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SPACE UPPER BOUND ANALYSIS FOR TRANSFORMATION FROM ELEMENTARY REFERENCE-NET SYSTEM TO LOW-LEVEL P/T NETS

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ABSTRACT
Elementary Reference-nett Systems (ERS) is a class of Object-Oriented Petri Nets that follows the nets-within-nets paradigm. It combines theoretical properties as well as numerous practical needs for multi-agent-systems specification. However, it comes with some constraints that limit their expressiveness for automatic verification purposes due to the highly expressive nature of the underlying class of Petri nets. This article presents a set of transformation procedure from ERS to basic Petri nets in order to make verification feasible. It further establishes the space upper bound for the transformation which shows that the state space of the transformed P/T net grows exponentially as the number of object nets increases.

Keywords: Petri nets, object orientation, agent systems, interaction, mobility, transformation

1, INTRODUCTION

Petri nets (Murata, 1989) are a graphical and mathematical modelling formalism for describing and studying systems characterized as being concurrent, asynchronous, distributed and nondeterministic.

Several results on the theory as well as on the practical applications exist in the literature. To be able to handle large-scale system development, Petri nets are still being extended in many directions: Several concepts proposed assigning firing times to transitions (Holliday, 1987), another proposal allows tokens of different data types, called Coloured Petri nets introduced by Jensen (1987, 2013). More recently, efforts towards aligning system modelling by Petri nets with object-oriented programming languages have led to the development of the so-called object-oriented Petri nets (OOP-nets) that follows nets-within-nets paradigm introduced by Valk (1998) and reviewed in Valk (2003). This formalism permits structured objects, specified as nets to be in another net as tokens, and consequently leads to a nested system of nets. These extensions are suitable for modelling hierarchical multi-agent distributed systems. Some examples are worth mentioning: (Masri et al 2009) modelled and analysed wireless network protocols with OOP-nets. In the field of control system and engineering, there are many researches with OOP-nets: manufacturing system scheduling (Chen (200), and Miyamoto (2000)). For satellite tracking: Rinkcs et al (2014) developed the methodology for analysis of the automated guided vehicle systems using OOP-nets, just to mention a few. The formalism described in this paper is based on the Elementary Reference-net System (ERS), which is a framework for modelling systems that capture both nesting, mobility and interaction of objects. The formal definition of ERS has been published elsewhere (Abdullahi & Mǘller 2016) and is not repeated here. However, some key aspects of the formal definition are discussed in Sect. III. The idea behind the approach described there was to provide a path to verification of properties of a slightly modified version of a class of OOP-nets, as a formal representation of previously studied formalisms of the nets in the Reference Nets by Kummer (2001). Although the theoretical results were still preliminary to verification, they demonstrated, the algorithm for transforming ERS into a single low-level Petri nets in order to make verification feasible.

The objective of this article is to establish isomorphism between the states of an ERS and that of the generate Petri net and to analyse the transformation algorithm in terms of the state space upper bound. In Sect. II we introduce definitions from theory of Petri net that seem relevant in establishing relationship between the isomorphic properties of state spaces of ERS and the transformed P/T net. Formal definition of ERS is presented in III. We recapitulate the transformation procedure illustrated with an example in Sect. IV. In Sect. V, we summarise the present contributions of analysing the transformation of ERS to a behavioural equivalent low-level P/T net, and conclude with an outlook.

2. Fundamentals of Petri Nets

Here we give some definitions from theory of P/T-net, Relevant for our study.

Definition 1 (P/T net). *A place/transition (P/T net for short) is a tuple* $N = (P, T, F, W)$ *where P is a finite set of places, T is a finite set of transitions, disjoint from* P, $F \subseteq (P \times T) \cup (T \times P)$ *is the flow relation, and* $W: F \longrightarrow \mathbb{N}\backslash\{0\}$ *is the arc weight function. If P and T are finite, the net* N *is said to be finite. If* $W(x, y) = 1$ *for all arcs* $(x, y) \in F$ *we usually omit W in the tuple and simply write (P, T, F) for a P/T net. The preset of a node* $x \in P \cup T$*, denoted* $\bullet x$*, is the set containing the elements that immediately precede* x *in the net i.e.* $\bullet x = \{y \in P \cup T | (y, x) \in F\}$. *Analogously, the postset of a node, denoted* $x\bullet$*, can be defined.*

Definition 2 (Marking and Enabled transition). *A marking of a P/T net* $N = (P, T, F, W)$ *is a function* $m: P \longrightarrow \mathbb{N}$. *A P/T net system* $\Sigma=(N, m_0)$ *is a net* $N = (P, T, F)$ *together with an initial marking* m_0. *The zero marking with* $m(p) = 0$ *for all* $p \in P$ *is denoted by* $\mathbf{0}$. *A transition* $t \in T$ *is enabled in a marking* m *iff* $m(p) \geq W(p, t)$ *for all* $p \in \bullet t$. *Enablement of transition* t *in marking* m *is denoted by* $m[t >$. *A transition that is enabled* *may or may not fire. Firing removes tokens from places in the preset of t and puts new tokens onto places in the postset of t according to the arc weight function. A transition t that is enabled in a marking m i.e. $m[t >$, may fire. The successor marking m' is defined as $m'(p) = m(p) - W(p,t) + W(t,p)$. We denote this by $m[t > m'$. The set of reachable markings of Σ is the smallest (w.r.t. ⊆) set $RM(\Sigma)$ with $m_0 \in RM(\Sigma)$ and $m \in RM(\Sigma)$ iff $m'[t > m$, for some $t \in T$ and $m' \in RM(N)$. For a finite sequence of transition $\sigma = t_1, \dots t_k$, we write $m[\sigma > m'$ if there are markings $m_1, \dots m_{k+1}$ such that $m_1 = m, m_{k+1} = m'$ and $m_i[t_i > m_{i+1}$, for all $i = 1, \dots, k$. Σ is k-bounded if, for every reachable marking m and every place $p \in P$, $m(p) \le k$, and Σ is safe if it is 1-bounded. Moreover, Σ is bounded if it is k-bounded, for some $k \in N$. One can show that the set $RM(\Sigma)$ is finite if Σ is bounded i.e. if $|RM(\Sigma)| < \infty$. A P/T net is 1-safe if all arc weights are 1 and there is at most one token in each place in every reachable marking.*

3. Formal Definition of Ers

In this section, we recapitulate some key aspect of the formal definition of Elementary Reference-net System (Abdullahi & Bertie 2016). An ERS is comprised of a system net, a set of uniquely named object nets, variables labelling the arcs of nets and a synchronisation labelling for transitions. By convention, the components of the system net carry a hat: $\hat{P}, \hat{T}, \hat{p}, \hat{t}$, ... etc.

A. Static Structure

Definition 3 (Static Structure). *Let the triple $\eta_i = (i, N_i, m_i)$ be a named marked object net, where i, is a unique name of an object net; N_i is a structure of the object net, and m_i is a marking in N_i. (Let $\Sigma = \{(i_1, N_1, m_1), \dots, (i_k, N_k, m_k)\}$ be a finite set of unique marked named object nets). The structure of an object net with a unique name $i \in \Sigma$ is a p/t net $N_i = (P_i, T_i, F_i)$, where P_i, is the set of places of the object net, T_i is the set of its transitions and $F_i \subseteq (P_i \times T_i) \cup (T_i \times P_i)$ is the flow relation. Moreover, we assume that all sets of nodes (places and transitions) are pairwise disjoint and set $P_\Sigma := \bigcup_{\eta_i \in \Sigma} P_{\eta_i}$ and $T_\Sigma := \bigcup_{\eta_i \in \Sigma} T_{\eta_i}$. By $N_\bullet$ we denote the name of the object net which has no places or transitions so that we can have name for ordinary black tokens in our nets.*

Definition 4 (Elementary Reference-net System (ERS)). *Let Var be a finite set of named variables. An elementary reference-net system is a tuple $RS = (\hat{N}, \Sigma_{m^0}, \ell, \mathcal{w}, \boldsymbol{R^0})$ where*

- *$\hat{N} = (\hat{P}, \hat{T}, \hat{F})$ is a p/t net called a system net, where $\hat{P}$is its set of places, $\hat{T}$is its set of transitions and $\hat{F} \subseteq (\hat{P} \times \hat{T}) \cup (\hat{T} \times \hat{P})$ is the flow relation.*
- *$\Sigma_{m^0} := \{(i_1, N_1, m_1^0), \dots, (i_k, N_k, m_k^0)\}$, is a finite set of marked named object nets.*
- *$\ell \subseteq (\hat{T} \cup \{\hat{\tau}\}) \times (T_{1i} \cup \{\tau\}) \times, \dots, (T_k \cup \{\tau\}) \backslash \{\hat{\tau}, \tau, \dots, \tau)\}$, is the synchronisation relation, where $\hat{\tau}$ and τ are special symbols intended to denote inactions at the system and the object net levels respectively. If $\boldsymbol{t} = (\hat{t}, t_1, \dots, t_k)$ and $\hat{t}, \neq \tau$ and $\exists i \in \{1, \dots, k\}$ such that $t_i \neq \tau$, then we say that $\hat{N}$ and $N_i \in \Sigma$ for every $i \in \{1, \dots, k\}$ with $k = |\Sigma|$, participate in $\boldsymbol{t}$. This is the reason why $(\hat{\tau}, \tau, \dots, \tau)$ is excluded from the set of synchronisation relation: at least one object net must participate in every synchronisation action with the system net.*
- *$\mathcal{w}: \hat{F} \longrightarrow Var \cup \{N_\bullet\}$ is an arc labelling function such that for an arc $\hat{a} \in (\hat{F})$ adjacent to a place $\hat{p}$ the inscription of $\mathcal{w}(\hat{a})$ matches the name of object net in $\hat{p}$*
- *$\boldsymbol{R^0}$ specifies the initial making, where $\boldsymbol{R^0}: \hat{P} \to \mathbb{N} \cup MS(\Sigma)$ with $\Sigma = \{(i_1, N_1, m_1), \dots, (i_k, N_k, m_k\}$. It has to satisfy the condition $\boldsymbol{R^0}(\hat{p}) \in \mathbb{N} \Leftrightarrow \boldsymbol{R^0}(\hat{p}) \in \{N_\bullet\}$.*

In the example of Fig. 1 an $RS = (\hat{N}, \Sigma, \ell, \mathcal{w}, \boldsymbol{M^0})$ is shown, where $\Sigma = \{N_1, N_2\}$. Arcs of $\hat{N}$ can be identified by their labelling from $\mathcal{w}(\hat{t})$. Hence {x, y,} can be bound to marked named object nets in places $\hat{p}_1$ and $\hat{p}_2$ adjacent to transition $\hat{t}'$ to enable it. In the initial marking, places $\hat{p}_1$ and $\hat{p}_2$ contain references to the marked named object nets N_1 and N_2 respectively. They have the same structure and could be generated from a type N. $\mathcal{N} = \{i | (i, N_i, m_i) \in \Sigma\}$, is a finite set of object nets names.

More to the definition ERS, variables appearing on arcs adjacent to a transition $\hat{t}$ of the system net must satisfy the following conditions: Condition (1) variable appearing in the incoming arc of a system net transition $\hat{t}$ must appear in an outgoing arc of $\hat{t}$ or no such variable exist. Condition (2) variable appearing in an outgoing arc of a system net transition $\hat{t}$ must appear in an incoming arc of $\hat{t}$ or no such variable exist. Conditions (1) and (2) are imposed to prevent the creation of object net at run time and to prevent the destruction of existing ones when a system net transition fires. Condition (3) joining of two object nets is not permitted, and (4) No splitting of an object nets. (This is because in reality, complex physical entities cannot be cloned at run time). With these structural restrictions, ERS still retain the capability to describe nesting of object nets, synchronisation, and mobility, but do not allow splitting of the inner marking of an object net or joining the inner marking of several object nets. For instance, if assuming these inner markings as modelling the inner state of an agent, this is a reasonable restriction and shows that ERSs are well suitable to model physical entities

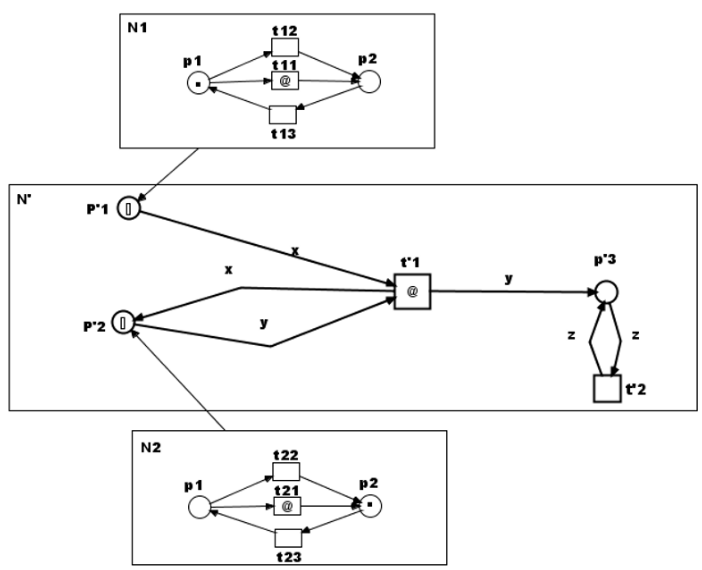

Fig. 1: An example of an ERS

B. Dynamic Behaviour

For the dynamic behaviour, we start by introducing the notion of marking for elementary reference-net system ERS under *reference semantics.* By Definition 4, an ERS contains a set $\Sigma_{m^0} := \{(i_1, N_1, m_1^0), \dots, (i_k, N_k, m_k^0)\}$ of marked named object nets. By ignoring the marking we obtain the set of (unmarked) named object nets $\Sigma := \{(i_1, N_1), \dots, (i_k, N_k)\}$. Hence in general a marking is given by:

1. a distribution of object nets or black tokens $\boldsymbol{R}: \hat{P} \to \mathbb{N} \cup MS(\Sigma)$ and
2. The vector $\boldsymbol{M} = (m_1, \dots, m_k)$ with the current marking of each N_i $(1 \le i \le k)$.

$\boldsymbol{R}$ specifies for each system net place $\hat{p}$, a number of black tokens (if $\hat{p}$ contains a black token or a multiset of unmarked named object nets we obtain the following Definition 5 below)

Sometimes by abuse of notation, for a named object net (i, N_i, m_i) in a place $\hat{p}$ of a marking $\boldsymbol{R}$ of the system net we write $\boldsymbol{R}(\hat{p}) = i$ meaning $\boldsymbol{R}(\hat{p}) = \{(i, N_i, m_i)\}$.

Definition 5. Given *an elementary reference-net system* $RS = (\hat{N}, \Sigma_{nt}, \ell, \mathcal{w}, \boldsymbol{R^0})$ *we define* $\mathcal{M} := \{M | M = (m_1, \dots, m_k) \wedge m_i \in MS(P_i)\}$. *Then a marking of an elementary reference-net system is a pair* (R, M) *where* $M \in \mathcal{M}$ *and* $\boldsymbol{R}: \hat{P} \to MS(\Sigma_{nt})$. *Specifying* M^0 *by the initial markings of the marked named object nets* $M^0 = (m_1^0, \dots, m_k^0)$ *we obtain the initial marking* $(\boldsymbol{R^0}, \boldsymbol{M^0})$ *of* RS.

The set of all markings of RS *is denoted by* $\mathcal{M}_{\mathrm{r}}$.

Like in other classes of High-level nets, variables are bound to names that are in a place $\hat{p}$ of a marking $\boldsymbol{R}$ of the system net when firing a transition in order to determine which tokens are removed from preset and which are added to postset.

Let $\hat{t} \in \hat{T}$ be a transition in the system net $\hat{N}$, then $\bullet\hat{t} = \{\hat{p} | (\hat{p}, \hat{t}) \in \hat{F}\}$, and $\hat{t}\bullet = \{\hat{p} | (\hat{t}, \hat{p}) \in \hat{F}\}$ are sets of its preset and postset. We denote by $\mathcal{w}(\hat{t}) := \{\mathcal{w}(\hat{p}, \hat{t}) | (\hat{p}, \hat{t}) \in \hat{F}\} \cup \{\mathcal{w}(\hat{t}, \hat{p}) | (\hat{t}, \hat{p}) \in \hat{F}\} = \bullet\hat{t} \times \{\hat{t}\} \cup \{t\} \times \hat{t}\bullet$ the set of all variables on arcs adjacent to $\hat{t}$.

A binding β *specifie*s which variables are bound to names, *where* $\beta: \mathcal{w}(\hat{t}) \cup \{\bullet\} \longrightarrow \mathcal{N} \cup \{N_\bullet\}$ *with* $\mathcal{N} = \{i | (i, N_{i,}, m_i) \in \Sigma\}$ *satisfying the conditions*: for each $x \in \mathcal{w}(\hat{t}) \cup \{\bullet\}$, there exist $i \in \mathcal{N}$ *such that* $\beta(x) = i$ *and if* $x = \bullet$ *then* $\beta(x) = N_\bullet$.

The *firing rule* will be introduced in three modes: First, we consider the mode when synchronisation occurs. In this mode we assume that a system net transition $\hat{t} \in \hat{T}$ and one or more object nets transition $t_i \in T_\Sigma$ of some object nets $N_i (i \ge 1)$ are activated and all transitions are related by the synchronisation relation ℓ. That is, $(\hat{t}, t_i) \in \ell$. This mode of the firing rule is called a synchronisation firing mode.

Definition 6 (synchronisation firing mode) *Let* $(\boldsymbol{R}, \boldsymbol{M})$ *be a marking of an elementary reference-net system* $RS = (\hat{N}, \Sigma_{nt}, \ell, \mathcal{w}, \boldsymbol{R^0})$, $\hat{t} \in \hat{T}$ *a transition of* $\hat{N}$ *and let* β *be a variable binding defined for all* $x \in \mathcal{w}(\hat{t}) \cup \{\bullet\}$. *Let* $\alpha_1, \dots, \alpha_k \in \Sigma_{nt}$ *be object nets involved in the firing of* $\hat{t}$. *Then* $\hat{t}$ *can fire provided that in each* $\alpha_i \in \Sigma_{nt}$ *for every* $i \in \{1, \dots, k\}$ *a transition* $t_i \in T_\Sigma$ *such that* $(\hat{t}, t_1, \dots, t_k) \in \ell$. *Then* $(\hat{t}, t_1, \dots, t_k)$ *is activated in* $(\boldsymbol{R}, \boldsymbol{M})$ *if:*

$\forall \hat{p} \in \hat{P}$: $(\beta(\mathcal{w}(\hat{p}, \hat{t})), N_{\beta(\mathcal{w}(\hat{p}, \hat{t}))}, m_{\beta(\mathcal{w}(\hat{p}, \hat{t}))}) \in \boldsymbol{R}(\hat{p})$ *and*
$\forall p \in P_i : \Pi_i(\boldsymbol{M}) \ge F_i(p, t_i),$. (1)

This is denoted by $(\boldsymbol{R}, \boldsymbol{M})[\hat{t}, t_i >$ *Let be* $m_i[t_i > m'_i$ *(w.r.t* N_i*). The successor marking* $(\boldsymbol{R'}, \boldsymbol{M'})$ *is defined by*

$\boldsymbol{R'}^{(\hat{p})} = \boldsymbol{R}(\hat{p}) \setminus \left(\beta(\mathcal{w}(\hat{p}, \hat{t})), N_{\beta(\mathcal{w}(\hat{p}, \hat{t}))}, m_{\beta(\mathcal{w}(\hat{p}, \hat{t}))}\right) \cup \left(\beta(\mathcal{w}(\hat{t}, \hat{p})), N_{\beta(\mathcal{w}(\hat{t}, \hat{p}))}, m_{\beta(\mathcal{w}(\hat{t}, \hat{p}))}\right) : \forall \hat{p} \in \hat{P}$ *and* $\boldsymbol{M'} = \boldsymbol{M_{i \to m_i}}$. (2)

This is denoted by $(R, M)[\hat{\boldsymbol{t}}, \boldsymbol{t_i} > (\boldsymbol{R'}, \boldsymbol{M'})$

If a system net transition is activated and without being included in the synchronisation relation, a chosen net-token does not change it current marking as it changes its location in the system net. Such a firing mode is called system-autonomous. The following definition can be seen as a special case of Definition 6 where the involved object net markings are not changed, i.e. $\boldsymbol{M'} = \boldsymbol{M}$.

Definition 7 *(system-autonomous firing mode).* *Let* $(\boldsymbol{R}, \boldsymbol{M})$ *be a marking of an elementary reference-net system* $RS = (\hat{N}, \Sigma_{nt}, \ell, \mathcal{w}, \boldsymbol{R^0})$ *and* $\hat{t} \in \hat{T}$ *a transition of* $\hat{N}$ *with a binding* β *such that* $\nexists(\hat{t}, x_i, \dots, x_k) \in \ell : \exists i \in \{1, \dots, k\} : x_i \ne \tau$. *Then* $\hat{t}$ *is activated in* $(\boldsymbol{R}, \boldsymbol{M})$ *if there is a net token such that:*

$(\beta(\mathcal{w}(\hat{p}, \hat{t})), N_{\beta(\mathcal{w}(\hat{p}, \hat{t}))}, m_{\beta(\mathcal{w}(\hat{p}, \hat{t}))}) \in \boldsymbol{R}(\hat{p}) \forall \hat{p} \in \hat{P}$. (3)

Since we use τ, *for inaction, this is denoted by* $(\boldsymbol{R}, \boldsymbol{M})[(\hat{t}, \tau) >$. *The successor marking* $(\boldsymbol{R'}, \boldsymbol{M'})$ *is defined by*

$$\forall \hat{p} \in \hat{P} : \boldsymbol{R'}(\hat{p}) = \boldsymbol{R}(\hat{p}) \setminus \left(\beta(\mathcal{w}(\hat{p}, \hat{t})), N_{\beta(\mathcal{w}(\hat{p}, \hat{t}))}, m_{\beta(\mathcal{w}(\hat{p}, \hat{t}))}\right) \cup \left(\beta(\mathcal{w}(\hat{t}, \hat{p})), N_{\beta(\mathcal{w}(\hat{t}, \hat{p}))}, m_{\beta(\mathcal{w}(\hat{t}, \hat{p}))}\right)$$
$$\boldsymbol{M'} = \boldsymbol{M}. \quad (4)$$

This is denoted by $(\boldsymbol{R}.\boldsymbol{M})[(\hat{t}_1, \tau) > (\boldsymbol{R'}, \boldsymbol{M'})$.

Definition 8 (object –autonomous firing mode) *Let* $(\boldsymbol{R}, \boldsymbol{M})$ *be a marking of an elementary reference-net system* $RS = (\hat{N}, \Sigma_{nt}, \ell, \mathcal{w}, \boldsymbol{R^0})$ *and* $t_i \in T_i$ *a transition of a net-token* $i = (i, N_i, m_i) \in \boldsymbol{R}(\hat{p})$ *for some* $\hat{p} \in \hat{P}$, *such that* $\nexists(\hat{t}, x_i, \dots, t_i, \dots, x_k) \in \ell$, *and* t_i *is activated in* N_i. *Then we say that* $(\hat{\tau}, t_i)$ *is activated in* $(\boldsymbol{R}, \boldsymbol{M})$ *(denoted* $(\boldsymbol{R}, \boldsymbol{M})[(\hat{\tau}, t_i) >$*]. The successor marking* $(\boldsymbol{R'}, \boldsymbol{M'})$ *of RS is defined by*

$\boldsymbol{R'} = \boldsymbol{R}$ *and*
$\boldsymbol{M'} = \boldsymbol{M}_{1 \to \mathrm{m_i}}$ *if* $m_i[t_i > m'_i$ *for* $\Pi_i(\boldsymbol{M}) = m_{\boldsymbol{i}}$. (5)

This is denoted by $(\boldsymbol{R}, \boldsymbol{M})[(\hat{\tau}, t_i) > (\boldsymbol{R'}, \boldsymbol{M'})$.

Definitions 6, to 8 could be easily merged. This is not done here to emphasise the differences.

To introduce the occurrence sequences for ERS *we assume an* ERS *as defined in Definition 4. Let* RS *be an* ERS *and* $(\boldsymbol{R}, \boldsymbol{M}), (\boldsymbol{R'}, \boldsymbol{M'}) \in \mathcal{M}_r$

Definition 9. *For a new alphabet* $\Gamma := (\hat{T} \cup \{\hat{\tau}\}) \times (T_1 \cup \{\tau\}) \times, \dots, (T_k \cup \{\tau\}) \setminus (\hat{\tau}, \tau, \dots, \tau)$ *where* $(\hat{\tau}, \tau, \dots, \tau)$ *denotes the neutral element of the free monoid* Γ^*, *we define:*

$(\boldsymbol{R}, \boldsymbol{M})[(\hat{\tau}, \tau, \dots, \tau) > (\boldsymbol{R'}, \boldsymbol{M'})$ *if* $(\boldsymbol{R}, \boldsymbol{M}) = (\boldsymbol{R'}, \boldsymbol{M'})$ *and*
$(\boldsymbol{R}, \boldsymbol{M})[\breve{w}(\hat{t}, \alpha) > (\boldsymbol{R'}, \boldsymbol{M'})\ if\ \exists(\boldsymbol{R''}, \boldsymbol{M''}) : (\boldsymbol{R}, \boldsymbol{M})[\breve{w} > (\boldsymbol{R''}, \boldsymbol{M''})\ and\ (\boldsymbol{R''}, \boldsymbol{M''})[(\hat{t}, \alpha) > (R', M')\ for\ some\ \breve{w} \in \Gamma^*, \hat{t}, \in \hat{T} \cup \{\hat{\tau}\}\ and\ \alpha \in ((T_1 \cup \{\tau\}) \times, \dots, (T_k \cup \{\tau\})$. (6)

To denote that $(\boldsymbol{R'}, \boldsymbol{M'})$ *is reachable from* $(\boldsymbol{R}, \boldsymbol{M})$ *by some occurrence sequence of actions we write* $(\boldsymbol{R}, \boldsymbol{M}) \overset{*}{\to} (\boldsymbol{R'}, \boldsymbol{M'})$.

The set of reachable markings of a reference system RS from a marking $(\boldsymbol{R}, \boldsymbol{M})$ *is denoted by* $R(RS, (\boldsymbol{R}, \boldsymbol{M}))$. $R(RS)$, *is the set of markings reachable from the initial marking* $(\boldsymbol{R^0}, \boldsymbol{M^0})$, *i.e.* $(R(RS) \coloneqq R(RS, (\boldsymbol{R^0}, \boldsymbol{M^0}))$. *The reachability graph* $(RG(RS)$ *is obtain as for P/T-net systems, i.e. RG(RS) is a labelled directed graph whose nodes is the set of reachable markings and edges are the tuples* $((\boldsymbol{R}, \boldsymbol{M}), (\hat{t}, \alpha), (\boldsymbol{R'}, \boldsymbol{M'})) \in \mathcal{M}_r \times (\hat{t}, \alpha) \times \mathcal{M}_r$ *where* $(\boldsymbol{R}, \boldsymbol{M}) \xrightarrow{(\hat{t},\alpha)} (\boldsymbol{R'}, \boldsymbol{M'})$.

We now extend the definition of 1-safe P/T-net to ERS. Safeness guarantees that the state space of a P/T-net is finite and for a 1-safe net, on each place at most one token resides. Many problems for 1-safe nets e.g., reachability, and liveness become decidable in polynomial space. A P/T net is 1-safe if and only if for all reachable marking there is at most one token on each place. In a 1-safe P/T-net all reachable markings can be interpreted as set (of marked places).

We introduce two conditions for safeness of ERS in Definition 10 as a generalisation of the safeness notion for P/T-nets.

Definition 10 (1-safe ERS) $Let\ RS = (\widehat{N}, \Sigma, \ell, \mathcal{w}, \boldsymbol{R^0})$ $be\ an\ ERS$. *RS is 1-safe if and only if all reachable markings are 1-safe and if and only if in all reachable markings there is at most one net-token on each system net place and each net-token is 1-safe:*

- $\forall (\boldsymbol{R}, \boldsymbol{M}) \in R(RS), \forall \hat{p} \in \hat{P}: (R(\hat{p}),) \leq 1$ *and*
- $\forall (i, N_i, m_i) \in \boldsymbol{R}(\hat{p})$: $\forall p_i \in P_i : \forall \hat{p} \in \widehat{P}\ (\boldsymbol{R}(\hat{p}), \Pi_i(\boldsymbol{M}(p_i)) > 0 \Longrightarrow \Pi_i(\boldsymbol{M}(p_i)) \leq 1.$

Observe that by this definition in the reachable marking of safe ERS each system net place is marked with at most one object net. The set $\{\boldsymbol{R}(\hat{p}) | \boldsymbol{R}(\hat{p}) \in R(RS), \hat{p} \in \hat{P}\}$ is thus a finite set and similar to the reachability set of safe P/T-net. Furthermore, the net-tokens are also safe by the definition and thus the set of reachable markings associated with them is finite too. This combination of the finite set of reachable markings associated with the system net and finite set of reachable markings associated the object net results in a finite state space for the ERS.

Observation 1. *Given an ERS if for all reachable markings there is at most one token on each system net place and each net-token is 1-safe, then all reachable markings are 1-safe.*

Theorem 1. *If an ERS is safe, then its set of reachable markings is finite.*

Proof. Let RS be a safe ERS. Let $m \coloneqq |\hat{P}|$ and $n \coloneqq max\{|P_i| \,|(i, (P_i.T_i, F_i), m_i) \in \mathcal{N}\}$ be the number of system net places and the maximum number of places present in an object net, respectively.

By definition of safe ERS each object net is 1-safe and hence there are at most 2^n different markings an object net may have. By definition of safe ERS each system net place is either marked or unmarked with an object net with one of these markings, thus there are up to $(1 + 2^n)^m$ different markings of RS, i.e. $|R(RS)| \leq (1 + 2^n)^m$.

□

4. The Transformation Procedure

In the following, we represent the set of transformation of Elementary Reference-net system into P/T nets as demonstrate in (Abdullahi & Bertie 2016). There are five rules which must be applied in sequence from Rule 1 to Rule 5. The first rule generates the set of places of the target P/T-net. The second rule defines the initial marking for the P/T-net. The third rule generates a family of transitions and arcs for each autonomous transition in the system net, and rule 4 generates family of transitions and arcs for each autonomous transition in an object net from the set of all marked named object net. The fifth rule creates a family of synchronisation transitions which belongs to the system net, and synchronization transitions in each object net by combining Rule 3 and Rule 4.

The set of transformation rules will be illustrated with the example of an elementary reference-net system shown in Fig. 1. In the figure, there is the system net N' represented by a net in the middle. Tokens residing in places p'_1 and p'_2 are references to marked named object nets N_1 and N_2. Their structures and inner markings are shown on the top and bottom of the system net. This net system will be translated into a P/T net system N^*.

The main idea is to construct a simulating P/T-net where the set of places consists of tuples $(\hat{p}, i)$ where $\hat{p}$ is a system net place and i ranges over all marked object net names that resides on $\hat{p}$. This set is finite due to the boundedness of the ERS. The set of transitions coincides with the set of transition relation $(\hat{t}, t_1, \ldots, t_k) \in \ell$ in the marking $(\boldsymbol{R}, \boldsymbol{M})$ that is encoded in the connections from the places to these transitions. Also since ERS is finite there are only *finitely many firing modes and hence the set of transition is also finite.*

A. Transformation Rules

Let $RS = (\widehat{N}, \Sigma, \ell, \mathcal{w}, \boldsymbol{R^0})$ *be an ERS with a set* Σ_{nt} *of all marked named object nets in the initial marking. By* R *we denote the set of all names used in* Σ_{nt} *and by* $R_i \subseteq R$ *the subset of all names for marked object nets* i. *The net system* RS *will be translated into a flat P/T-net system* $N^* = (P^*_{N^*}, T^*_{N^*}, F^*_{N^*}, M^*_0)$ *where* M^*_0 *is the initial marking.*

Rule 1: *Generate places. To generate the set* $P^*_{N^*}$ *of places of a P/T-net* N^* *we define two separate sets. The first, is the set* P'_{N^*} *of places from the system net* $\widehat{N}$, *and the second the set* P_{N^*} *of all places of each object net in the initial marking of the system net. Finally, we take the union of these set as the set* $P^*_{N^*}$ *of a target P/T-net* N^*, *with the assumption that* $P'_{N^*} \cap P^*_{N^*} = \emptyset$.

he first set of places of N^* *is generated by duplicating all places of the system net for each net-token name* $(i, N_i, m_i), (i \geq 1)$ *used in the initial marking of the system net and labelled it with a pair* (p', i) *where* p'*is a place in* $\widehat{P}$ *and* i *is the name of the possible net-token that reside on* p'. *Thus the set* P'_{N^*}*of places of* N^**from place of the system net is defined as follows:*

$$P'_{N^*} \coloneqq \cup_{p' \in \hat{P}} \{(p', i) | i \in R, i \geq 1\}\,. \qquad (7)$$

The second set of places of N^* *is generated by taking a copy of each place in the set* P_i *for each net-token and labelled it with a pair* (p_i, i) *where* p_i *is a place in* P_i *and* i *is the name of the net-token. Thus the set* P_{N^*}*of* N^* *from places of each net-token is defined as follows:*

$$P_{N^*} \coloneqq \cup_{i \in \Sigma_{nt}} \{(p_i, i) | p_i \in P_i, i \in R, i \geq 1\}\,. \qquad (8)$$

Therefore the set $P^*_{N^*}$ *of a target P/T-net* N^**is the union of these set, namely*

$$P^*_{N^*} \coloneqq P'_{N^*} \cup P_{N^*}\,. \qquad (9)$$

For the example ERS in Fig. 1, each place in system net is duplicated and labelled with each net-token name represented in the initial marking as $(p'_1, 1), (p'_1, 2), (p'_2, 1), (p'_2, 2), (p'_3, 1), (p'_3, 2)$ and one copy of places in the set P_i for each net-token N_i as

$(p_1, 1), (p_2, 1), (p_1, 2), and\ (p_2, 2)$ in P/T-net N^* from Rule 1 as shown in Fig. 2 below.

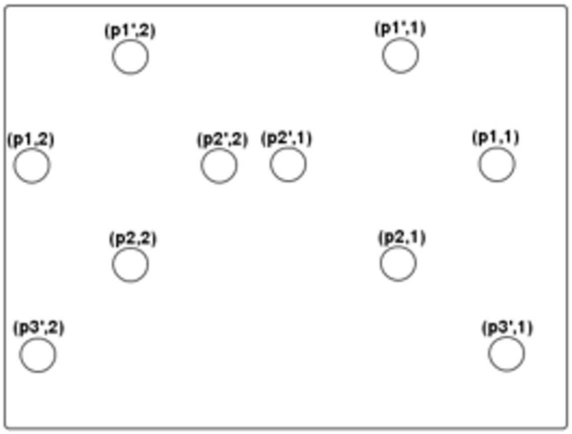

Fig. 2: Set of places of P/T net

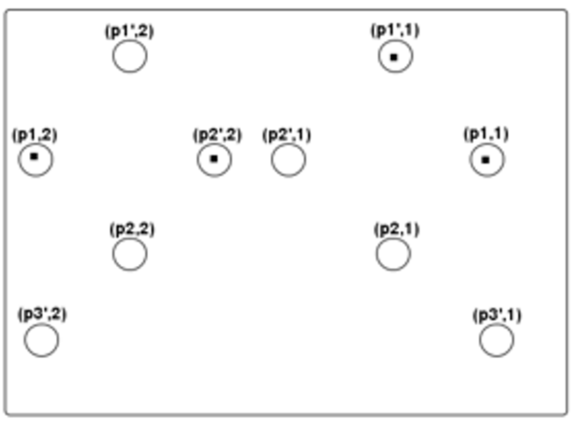

Fig. 3: Set of places with initial marking

Rule 2: *Define the initial marking for N^*. For a P/T-net N^* we define an encoding of markings on places from the set of places $\hat{P}$ in an ERS by markings on the generated places from $P^*_{N^*}$. If an object net with name $i \in R$ resides in a place $\hat{p}$ in an initial marking $R^0(\hat{p})$ of the system net, then the number of black tokens in place $(\hat{p}, i) \in P^*_{N^*}$ in the initial marking M^*_0 in the constructed net is the number of appearances of name i in the multiset $R^0(\hat{p})$, namely*

$$M^*_0(\hat{p}, i) = R^0(\hat{p}). \tag{10}$$

Similarly, we define an encoding of markings on places from the set of places P_i on the generated places from $P^*_{N^*}$. If all places (p, i) for all p such that $(p, i) \in P^*_{N^*}$ is marked in the initial marking M^0 of the net-token $i \in \mathbb{R}_i$, then the number of black tokens in place $(\hat{p}, i) \in P^*_{N^*}$ in M^*_0 is the number black tokens in $M^0(p)$, namely

$$M^*_0(p, i) = M^0(p). \tag{11}$$

If a place in the system net is a place that contains a black token, then the unique copy corresponding to the place in N^* is also marked with a black token. It is easy to see that this encoding defines a one-to-one correspondence between marking in ERS and 1-safe markings in N^*. In the given ERS, reference to the object net N_1 resides in $\hat{p}_1$, and reference to the object net resides in $\hat{p}_2$. Hence, we have tokens in $(p'_1, 1)$ and $(p'_2, 2)$ for N^*. Likewise, we define the markings for places $(p_1, 1)$ and $(p_1, 2)$. This is illustrated in Fig. 3.

Rule 3: *Generate a family of P/T-net transitions from a system net. We define a set T^*_{sat} of transitions of N^* obtained from each autonomous transition of the system net $\hat{N}$ by duplicating each autonomous transition for each input arc variable of $\hat{t}$ that may be bound to any of the named net-token name in each place adjacent to $\hat{t}$ with appropriate input and output arcs, in N^*.*

$$T^*_{sat} := \bigcup_{\hat{t}\in\hat{T}}\{t'_{\beta_i(x)} | x \in w(\hat{t}): \hat{t}\ is\ a\ system\ autonomous\ transition\} \tag{12}$$

In the example ERS, the set $w(\hat{t})$ of input arc variables that can be bound to a named net-token for t'_2 is as follows:

$$\beta(w(t'_2)) = \{\beta_1 = (z = 1)\ \beta_2 = (z = 2) \tag{13}$$

Where in binding β_1 the named object net 1 is bound to the input arc variable z and in binding β_2, the named object net 2 is also bound to the input arc variable z, respectively.

Therefore, two transitions $t'_{21}\ and\ t'_{22}$ are generated for transition t'_2 from Rule 3.

The appropriate input and output arcs are added. We define a set F^*_{sat} representing arcs of system autonomous transitions in N^*as follows:

$$F^*_{sat} = \bigcup_{\hat{a}\in\hat{F}}\{(x', y' | (x, y) = w(\hat{a}), x' \in P'_{N^*}(x) \cup T^*_{sat}(x), y' \in P'_{N^*}(y) \cup T^*_{sat}(y)\} \tag{14}$$

The set F^*_{sat} gives all possible pairs of a place and a transition representing each system autonomous transition to be drawn in N^*. This is shown in Fig. 4.

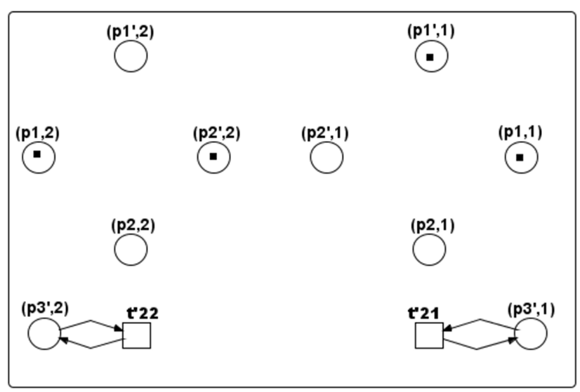

Fig. 4: Transitions and arcs generated by Rule 3

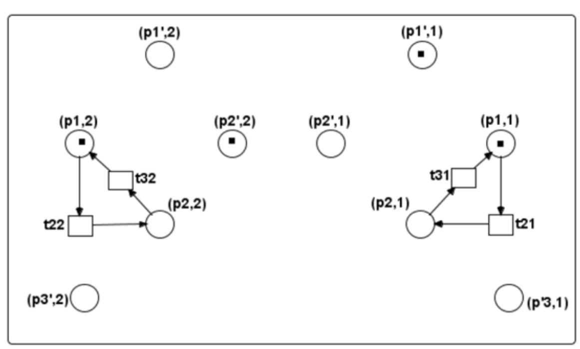

Fig.5: Transitions and arcs generated after Rule 4

Rule 4: Generate a family of transitions representing autonomous transitions in each net-token. For a *set* T^*_{nat} of transitions of N^* obtained from each autonomous transition in the object net with name $(i, N_i, m_i) \in \Sigma_{nt}$ we define a set of similar autonomous transitions as follows.

$$T^*_{nat} \coloneqq \cup_{i \in \Sigma_{nt}} \{t | t_i \in T_i \wedge t_i \notin \boldsymbol{\ell}\} \quad (15)$$

In the example four transitions $t_{21}, t_{22}, t_{31}, t_{32}$ are generated. For each autonomous transition $t_{21} and\ t_{31}$ in object net $i = 1$, similar transitions $t_{11} and\ t_{31}$ are generated in N^*, also, for each autonomous transition $t_{22} and\ t_{32}$ in object net $i = 2$, similar transitions $t_{22} and\ t_{32}$ are generated in N^* from Rule 4.

Once more, appropriate input and output arcs are drawn so as to keep input and output places for each transition representing an object net autonomous transition $t \in T^*_{nat}$ *in* N^*. Let us define a set F^*_{nat} *of arcs of object nets autonomous transitions in* N^*as follows*:*

$$F^*_{nat} = \cup_{a_i \in F_i} \{ (x'_i, y'_i) | \left(x_i, y_i\right) \in F_i, x'_i\ P_{N^*}(x_i) \cup T^*_{nat}\left(x_{i,}\right), y'_i\ \in P_{N^*}(y_i)\ \cup T^*_{nat}\left(y_{i,}\right)\} \quad (16)$$

The set F^*_{nat} gives all possible pairs of a place and a transition representing each object net autonomous transition to be constructed n in N^*. This is depicted in *Fig. 5 above, for the example ERS*

Rule 5: *Generate a family of transitions representing synchronisation transitions obtained from the system net and object nets. An occurrence of a synchronous firing presumes simultaneous occurrence of a transition $\hat{t} \in \hat{T}$ with a set of transitions given by a binding β in system net, and some net-tokens transitions $(t_1, \ldots, t_k) \in \ell$. Therefore, a corresponding synchronisation transition in the P/T-net N^*, is composed of each synchronous transitions in each possible object net referenced in the initial marking of the system net that can occur synchronously with the system net together with a synchronous transition in the system net. This can be viewed as a combination of Rule 3 and Rule 4 with the condition that all involved transitions must be elements in the transition relation ℓ of an ERS.*

Transitions $(t_1, \ldots, t_k)$ occur simultaneously with $\hat{t} \in \hat{T}$ of a system net, if $(\hat{t}, (t_i, \ldots, t_k)) \in \boldsymbol{\ell}$. We generate synchronisation transitions from an ERS in a P/T-net N^* accordingly. This implies that we will have $|\ell|$ such transitions in N^*. Each of these transitions is composed of a system net transition $\hat{t} \in \hat{T}$, and some transitions of net-tokens that participate in synchronous firing of $\hat{t}$. The family of these transitions sets is defined as follows.

$$T^*_{sync_i} \coloneqq \cup^k_{i=1} \{t_{i.\beta_i(x)} = \{\hat{t}, t_1, \ldots, t_k\} | x \in w(\hat{t}), \hat{t} \in \hat{T}, t_1 \in T_1, \ldots, t_k \in T_k\}. \quad (17)$$

In our example two places $\hat{p}_1\ and\ \hat{p}_2$ are marked with one net-token each in the initial marking. We add two transitions $t_1 = \{\{\hat{t}_1, t_{21}, \tau\}$ *and* $t_2 = \{\hat{t}_1, \tau, t_{22}\}$ annotated with *@1* and *@2*, which is shown in Fig. 6.

The result of transforming ERS into P/T-net is shown in Fig. 7

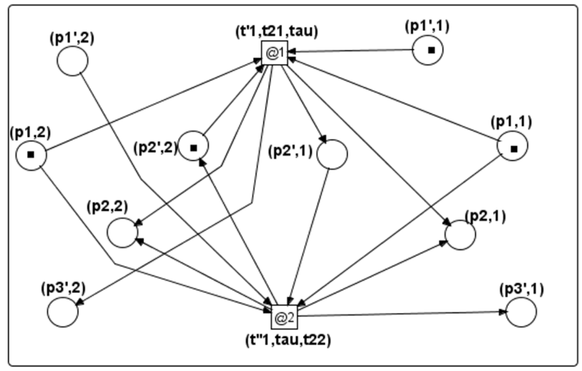

Figure 6: Synchronous firing transitions and arcs

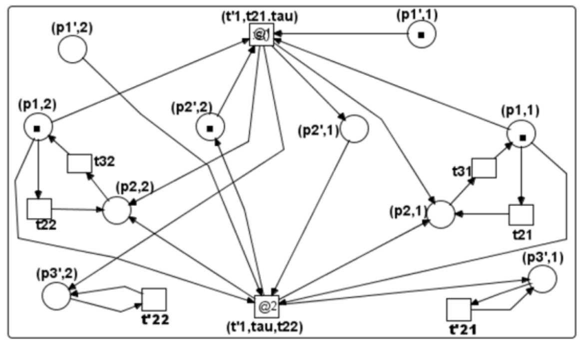

Figure 7: Result of transforming ERS in Fig. 1 into a P/T net

5. Isomorphic Property of the State Spaces and Analysis of Space Upper Bound

A. Isomorphic Property of State Space Spaces

In this subsection we establish an isomorphism between the states of an ERS and the generated 1-safe P/T-net. In Rule 2 we defined two separate initial markings for the P/T-net N^*: $\mathrm{M}^*_0(\hat{\mathrm{p}}, \mathrm{i})$ and $\mathrm{M}^*_0(\mathrm{p}, \mathrm{i})$. The former is an encoding of markings from the set of places $\hat{\mathrm{P}}$ of the system net in an ERS and the latter is an encoding of markings from the set of places $\mathrm{P_i}$ of an object nets i. Likewise, we defined three sets of transitions: $\mathrm{T}^*_{\mathrm{sat}}$, $\mathrm{T}^*_{\mathrm{nat}}$, and $\mathrm{T}^*_{\mathrm{sync_i}}$ from Rule 3, Rule 4 and Rule 5 respectively in N^*. In the following, we define some mappings from the P/T-net to an ERS.

Definition 11. *A mapping $\hat{f}$ maps a marking M^* of a P/T-net N^* from the set of places $\hat{P}$ to markings R of a system net of an ERS as follows:*

$$\hat{f}(M^*)(\hat{p}, i) = R(\hat{p})\ such\ that\ (\hat{p}, i) \in P^*_{N^*}: \hat{p} \in \hat{P}: i \in R \quad (18)$$

Definition 12 *A mapping f maps a marking M^* of a P/T-net N^* from the set of places P_i of net-token i of an ERS to a marking M of an object net of an ERS as follows:*

$$f(M^*)(p, i) = M(p)\ such\ that\ (p, i) \in P^*_{N^*}: p \in P_i: i \in R \quad (19)$$

Definition 13 *$\hat{g}$ is a mapping that maps a transition $t'_{\beta_i(x)} \in T^*_{sat}$ of P/T-net N^* to a system-autonomous firing mode $(\hat{t}, \tau) \notin dom(\ell)$ of an ERS as follows:*

$$\hat{g}\left(t'_{\beta_i(x)}\right) = (\hat{t}, \tau)\,, \quad (20)$$

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Definition 14 *g is a function that maps a transition $t \in T^*_{nat}$ of P/T-net N^* to an object-autonomous firing mode $(\tau, t_i) \notin dom(\ell)$ of an ERS as follows.*
$g(t) = (\tau, t_i)$. (21)

Definition 15 *g_s is a mapping function that maps a transition $t_{i.\beta_i(x)} \in T^*_{sync_i}$ of P/T-net N^* to a synchronisation firing mode $(\hat{t}, t_1, \dots, t_k) \in \ell$ of an ERS as follows:*
$g_s(t_{i.\beta_i(x)}) = \{(\hat{t}, t_1, \dots, t_k)\}$ (22)
With respect to these definitions, the following lemmas related to ERS and a P/T-net constructed by Rules 1 to 5, hold.

Lemma 1 *For the initial marking at the system net level, the following equality holds:*
$R^0(\hat{p}) = \hat{f}\,(M_0^*)(\hat{p}, i)$. (23)

Lemma 2 *Suppose that $R = \hat{f}(M^*)$ and $(\hat{t}, \tau) = \hat{g}(t'_{\beta_i(x)})$. The following proposition holds:*
$M^*[t'_{\beta_i(x)} > \Leftrightarrow R[(\hat{t}, \tau) >$. (24)

Lemma 3 *Suppose that $R_1 = \hat{f}(M_1^*)$, $M_1^*[t'_{\beta_i(x)} > M_2^*$, and $R_1[\hat{g}(t'_{\beta_i(x)}) > R_2$. The following equality holds.*
$R_2 = \hat{f}(M_2^*)$. (25)

Lemma 4 *For the initial marking of the object net, the following holds:*
$M^0(p) = f(M_0^*)(p, i)$. (26)

Lemma 5. *Suppose that $M = f(M^*)$ and $(\tau, t_i) = g(t)$. The following proposition holds:*
$M^*[g(t) > \Leftrightarrow M[((\tau, t_i)) >$. (27)

Lemma 6. *Suppose that $M_1 = f(M_1^*)$, $M_1^*[t > M_2^*$, and $M_1[g(t) > M_2$. The following equality holds:*
$M_2 = f(M_2^*)$. (28)

Lemma 7. *Suppose that $(R_1, M_1) = f_s(M_1^*)$ and $t_s = g_s(t_{i.\beta_i(x)})$. The following proposition holds:*
$M_1^*[g_s(t_{i.\beta_i(x)}) > \Leftrightarrow (R_1, M_1)[t_s >$. (29)

Lemma 8 *Suppose $(R_1, M_1) = f_s(M_1^*)$, $M_1^*[t_{i.\beta_i(x)} > M_2^*$ and $(R_1, M_1)[g_s(t_{i.\beta_i(x)}) > (R_2, M_2)$. The following holds:*
$(R_2, M_2) = f_s(M_2^*)$. (30)

From the above Lemmas, the following theorem holds.

Theorem 2 Let RS be a 1-safe ERS. Let also N^* be a 1-safe P/T-net obtained from RS by the set of transformation Rules 1 to 5 above. Then state spaces of RS and N^* are isomorphic*.*

Proof: Lemmas 1 and 4 defines a one-to-one mapping between the initial markings of the 1-safe P/T-net $N^*$and the initial marking in RS. From Lemma 2 a system-autonomous firing mode $(\hat{t}, \tau)$ is enabled in a marking (R, M) if, and only if, the corresponding transition $t'_{\beta_i(x)}$ is enabled in the corresponding marking M^*. Similarly, from Lemma 5 an object-autonomous firing mode (τ, t_i) is enabled in a marking (R, M) if, and only if, the corresponding transition t is enabled in the corresponding marking M^*. Again, from Lemma 7 a synchronous firing mode $(\hat{t}, t_1, \dots, t_k)$ is enabled in a marking (R, M) if, and only if, the corresponding transition $t_{i.\beta_i(x)}$ is enabled in the corresponding M^*. Finally from Lemmas 3, 6 and 8, the generated markings in the 1-safe P/T-net can be mapped to the generated markings in the RS. □

Thus we have shown that every ERS can be transformed to behaviourally equivalent 1-safe P/T-net. In the next subsection we analyse the space upper bound associated with transforming an ERS into 1-safe P/T-net.

B. Space Upper Bound Analysis for the Transformation

In this subsection we discuss the space upper bound analysis associated with translating ERS into a 1-safe P/T-net. Let us suppose that the number of adjacent places for transition at the system net level is at most ρ, the number of places for each transition of object nets is at most γ, the number of object nets is k, and the number of to tokens in the initial marking of the system and object nets is δ.

The space complexity of generating places at Rule 1 is $O(k|\hat{P}| + |P_\Sigma|)$ because at most k copies of object net places are generated for each place of the system net, and at most $|P_\Sigma|$ are generated for each place of object nets. The space complexity of generating initial marking at Rule 2 is $O(\delta)$.The number of bindings for each arc variable adjacent to each transition of the system net is at most k^ρ; therefore, the space complexity of generating transitions at Rule 3 is $O(k^\rho|\hat{T}|)$. The number of arcs for each transition is at most ρ; therefore, the complexity of generating arcs at Rule 3 is $O(k^\rho \rho|\hat{T}|)$. The space complexity of generating transitions at Rule 4 is $O(|T_\Sigma|)$. The number of arcs for each transition of object nets is at most γ; therefore the space complexity of generating arcs at Rule 4 is $O(\gamma|T_\Sigma|)$.

One transition representing a synchronous transition is composed of at most $k+1$ transitions because of transition relation function; therefore, the number of transitions represented in the set of synchronous transitions will grows to the $k+1$ power of the number of transitions in the ERS. Consequently, the complexity of generating transitions at Rule 5 is $O(k^{(k+)\rho}|\hat{T}|)$.

In the worst case, ρ and γ equals the number of places and k grows as the number of object nets increases. Thus, the complexity of complete transformation is exponential with the size of the ERS. Moreover, due to conditions (1), (2), (3) and (4) imposed on variables appearing on arcs adjacent to a transition $\hat{t}$ of the system net in definition 4, the space upper bound for complete transformation is exponential with the size of the components of the ERS. As a result, the space requirement for the transformation from ERS to a low-level Petri nets is very huge.

6. Conclusion

As introduced in this article, various classes of OOP-nets are proposed so far. ERS follows the net-within-net paradigm by considering a Petri net as an object. Such a formalism is suitable to model applications in, for example, an agent context but further applications are expected. Not only simulation techniques but analytical schemes are important, however only studies about decidability of the reachability have been carried out so far. The need for application analytical method has encourage and driven the development of a set of transformation rule from ERS to Petri nets. Analysis shows that states space of ERS and the resulting

Petri net are isomorphic, however the space required by the algorithm grows exponential with the size components of an ERS. Future developments are expected to implement and bundle the transformation algorithm into a software tool.

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