures it is observed that as Grashof numbers increase, which literally means increase in buoyancy force over viscous force, the velocity increases in both regions with almost equal strength. However, the Grashof number for mass transfer increases the velocity more than the Grashof number for mass transfer.

Figure 6 and 7 depict the effect of Prandtl number (Pr) and Porosity Parameter (K) on the velocity profile. It is observed in figure 3 that as molecular diffusion on momentum over molecular diffusion of heat increases, the velocity increases in both regions. It is also observed from figure 4 that the velocity tends to its maximum as K tends to zero. This implies that an increase in K leads to a decrease in velocity in Region I (porous region) & also in Region II. However, the velocity drag in Region I is very large for large value of K. A similar result was observed by Umavathi, Liu, Prathap and Shaik-Meera for porous media sandwiched between viscous fluids.

Figure 8, 9, and 10 shows the effect of viscosity ratio (α), Radiation parameter (F), and Hartman number (M) on the velocity profile. It is found on figure 8 that as viscosity ratio increases the velocity decreases. Figure 9 shows that increase in the radiation parameter results to a decrease in the velocity profile in both regions. Increase in the Hartman number also results to a decrease in the velocity profile in both regions as depicted on figure 10. As for Hartman number, similar result was obtained by Muteen for two phase MHD fluid flow and Krishnamurthy et al for MHD boundary layer flow.
Figure 11 and 12 depict the effect of Schmidt number (Sc) and Diffusivity ratio (γ) on the velocity profile. It is observed on figure 11 that domination of viscous diffusion rate over molecular diffusion rate leads to an increase in the flow velocity profile in both regions. A similar behavior is observed on figure 12 as increase in diffusivity ratio leads to increate in velocity profile.
Figure 13, 14 and 15 portray the effect Prandtl number (Pr), Radiation parameter (F) and Conductivity ratio on the temperature profile. It is observed that increase in both Prandtl number and conductivity ratio lead to an increase in the temperature profile. However, increase in the Radiation parameter cause decrease in the temperature profile as shown on figure 14.

Figure 16 and 17 demonstrate the effect of Schmidt number (Sc) and Diffusivity ration (γ) on the concentration profile. It is observed on figure 16 that as viscous diffusion rate dominates the molecular diffusion rate, the concentration profile increase in both regions. A similar result is observed on figure 17 where increase in Diffusivity ratio causes an increase in the concentration profile in both regions.
When the chemical parameter $K_c = 0$ and ratio of diffusivity $\gamma = 0$, then our results will be in total agreement to the results of Joseph *et al* (2015)

Table 1: Effect of Flow Parameters on Sherwood Number

ε	Sc	L	Kc	Sh(U)	Sh(L)
0	0.78	1	1	-0.5549	-0.7131
0.025	0.78	1	1	-0.5511	-0.7666
0.025	2.62	1	1	-1.6711	-0.4385
0.025	0.78	2	1	-0.6218	-0.8093
0.025	0.78	1	2	-0.2223	-1.6913

Table 2: Effect of Flow Parameters on Nusselt Number

E	Pr	F	B1	Nu(L)	Nu(U)
0.001	1	3	1	-0.2009	-1.014
0.025	1	3	1	-0.2606	-0.9033
0.025	2	3	1	-0.233	-1.4215
0.025	1	5	1	-0.2606	-0.9033
0.025	1	3	2	-0.2383	-0.9037

Table 3: Effect of Flow Parameters on Coefficient of Skin Friction

E	Gr	Gc	Pr	F	K	M	Kc	Sc	α_1	β_1	Υ	$C_f(U)$	$C_f(L)$
0.001	5	5	0.5	1	2	1	1	0.78	1	1	1	-7.9776	9.7984
0.025	5	5	0.5	1	2	1	1	0.78	1	1	1	-7.2702	9.7239
0.025	10	5	0.5	1	2	1	1	0.78	1	1	1	-11.3199	14.7027
0.025	5	10	0.5	1	2	1	1	0.78	1	1	1	-10.0529	14.6298
0.025	5	5	1.5	1	2	1	1	0.78	1	1	1	-7.5382	9.7244
0.025	5	5	0.5	2	2	1	1	0.78	1	1	1	-7.0916	9.7177
0.025	5	5	0.5	1	5	1	1	0.78	1	1	1	-1.2422	9.7239
0.025	5	5	0.5	1	2	3	1	0.78	1	1	1	-2.9471	9.6758
0.025	5	5	0.5	1	2	1	2	0.78	1	1	1	-6.338	9.6146
0.025	5	5	0.5	1	2	1	1	2.62	1	1	1	-8.1267	9.767
0.025	5	5	0.5	1	2	1	1	0.78	2	1	1	-7.0879	9.6484
0.025	5	5	0.5	1	2	1	1	0.78	1	2	1	-7.2618	9.7266
0.025	5	5	0.5	1	2	1	1	0.78	1	1	2	-7.3507	9.7183

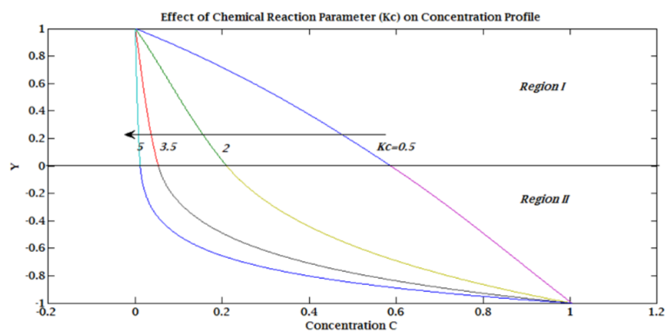

Figure 2: Effect of Chemical Reaction Parameter (Kc) on Concentration Profile

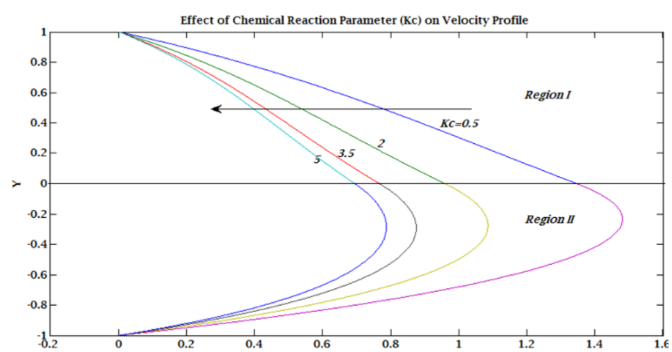

Figure 3: Effect of Chemical Reaction Parameter (Kc) on Velocity Profile

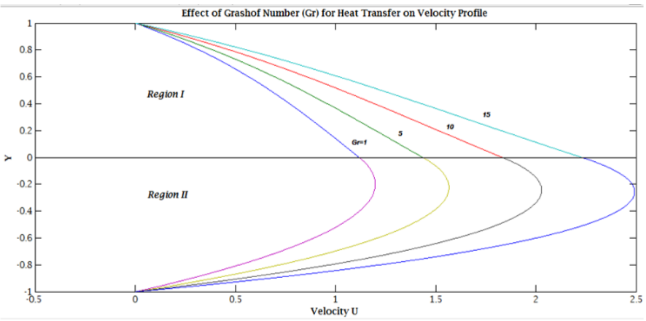

Figure 4: Effect of Grashof Number (Gr) for Heat Transfer on Velocity Field

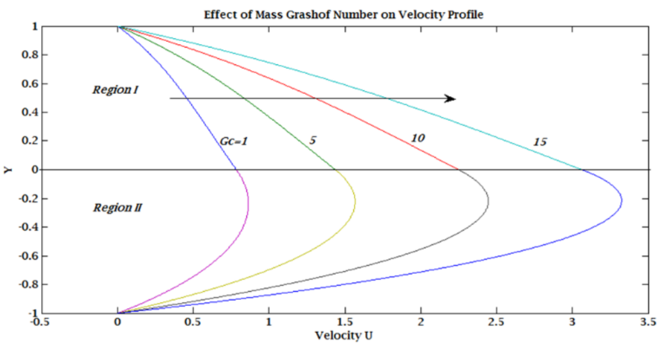

Figure 5: Effect of Grashof Number (Gc) for Mass Transfer on Velocity Field

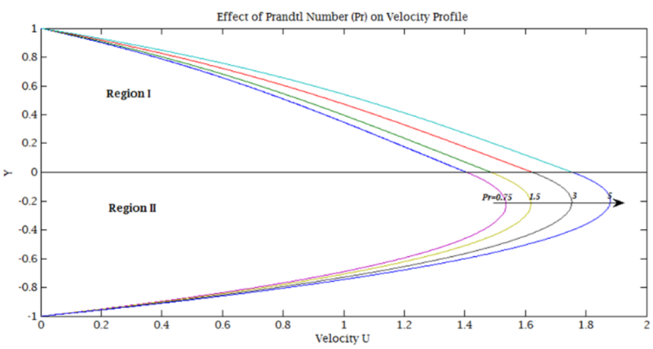

Figure 6: Effect of Prandtl Number (Pr) on Velocity Field

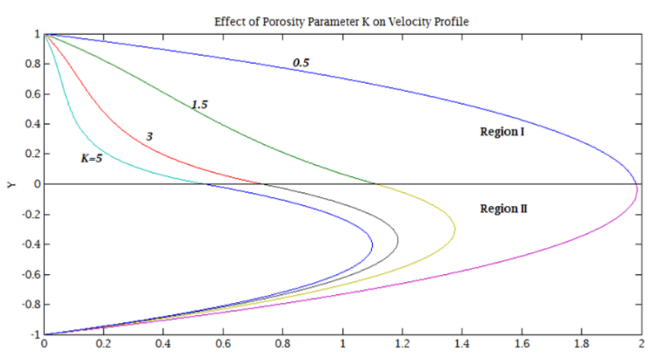

Figure 7: Effect of Porosity Parameter on Velocity Field

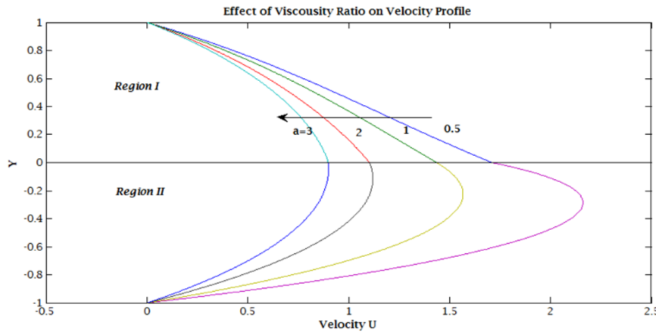

Figure 8: Effect of Viscosity Ratio on Velocity Profile

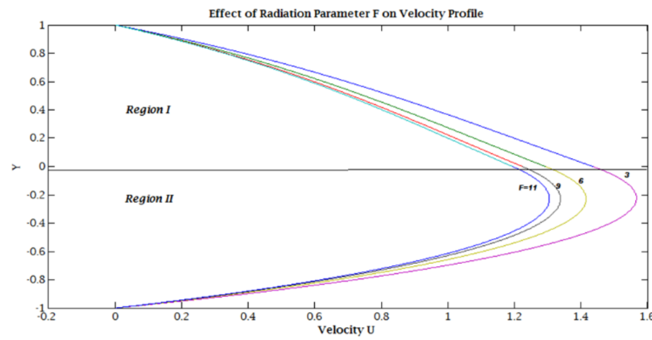

Figure 9: Effect of Radiation Parameter (F) on Velocity Profile

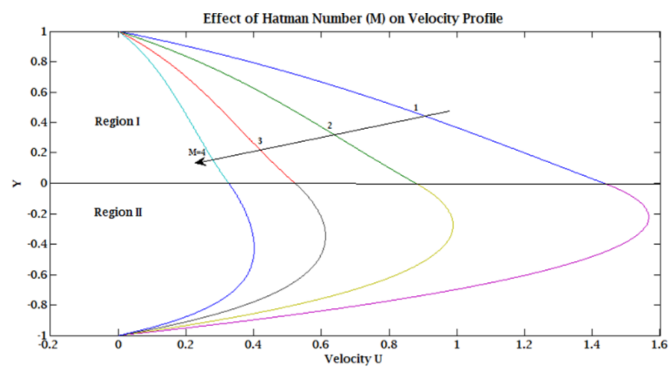

Figure 10: Effect of Hartman Number (M) on Velocity Profile

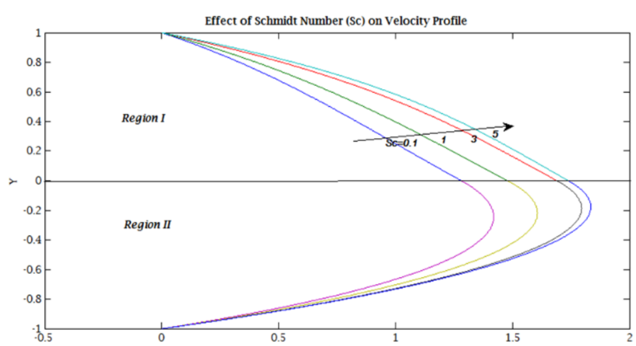

Figure 11: Effect of Schmidt Number on velocity Profile

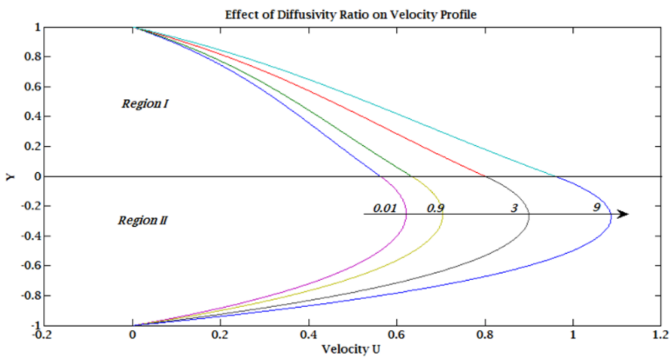

Figure 12: Effect of Diffusivity Ratio of Velocity Profile

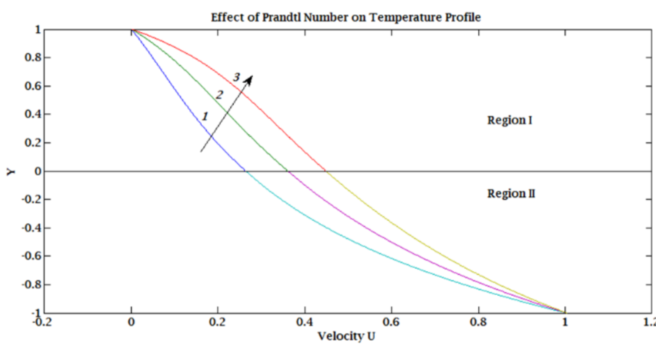

Figure 13: Effect of Prandtl Number (Pr) on Temperature Profile

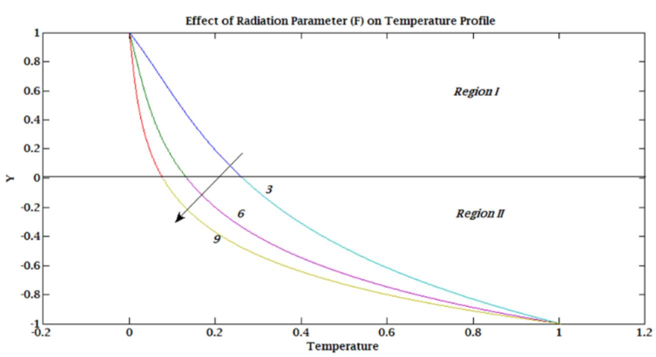

Figure 14: Effect of Radiation Parameter (F) on Temperature Profile

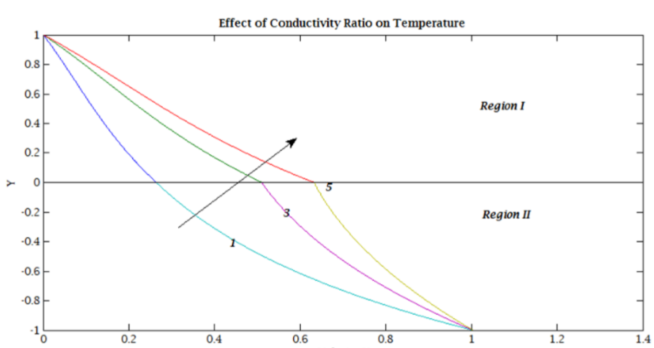

Figure 15: Effect of Conductivity Ratio on Temperature Profile

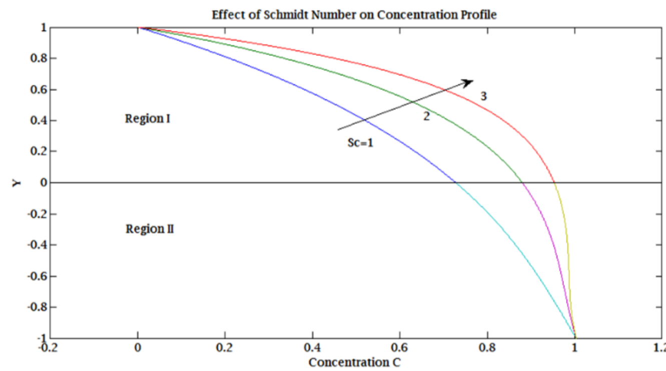

Figure 16: Effect of Schmidt Number (Sc) on Concentration Profile

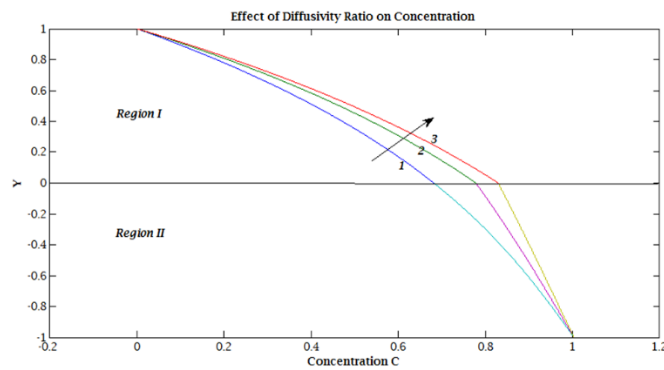

Figure 17: Effect of Diffusivity Ratio on Concentration Profile

Summary and Conclusion

In this paper, the effect of first order chemical reaction on unsteady MHD free convective two immiscible fluid flows with heat and mass transfer has been studied. The governing equations of the flow were converted to dimensionless form using appropriate non dimensional parameters. The resulting partial differential equations were transformed to ordinary differential equation by regular perturbation method. The velocity, concentration and temperature fields were obtained from the resulting ODEs. Graphs depicting the effect of chemical reaction and other flow parameters on the velocity, temperature and concentration profiles were obtained. The effect of flow parameters on coefficient of skin friction, Nusselt number and Sherwood number were also studied and tabulated appropriately.

It was concluded that the increase in chemical reaction coefficient/parameter K_c suppresses both velocity and concentration profiles.

Nomenclature

T_{w1}, T_{w2}	Wall Temperature
C_{w1}, C_{w2}	Wall Concentration
U, V	Velocity Components
t	Time
ʋ	Kinematic Viscosity
P	Pressure
K	Permeability of Porous Medium
α	Viscosity Ratio
B_o	Coefficient of Electromagnetic Field
M	Hartman Number
θ	Dimensionless Temperature
F	Radiation Parameter

Pr	Prandtl Number
C_p	Specific Heat at Constant Pressure
G_r	Grashof Number for Heat Transfer
G_c	Grashof Number for Mass Transfer
Sc	Schmidt Number
γ	Ratio of Diffusivity
A	Positive Real Number
g	Acceleration due to gravity
ε	Coefficient of periodic parameter
ω	Frequency Parameter
β	Thermal Conductivity

$V_1 = F + i\omega Pr$; $V_2 = i\omega Sc + KC_1{}^2$; $V_3 = M^2 + K^2$; $V_{30} = M^2 + K^2 + i\omega$; $V_4 = \frac{Pr}{(B_1E_1)}$;

$V_5 = \frac{F}{B_1}$; $V_6 = \frac{Sc}{L}$; $V_{60} = \left(Kc_2{}^2\right)$; $V_7 = \frac{1}{(a_1E_1)}$; $V_8 = \frac{M^2}{a_1}$;

$V_9 \frac{(FE_1+Pri\omega)}{(B_1E_1)}$; $V_{10} = \frac{(Sci\omega)}{L+Kc_1{}^2}$; $V_{11} = \frac{(E_1M^2+i\omega)}{(a_1E_1)}$;

$m_1 = \frac{Pr+\sqrt{Pr^2+4F}}{2}$; $m_2 = \frac{Pr-\sqrt{Pr^2+4F}}{2}$; $m_3 = \frac{Sc+\sqrt{Pr^2+4Kc_1{}^2}}{2}$;

$m_4 = \frac{Sc-\sqrt{Pr^2+4Kc_1{}^2}}{2}$; $m_5 \frac{1+\sqrt{1+4V_3}}{2}$;

$m_6 \frac{1-\sqrt{1+4V_3}}{2}$; $m_7 = \frac{Pr+\sqrt{Pr^2+4V_1}}{2}$; $m_8 = \frac{Pr-\sqrt{Pr^2+4V_1}}{2}$;

$m_9 = \frac{Sc+\sqrt{Pr^2+4V_2}}{2}$; $m_{10} = \frac{Sc-\sqrt{Pr^2+4V_2}}{2}$;

$m_{11} = \frac{1+\sqrt{1+4V_{30}}}{2}$; $m_{12} = \frac{1-\sqrt{1+4V_{30}}}{2}$; $m_{13} = \frac{V_4+\sqrt{V_4{}^2+4V_5}}{2}$;

$m_{14} = \frac{V_4-\sqrt{V_4{}^2+4V_5}}{2}$;

$m_{15} = \frac{V_6+\sqrt{V_6{}^2+4V_{60}}}{2}$; $m_{16} = \frac{V_6-\sqrt{V_6{}^2+4V_{60}}}{2}$; $m_{17} = \frac{V_7+\sqrt{V_7{}^2+4V_8}}{2}$; $m_{18} = \frac{V_7-\sqrt{V_7{}^2+4V_8}}{2}$;

$m_{19} = \frac{V_4+\sqrt{V_4{}^2+4V_9}}{2}$; $m_{20} = \frac{V_4-\sqrt{V_4{}^2+4V_9}}{2}$; $m_{21} = \frac{V_6+\sqrt{V_6{}^2+4V_{10}}}{2}$; $m_{22} = \frac{V_6-\sqrt{V_4{}^2+4V_{10}}}{2}$;

$m_{23} = \frac{V_7+\sqrt{V_7{}^2+4V_{11}}}{2}$; $m_{23} = \frac{V_7-\sqrt{V_7{}^2+4V_{11}}}{2}$; $r_1 = e^{m_{14}}$; $r_2 = e^{(m_{14}-m_{13})-1}$;

$r_3 = 1 - e^{(m_1-m_2)}$; $r_4 = B_1m_{14}e^{m_{14}}$; $r_5 = m_1 - m_2e^{m_1-m_2}$; $r_6 = B_1m_{14}e^{(m_{14}-m_{13})-(B_1m_{13})}$;

$r_7 = e^{m_{16}}$; $r_8 = 1 - e^{m_3-m_4}$; $r_9 = e^{(m_{16}-m_{15})-1}$; $r_{10} = Lm_{16}e^{m_{16}}$; $r_{11} = m_3 - m_4e^{m_3-m_4}$;

$r_{12} = Lm_{16}e^{(M_{16}-M_{15})-LM_{15}}$; $r_{13} = e^{(m_{18}-m_{17})-1}$; $r_{14} = 1 - e^{m_5-m_6}$; $r_{15} = m_5 - m_6e^{m_5-m_6}$;

$r_{16} = a_1m_{18}e^{(m_{18}-m_{17})-a_1m_{17}}$; $r_{17} = e^{(m_{20}-m_{19})-1}$; $r_{18} = 1 - e^{m_7-m_8}$; $r_{19} = m_7 - m_8e^{m_7-m_8}$;

$r_{20} = B_1m_{20}e^{(m_{20}-m_{19})-B_1m_{19}}$; $r_{21} = 1 - e^{m_9-m_{10}}$; $r_{22} = e^{(m_{22}-m_{21})-1}$; $r_{23} = m_9 - m_{10}e^{m_9-m_{10}}$;

$r_{24} = Lm_{22}e^{(M_{22}-M_{21})-LM_{21}}$; $r_{25} = e^{(m_{24}-m_{23})-1}$; $r_{26} = 1 - e^{m_{11}-m_{12}}$; $r_{27} = a_1m_{24}e^{(m_{24}-m_{23})-a_1m_{23}}$; $r_{28} = m_{11} - m_{12}e^{m_{11}-m_{12}}$; $c_{13} = \frac{r_3r_4-r_1r_5}{r_3r_6-r_3r_5}$; $c_1 = \frac{r_1-c_{13}r_2}{r_3}$;

$c_2 = -c_1e^{(m_1-m_2)}$; $c_3 = \frac{r_9r_{10}-r_7r_{12}}{r_9r_{11}-r_8r_{12}}$; $c_4 = -c_3e^{m_3-m_4}$;

$c_{14} = e^{m_{14}-c_{15}}e^{m_{14}-m_{13}}$; $c_{15} = \frac{r_7-c_3r_8}{r_9}$;

$c_{16} = e^{m_{16}-c_{15}}e^{m_{16}-m_{15}}$; $K_1 = \frac{P}{V_3}$

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