

# SELF-EXCITING THRESHOLD AUTOREGRESSIVE MODEL AND ENDOGENOUS STRUCTURAL BREAKS OF EXCHANGE RATE DYNAMICS IN NIGERIA: A BAI AND PERRON SEQUENTIAL METHOD

C. L. Ani<sup>1</sup> and U. M. Hassan<sup>2</sup>

<sup>1</sup>Department of Statistics, Air Force Institute of Technology, Kaduna, Nigeria.  
E-mail: [emekaani605@gmail.com](mailto:emekaani605@gmail.com)

<sup>2</sup>Department of Mathematical Sciences, University of Maiduguri, Maiduguri, Nigeria.  
E-mail: [umarstat3@gmail.com](mailto:umarstat3@gmail.com)

## ABSTRACT

This paper empirically examines structural breaks in Nigeria's exchange rates of the Naira to GBP and USD datasets using average monthly frequency covering from January, 2004 to January, 2020. The study adopts a Bai and Perron (1998) sequential procedure. The study revealed that both series followed a nonlinear process. The Augmented Dickey-Fuller test showed that the conventional unit root test is biased towards the non-rejection of the null hypothesis of unit root presence in the series. While, the unit root breakpoint test by Vogelsang and Perron (1998) shows a contrary view as it rejected the null hypothesis. The NGN/GBP series generated a three (3) regime process with two significant threshold values of 259.96 and 388.93 for the period under review. While, NGN/USD series generated a two (2) process with a single threshold value of 359. The generated SETAR models for NGN/GBP and NGN/USD are SETAR(3;5,3) and SETAR(2;5,4), respectively. The diagnostic test (i.e. Breusch-Godfrey Serial Correlation LM Test and ARCH Heteroscedasticity Test) conducted on both generated models showed that, the models are free from serial correlation and heteroscedasticity which implies that these models are adequate for forecasting NGN/GBP and NGN/USD exchange rates.

**Keywords:** Nigeria Exchange Rate, Structural Breaks, Unit Root, Nonlinearity, SETAR Model

## INTRODUCTION

Most financial data exhibit structural breaks, asymmetry, jumps, instability etc. Tong (1990), and exchange rate (ER) is one that is characterized by these nonlinearity features. Exchange rate maybe define as the rate at which one country's currency is exchanged for other country's currency. An economy is vivacious if it experiences stability in its exchange rate. But, when there is fluctuations in the exchange rate, then there will definitely be frequent inflow and outflow of funds which affect foreign investment as investors will be unwilling to invest. Sustaining a steady exchange rate is somewhat a difficult task for both government financial institutions, private and corporate bodies, which is not at all times possible to guarantee. Exchange rate increases more sharply during recession than they increase during recoveries (Bakare, 2011).

From 2002 to 2016 Nigeria has seen a series of policy modification and changes in government and private financial institutions, commercial banks, shipping agencies etc. For

example, the structure of the Nigerian financial sector changed within these periods in terms of the number of institutions, capital requirements and licensing of new institutions. Specifically, in the banking sub-sector, the number of DMBs declined to twenty (20) from twenty-four (24), following the mergers/acquisitions of four (4) of the intervened banks by four (4) healthy banks (CBN, 2010). The number of DMBs' branches, however, increased to 5,810, from 5,799 in 2012 indicating an increase of 0.2 per cent (CBN Annual Abstract, 2013). In structural term, changes account for approximately one-fifth of the total change in labour productivity in Nigeria between 2001 and 2013, as well as the inclusion of the Chinese Renminbi to compliment other currencies in the country's foreign reserves (CBN, 2013) was to curtail the frequent fall of the Naira each time it suffers against the GBP, US \$ and Euro €. And, in recent years, the acquisitions of banks by both the Asset Management Company of Nigeria (AMCON) and other commercial banks/individuals, as well as changes in policies, policy shifts by different administration and global market crisis. But more importantly, if structural changes exist in the dataset and not captured in the unit root test specification then, the results may be misleading towards the non-rejection of the non-stationarity hypothesis (Perron, 1989; Perron, 1997; Leybourne and Newbold, 2003). As a result, any economic shock whether demand, supply or policy-induced to the variable will certainly have adverse effects on the variable in long-run. Thus, forecasting such variable may practically be wrong as model is miss-specified (Chukwu, Agu & Onah, 2010). According to Perron (2018), a complex view between structural break and unit root exist when the exact system is subjected to structural changes, then the tests that tries to differentiate amid a unit root and a trend stationarity procedure tends to favor the unit root model. Nonetheless, the break dates can order the process within regimes. Most researchers try to evaluate unit root in the occurrence of structural break there-by wanting to know if conventional unit root tests are influenced towards the rejection of unit root. Bauwens *et al.* (2010) assessed the precision measure of several predicting models and stated that the results obtained depend largely on how the structural breaks are treated. Roberto (2015) states that, in applying the threshold model to the US current account imbalances, threshold variable is an important component in the model to evaluate if the current account differs during deficits and surpluses or whether the size of the imbalance matters. Solarin and Anoruo (2015) conducted a study to investigate whether policies stabilization can effectively enhance

the GDP in African countries. In examining the effectiveness of these policies, the unit root features of per capita GDP in 52 African countries that proposed that per capital GDP in 52 countries been investigated are nonstationary. The results obtained after applying the nonlinear unit root test showed that, half of the whole sample of per capita GDP are non-stationary. This implies that, structural break in the presence of unit root as applied in the study shows significant improvement in the rejection of unit root for trend stationary data. Paresh and Stephan (2010) in their study proposed a new ADF-type test for unit roots which accounts for two structural breaks. They consider two different specifications: (a) two breaks in the level of a trending series; and (b) two breaks in the level and slope of trending data. The breaks whose time of occurrence is assumed to be unknown are modelled as innovational outliers and thus takes effect gradually.

This paper seeks to investigate whether nonlinearity exist in the series, stability or structural breaks in the mean level of the various exchange rates considered and, both the unit root test and the breakpoint unit root test as proposed by Vogelsang and Perron (1998) will be conducted since, Vogelsang and Perron (1998) points out that structural change and unit roots are closely related, and researchers should bear in mind that conventional unit root tests are biased towards a false unit root null when the data are trend stationary with a structural break. Also, it is paramount to note that exchange rate has huge impact on the nation's foreign reserves. So, the exchange rates here considered, are the two (2) major currencies in the nation's foreign reserves (i.e. GB Pounds and US Dollar). Also, we adopt the Bai and Perron (1998) method on structural change model in building the self-exciting threshold autoregressive (SETAR) model, there-by determining the number of regimes and threshold values that exist in each currency. Finally, provide one-step head forecast of the Naira exchange rates to each of the identified foreign currency.

## MATERIALS AND METHODS

### Preliminaries

Exchange rates are characterized by dramatic and abrupt fluctuations in behaviour. This study seeks to investigate the presence of nonlinearity in the datasets using Brock, Dechert and Scheinkman (BDS) nonlinearity test, the study also seeks to investigate the associated structural break dates in the datasets thereby associating the break dates to economic phenomena or downturn, global market crises or recession, crude oil prices and government policies or political administration etc. this is achieved using the Bai and Perron (1998) sequential method. Comparative analysis between the unit root test for stationarity using the Augmented Dickey-Fuller Test and Unit test in the presence of structural break. The Bai and Perron (1998) sequential method is also employed for regime determination and SETAR model parameter estimation. Furthermore, model diagnostic test is employed to investigate the presence of both serial correlation and heteroscedasticity so as to evaluate the adequacy of the model before using the generated models for prediction. The Breusch-Godfrey LM Test is used to test for Serial Correlation while, the Autoregressive Conditional Heteroscedasticity (ARCH) LM Test is employed to investigate each series of heteroscedasticity.

### Data Source and Description

The time series dataset employed in this study are exchange rates of the Naira to the Great Britain Pounds (GBP, £1.00) and US Dollar (USD, \$1.00). These exchange rates dataset here mentioned characterized Nigerian foreign reserves. The exchange rate datasets for NGN/GBP and NGN/USD are obtained as secondary data from the Central Bank of Nigeria (CBN). The CBN website from which data were read are [www.cbn.gov.ng/rates/exrate.asp](http://www.cbn.gov.ng/rates/exrate.asp). The exchange rates are to be interpreted as the amount of NGN in one GBP and USD. The data comprises of monthly frequency ranging from January, 2004 to January, 2020. The monthly datasets is utilised since structural breaks/shifts/change can be detected more evidently when a low frequency period is employed. E-View 9.0 statistical software was used to analyse the time series dataset.

### BDS Test for Nonlinearity

The nonlinearity test developed by Brock, Dechert and Scheinkman (BDS) is also a reliable and general test for nonlinearity. Initially the test was intended to test for the null hypothesis of independent and identical distribution (*iid*) for the purpose of identifying non-random chaotic dynamics. The null hypothesis tested under the BDS test claims that the time series has linear dependency. Employing the BDS test supports scholars to investigate whether the data set under study follows a nonlinear process.

Brock, Dechert, Scheinkman and LeBaron (1996) define the BDS statistic as follows:

$$V_{m,\epsilon} = \sqrt{T} \frac{C_{m,\epsilon} - C_{1,\epsilon}^m}{S_{m,\epsilon}} \quad (1)$$

where  $S_{m,\epsilon}$  is the standard deviation of  $\sqrt{T} C_{m,\epsilon} - C_{1,\epsilon}^m$  and can be estimated consistently as highlighted by Brock, Dechert, Scheinkman and LeBaron (1996).

### Augmented Dickey-Fuller (ADF) Unit Root Test

To compute the test statistics, the following augmented Dickey-Fuller regression model, which is a generalized autoregression model formulated by Dickey and Fuller (1979), is used. To differentiate a unit root, the following regression can be run:

$$\Delta Y_t = b_0 + \sum_{j=1}^k b_j \Delta Y_{t-j} + \beta_t + \gamma Y_{t-1} + \varepsilon_t \quad (2)$$

The regression comprises sufficient lags of  $\Delta Y_t$  as a result  $\varepsilon_t$  comprises no autocorrelation. If a time trend is not required then the model can be use without  $t$ . In the wake of a unit root, differentiating  $Y_t$  is paramount which should result in a white-noise series (no correlation with  $Y_{t-1}$ ). The Augmented Dickey-Fuller (ADF) test;  $H_0: \gamma = 0$  against  $H_1: \gamma \neq 0$ . Therefore, the test statistic for  $H_0: \gamma = 0$  is

$$Z_t = \frac{\hat{\gamma}}{\hat{\sigma}_\gamma} \quad (3)$$

Where  $\hat{\sigma}_\gamma$  is the standard error of  $\hat{\gamma}$ .

### Structural Break and Breakpoint Unit Root Test

This study follows the methodology as described in Perron and Vogelsang (1998) that treats the breakpoint as endogenous and allow for two structural breaks in the mean of the series. This method proposed by Perron and Vogelsang offers two models these includes; An additive outliers (AO) model, which captures a sudden/rapid change in the mean of a dataset; and an

innovational outliers (IO) model, which allows for a gradual shift in the mean of the dataset. In this study, we use the innovative outlier (IO) model, where a dummy for a break in the level is allowed along with a dummy for a break in the trend at an unknown period of time. Under the innovative outlier (IO) model, the break is modelled as evolving more slowly over time on Nigeria average monthly exchange rates of the Naira to GBP and USD.

The IO:

$$y_t = \hat{\mu} + \hat{\beta}_t + \hat{\gamma}DT_t(\hat{\lambda}) + \hat{\theta}DU_t(\hat{\lambda}) + \hat{\alpha}y_{t-1} + \sum_{i=1}^k \hat{C}_i \Delta y_{t-i} + \hat{e}_t \quad (4)$$

where  $DU = 1$  if  $t > T_b$ , and 0 otherwise,  $T_b =$  Break date and  $k =$  Number of lags

We test two restricted models as given by Zivot and Andrews, when only a break in the level or a break in the trend is included.

$$y_t = \hat{\mu} + \hat{\beta}_t + \hat{\theta}DU_t(\hat{\lambda}) + \hat{\alpha}y_{t-1} + \sum_{i=1}^k \hat{C}_i \Delta y_{t-i} + \hat{e}_t \quad (5)$$

$$y_t = \hat{\mu} + \hat{\beta}_t + \hat{\gamma}DT_t(\hat{\lambda}) + \hat{\alpha}y_{t-1} + \sum_{i=1}^k \hat{C}_i \Delta y_{t-i} + \hat{e}_t \quad (6)$$

For the error term to be serially independent in the case of the error term been autocorrelated, sufficient lagged difference terms need to be added.

### Self-Exciting Threshold Autoregressive (SETAR) Model Specification

Tong (1990) TAR models are generally stated to as piecewise linear or regime switching models. The self-exciting threshold autoregressive model is a special case of the TAR model where the threshold variable  $S_{t-d}$  is substituted by the past values of the series  $y$ , that is when  $S_{t-d} = Y_{t-d}$ ;  $d$  is the delay parameter, prompting the deviations between two or more different regimes. In other words, distinct from the TAR model, where the threshold variable is exogenous, the threshold variable of a SETAR model is endogenous. When the series exceed the threshold value, the process takes on a new value. SETAR model are often stated to as SETAR ( $k, \rho, d$ ) – where  $k$  is the number of regimes,  $\rho$  is the autoregressive order in each regime and  $d$  is the delay parameter or threshold value.

A two regime SETAR models is presented in Equations (7) below:

### SETAR model

$$Y_t = \begin{cases} \phi_{10} + \phi_{11}Y_{t-1} + \dots + \phi_{1p}Y_{t-p} + \varepsilon_{1t} & \text{if } Y_{t-d} < \gamma \\ \phi_{20} + \phi_{21}Y_{t-1} + \dots + \phi_{2p}Y_{t-p} + \varepsilon_{2t} & \text{if } Y_{t-d} \geq \gamma \end{cases} \quad (7)$$

Where  $d$  is the delay parameter, influencing the changes between two different regimes and the  $\phi'_s$ – represent the autoregressive parameters. The delay parameter or threshold variable is a positive integer. These models can be applied to the time series data which has a break or regime switching behavior (Tong and Lim, 1980).

### Bai and Perron Sequential Method for SETAR Regime Determination and Parameter Estimation

Recently, Bai and Perron (1998) offers the theoretical and analytical results that extend the Quandt-Andrews technique by permitting for multiple unknown breakpoints. The segment below provides a framework of the Bai and Bai- Perron method to structural break testing:

We consider the standard multiple regression models with  $T$  periods and  $m$  possible breaks (yielding  $m + 1$  regimes). For the observations  $T_j, T_j + 1, \dots, T_{j+1} - 1$  in regime  $j$  we have the model written as:

$$y_t = X'_t \beta + Z'_t \delta_j + \varepsilon_t \quad (8)$$

for the regimes  $j = 0, \dots, m$ ., the regressors are separated into two sets. Those whose parameters do not differ across regimes are the  $X$  variables. While, parameters that are regime-specific are the  $Z$  variables.

The technique of estimation considered here, is based on the least square's principle. For each  $m$ -partition  $(T_1, \dots, T_m)$ , the related least-squares estimates of  $\beta$  and  $\delta_j$  are gotten by minimizing the sum of squared residuals;

$$\begin{aligned} & (Y - X\beta - \bar{Z}\delta)'(Y - X\beta - \bar{Z}\delta) \\ & = \sum_{i=1}^{m+1} \sum_{t=T_{i-1}+1}^{T_i} [Y_t - X'_t \beta - Z'_t \delta_i]^2 \text{ or } S_T(T_1, \dots, T_m) \\ & = \sum_{i=1}^{m+1} \sum_{t=T_{i-1}+1}^{T_i} (y_t - \beta_i)^2 \end{aligned} \quad (9)$$

Let  $\hat{\beta}(\{T_j\})$  and  $\hat{\delta}(\{T_j\})$  denote the resulting estimates based on the given  $m$ -partition  $(T_1, \dots, T_m)$  denoted by  $\{T_j\}$  or  $\hat{\beta}(\{T_1, \dots, T_m\})$ , where  $\beta = (\beta_1, \dots, \beta_{m+1})'$ . Putting these into equation (9), the estimated breakpoints are as follows:

$$\begin{aligned} & (\hat{T}_1, \dots, \hat{T}_m) \\ & = \text{argmin}_{T_1, \dots, T_m} S_T(T_1, \dots, T_m) \end{aligned} \quad (10)$$

The minimization is taken over all partitions  $(T_1, \dots, T_m)$  such that  $T_i - T_{i-1} \geq q$ . Hence, the breakpoints estimators are global minimizers of the objective function. Lastly, the regression parameters estimates are gotten using the related least squares estimates at the estimated  $m$ -partition  $\{\hat{T}_j\}$ , i.e.  $\hat{\beta} = \hat{\beta}(\{\hat{T}_j\})$ ,  $\hat{\delta} = \hat{\delta}(\{\hat{T}_j\})$

### RESULT

This section presents the empirical findings of this study. Specifically, the section focuses on the presentation of results on time plots of the series under investigation, nonlinearity test, unit root test, breakpoint unit root test and break date identification, regime determination and estimation of SETAR models, model diagnostic tests as well as forecast performance measures.

### BDS Nonlinearity Test

This section presents the Brock, Dechert, Scheinkman and LeBaron (1996) nonlinearity test, Table 1 shows the results of the BDS nonlinearity test conducted on NGN/GBP and NGN/USD exchange rates, respectively.

**Table 1:** Brock, Dechert and Scheinkman (BDS) Test on NGN/GBP and NGN/USD

Dimension	BDS Statistic		Std. Error		Z-Statistic		Prob	
	NGN/GBP	NGN/USD	NGN/GBP	NGN/USD	NGN/GBP	NGN/USD	NGN/GBP	NGN/USD
2	0.196730	0.196751	0.006349	0.005686	30.98610	34.60414	0.0000*	0.0000*
3	0.331232	0.330432	0.010021	0.008985	33.05314	36.77574	0.0000*	0.0000*
4	0.423302	0.422412	0.011853	0.010638	35.71263	39.70598	0.0000*	0.0000*
5	0.486414	0.486487	0.012272	0.011025	39.63718	44.12642	0.0000*	0.0000*
6	0.531108	0.530297	0.011756	0.010571	45.17816	50.16476	0.0000*	0.0000*

\* denotes the significant of BDS Statistic at each dimension at 5% significance levels.  $H_0$ : Time series dataset has linear dependency,  $H_1$ : Time series dataset has nonlinear dependency

In Table 1 above, the result revealed that the reported z-statistic for both NGN/GBP and NGN/USD are high (i.e. the p-value of the respective z-statistics are small). By implication, since the p-values of the corresponding dimensions of both NGN/GBP and NGN/USD are less than 5% level of significance, we reject the null hypothesis of the time series datasets having linear dependency and conclude that, the exchange rates of the Naira to one Pounds Sterling and one US Dollar (i.e. NGN/GBP, £1.00 and NGN/USD, \$1.00) follows a nonlinear patterns

**Unit Root and Stationarity Test**

In this section, the graphical, unit root and stationarity properties of the study variable is examined using time series plots and Augmented Dickey Fuller (ADF) unit root test as presented in Figure 1 and Table 2, respectively.

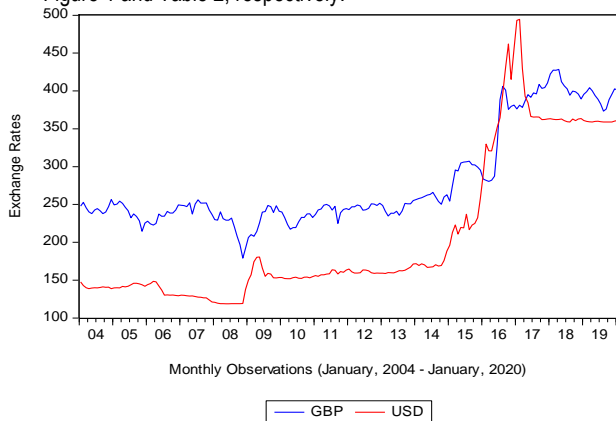


Figure 1: Time Plot of Nigeria Monthly Exchange Rate of the Naira (NGN/GBP £1.00 and NGN/USD \$1.00) from January, 2004 to January, 2020.

The visual plots as presented in Figure 1, the Nigeria monthly exchange rate of the Naira to one Pounds Sterling shows an upward trend than the exchange rate of the Naira to one US Dollar from January, 2004 to August, 2016. The Naira to the USD showed higher spike from September, 2016 to March, 2017 and, the exchange rate of the Naira to the US Dollar stabilized from June, 2017 to January, 2020. While, the Naira to the GBP showed higher spike from December, 2017 to April, 2018; for the period September, 2016 to January, 2020, the Naira/Pounds Sterling exchange rate recorded its lowest value of N373.00 as against on £1.00 in the month of August, 2019. Apart from the sharp increase in both exchange rates, instabilities in the rates under review have been established. From Figure 1, it is evident structural breaks exist in both NGN/GBP and NGN/USD.

**Table 2:** Augmented Dickey Fuller (ADF) Unit Root Test on NGN/GBP and NGN/USD.

Variable	Option	ADF		Variable	Option	ADF	
		Test Statistic	P-Value			Test Statistic	P-Value
NGN/GBP	Intercept	-0.619849	0.8621	NGN/USD	Intercept	-0.542214	0.8788
	Intercept and Trend	-2.380870	0.3884		Intercept and Trend	-2.354616	0.4022
$\Delta$ NGN/GBP	Intercept	-10.06991	0.0000*	$\Delta$ NGN/USD	Intercept	-10.21716	0.0000*
	Intercept and Trend	-10.11887	0.0000*		Intercept and Trend	-10.22776	0.0000*
<b>Asymptotic Critical Values</b>				<b>Asymptotic Critical Values</b>			
1%	Intercept	-3.464643		1%	Intercept	-3.464643	
5%		-2.876515		5%		-2.876515	
10%		-2.574831		10%		-2.574831	
1%	Intercept and Trend	-4.006824		1%	Intercept and Trend	-4.007613	
5%		-3.433525		5%		-3.433906	
10%		-3.140623		10%		-3.140847	

\* denotes the significant of ADF test statistic at 1%, 5% and 10% significance levels.  $\Delta$  is the first difference operator.  $H_0$ : Time series dataset

has a unit root,  $H_1$ : Time series dataset has a unit root.

Table 2 shows the unit root test of NGN/GBP and NGN/USD using the Augmented Dickey-Fuller test. For NGN/GBP series at level, the test statistic obtained for constant only is -0.6198 with a p-value of 0.8621 and the test statistic obtained for constant and linear trend is -2.3809 with p-value of 0.3884. While, for NGN/USD series the test statistic obtained for constant only is -0.5422 with a p-value of 0.8788 and the test statistic obtained for constant and linear trend is -2.3546 with p-value of 0.4022. This results revealed that, since the corresponding p-values obtained for each ADF statistic of both series is greater than 5% level of significance we do not reject the null hypothesis of presence of unit root and conclude that both series have the presence of unit root as established by the ADF test.

**Unit Root Test in the Presence of Structural Break**

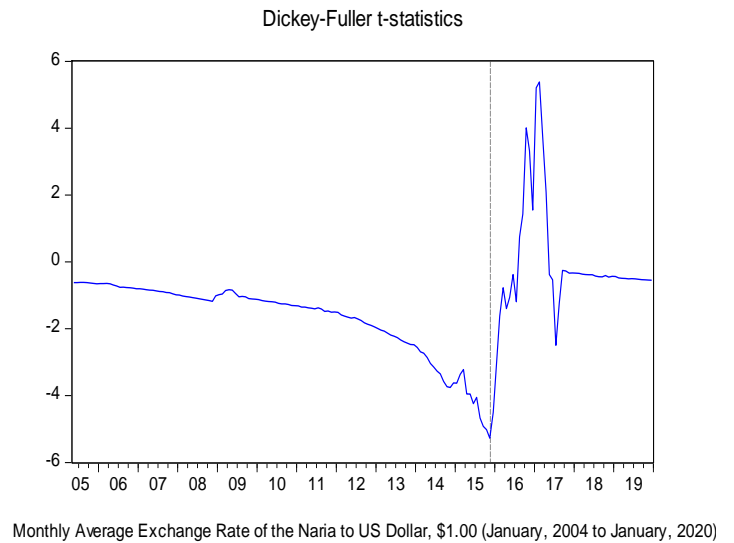
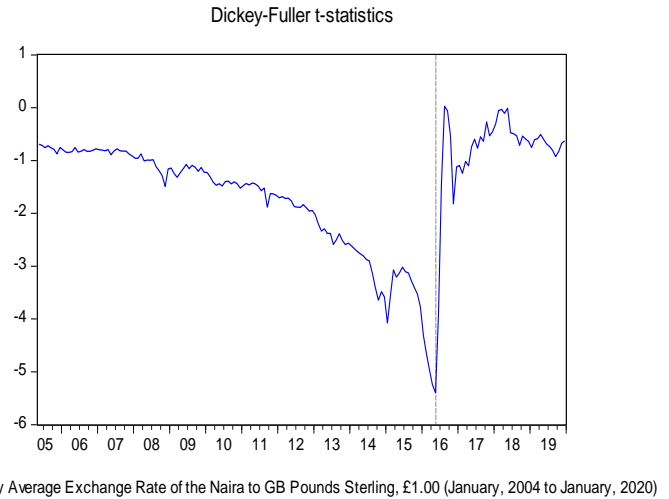
Table 3 shows the breakpoint unit root test, as well as the identifiable break dates and also, Figure 2 displays evidence of structural break for each trend specification of both series.

**Table 3: Breakpoint Unit Root Test on NGN/GBP and NGN/USD**

Structural Break Result (Breakpoints Break Dates)			ADF Test			
NGN/GBP	NGN/USD	Trend	NGN/GBP		NGN/USD	
		Specification	Test Statistic	P-Value	Test Statistic	P-Value
2016M05	2015M11	Intercept	-5.395367	< 0.01**(*)	-5.273918	< 0.01**(*)
		Intercept	-6.455066	< 0.01**(*)	-6.581902	< 0.01**(*)
		and Trend				

\* denotes the significant of ADF test statistic at 1% and \*\* at 5% significance levels.  $H_0$ : Time series dataset has a unit root,  $H_1$ : Time series dataset has a unit root.

From the results obtained in Table 3 above, the structural break point test reveals that only one significant break date exist for each series that is, the break date identified for NGN/GBP exchange rate is 2016M05 (i.e. May, 2016), while, the break date identified for NGN/USD exchange rate is 2015M11 (i.e. November, 2015) which is evident in Figure 2 below. For NGN/GBP series at level, the test statistic obtained for intercept is -5.3954 and -6.4551 for intercept and trend; since, both of their p-values are less than 1% level of significance we reject the null hypothesis of unit root presence, we can state clearly that, the series is free from unit root and can be used without differencing or transformation. Also, for NGN/USD series at level, the test statistic obtained for intercept is -5.2739 and -6.5819 for intercept and trend; so, since both of their p-values are less than 1% level of significance, we reject  $H_0$  and conclude that, the Naira/Dollar exchange rate is free from unit root.



**Figure 2:** Graphical Plot of Structural Break Date of NGN/GBP and NGN/USD

**Regime Determination and SETAR Model Estimation of NGN/GBP Using Bai and Perron Sequential Method**

The Bai and Perron (1998) sequential procedure is used to determine the threshold variable, threshold value and specify the number of regimes as shown in Table 4, while, the parameter estimation is shown in Table 5.

**Table 4:** Regime Selection using SSR and Identification of Threshold Variable (Delay Parameter) and Threshold Value

Threshold Variable	Sum of Squared Residuals (SSR)	Regimes
NGN/GBP(-3)	9645.819735	3
NGN/GBP(-4)	9877.944554	3
NGN/GBP(-1)	10845.301908	2
NGN/GBP(-5)	11313.983467	2
NGN/GBP(-2)	11439.105243	2

Summary				
Threshold Variable	Estimated No. of Thresholds	Threshold Data Value	Adjacent Data Value	Threshold Value used
NGN/EUR(-3)	2	259.96	258.95	259.9599
		388.93	388.43	388.9299

Method: Bai – Perron tests of  $L + 1$  vs.  $L$  sequentially determined thresholds. Maximum number of thresholds: 5

Table 4 above shows the results obtained for NGN/GBP series, the result indicates that, the selected threshold variable for the model is NGN/GBP(-3) since it has the lowest SSR coefficient of 9645.8197 compared to others and it specify the series to a three (3) regime process. The threshold values corresponding to the break are 259.9599 and 388.9299

**Table 5:** Parameter Estimates of the SETAR Model of NGN/GBP  
Dependent Variable: EXCHANGE RATE (NGN/GBP)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
<b>GBP(-3) &lt; 259.9599 -- 124 obs</b>				
Constant	33.01156	12.80562	2.577895	0.0108*
NGN/GBP (-1)	0.987461	0.100676	9.808301	0.0000*
NGN/GBP (-2)	-0.071550	0.147197	-0.486084	0.6275
NGN/GBP (-3)	0.020867	0.143575	0.145336	0.8846
NGN/GBP (-4)	0.050539	0.146854	0.344147	0.7312
NGN/GBP (-5)	-0.124948	0.105420	-1.185236	0.2376
<b>259.9599 &lt;= GBP(-3) &lt; 388.92999 -- 36 obs</b>				
Constant	22.96429	8.640268	2.657821	0.0086*
NGN/GBP (-1)	1.740884	0.113057	15.39834	0.0000*
NGN/GBP (-2)	-1.117287	0.221572	-5.042538	0.0000*
NGN/GBP (-3)	0.456227	0.246403	1.851544	0.0658
NGN/GBP (-4)	-0.651977	0.250242	-2.605389	0.0100*
NGN/GBP (-5)	0.509199	0.147871	3.443546	0.0007*
<b>388.92999 &lt;= GBP(-3) -- 28 obs</b>				
Constant	78.48234	56.16724	1.397297	0.1641
NGN/GBP (-1)	1.096636	0.207370	5.288298	0.0000*
NGN/GBP (-2)	-0.028895	0.283558	-0.101900	0.9190
NGN/GBP (-3)	-0.300785	0.325902	-0.922930	0.3574
NGN/GBP (-4)	0.161645	0.322610	0.501052	0.6170
NGN/GBP (-5)	-0.124819	0.139282	-0.896162	0.3714
R-squared	0.988497	F-statistic		859.3060
Adjusted R-squared	0.987346	Prob(F-statistic)		0.000000*

Trimming 0.15, Max. thresholds 5, \* Significant at the 0.05 level

The result of the SETAR model for NGN/GBP is presented in Table 5. The model comprises of three (3) regime parameter estimates. The model displays, two (2) threshold values which includes; 259.9599 and 388.92999, these threshold values prompted the generation of three (3) regimes for the series. The first, second and third regime comprises of 124, 36 and 28 observations, respectively. In the first regime, the process contains estimates of lags less than the threshold value (259.9599); In the second regime, the process contains estimates of lags that are greater or equal to the threshold value (259.9599) but less than the threshold value (388.92999) (i.e. the lag

estimates are within the two threshold values generated in the series). And, the third regime comprises of lags that are greater than or equal to the threshold value (388.92999). The first and third regime showed that, only the coefficient of the first lag (i.e. NGN/GBP(-1)) of the dependent variable is statistically significant at 0.05 level since their corresponding p-values are less than 5% level of significance, while other lags are not significant. While, in the second regime, the following lags are significant (NGN/GBP(-1, -2, -4, -5) since their corresponding p-values are less than 5% level of significance while, only the third lag (i.e. NGN/GBP(-3)) is statistically insignificant as its p-value is greater than 5% level of significance. The R-squared adjusted shows that about 98.8% variability in exchange rate of Naira to one Pounds Sterling is explained by changes in its lags. While, the F-statistics which shows the general significance of the model is given as (859.306) with probability value of 0.0000 indicating that the model or regressors are jointly significant at 5% level of significance. So, the estimates are valid for statistical explanation and policy inferences.

Thus, the fitted SETAR model established for Naira to one Pounds Sterling exchange rate is given as SETAR(3;5,3) and its process can be written as:

$$Y_t = \begin{cases} 33.01 + .98Y_{t-1} - 0.07Y_{t-2} + 0.02Y_{t-3} + 0.05Y_{t-4} \\ \quad - 0.12Y_{t-5} \quad \text{if } Y_{t-3} < 259.96 \\ 22.96 + 1.74Y_{t-1} - 1.12Y_{t-2} + 0.46Y_{t-3} - 0.65Y_{t-4} \\ \quad + 0.51Y_{t-5} \quad \text{if } 259.96 \leq Y_{t-3} < 388.93 \\ 78.48 + 1.10Y_{t-1} - 0.03Y_{t-2} - 0.3Y_{t-3} + 0.16Y_{t-4} \\ \quad - 0.12Y_{t-5} \quad \text{if } 388.93 \leq Y_{t-3} \end{cases} \quad (11)$$

**Regime Determination and SETAR Model Estimation of NGN/USD Using Bai and Perron Sequential Method**

The Bai and Perron (1998) sequential procedure is adopted to determine the threshold variable, threshold value and specify the number of regimes as shown in Table 7, while, the parameter estimation are shown in Table 8.

**Table 7:** Regime Selection using SSR and Identification of Threshold Variable (Delay Parameter) and Threshold Value

Threshold Variable	Sum of Squared Residuals (SSR)	Regimes
NGN/USD(-4)	13222.083201	2
NGN/USD(-5)	13222.083201	2
NGN/USD(-3)	14250.423473	2
NGN/USD(-2)	14467.440600	2
NGN/USD(-1)	14780.451207	2

Summary				
Threshold Variable	Estimated No. of Thresholds	Threshold Data Value	Adjacent Data Value	Threshold Value used
NGN/USD(-4)	1	359	351.82	359

Method: Bai – Perron tests of  $L + 1$  vs.  $L$  sequentially determined thresholds. Maximum number of thresholds: 5

Table 7 displays the results obtained for NGN/USD series, the result indicates that, the selected threshold variable for the model is NGN/USD(-4) since it has the lowest SSR coefficient of 13222.083201 when compared to other threshold variables; the identified threshold variable specifies the series to a two (2) regime process. The threshold value corresponding to the break is 359.



**Table 8:** Parameter Estimates of the SETAR Model of NGN/USD

Dependent Variable: EXCHANGE RATE (NGN/USD)				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
<b>USD(-4) &lt; 359 -- 149 obs</b>				
Constant	-11.55216	3.548740	-3.255285	0.0014*
NGN/USD (-1)	1.305720	0.111947	11.66374	0.0000*
NGN/USD (-2)	-0.212592	0.184472	-1.152434	0.2507
NGN/USD (-3)	-0.169552	0.184877	-0.917107	0.3603
NGN/USD (-4)	0.098146	0.187146	0.524437	0.6006
NGN/USD (-5)	0.059503	0.124744	0.477000	0.6340
<b>359 &lt;= USD(-4) -- 39 obs</b>				
Constant	80.90664	16.77435	4.823235	0.0000*
NGN/USD (-1)	0.985597	0.085765	11.49183	0.0000*
NGN/USD (-2)	-0.292865	0.126346	-2.317958	0.0216*
NGN/USD (-3)	0.051264	0.129755	0.395085	0.6933
NGN/USD (-4)	0.396523	0.124548	3.183683	0.0017*
NGN/USD (-5)	-0.360953	0.079371	-4.547692	0.0000*
R-squared	0.992934	F-statistic		2248.382
Adjusted R-squared	0.992492	Prob(F-statistic)		0.000000*

Trimming 0.15, Max. thresholds 5, \* Significant at the 0.05 level

The result of the SETAR model for NGN/USD is presented in Table 8. The model comprises of two (2) regime. The model displays, a single threshold value (359), the generated threshold value stimulated two (2) regimes for the series. The first regime comprises of 149 observations, while, the second regime comprises of only 39 observations. In the first regime, the process contains estimates of lags less than the threshold value 359 and the second regime contains estimates of lags that are greater or equal to the threshold value 359. The first regime showed that, only the coefficient of the first lag (i.e. NGN/USD(-1)) of the dependent variable is statistically significant at 0.05 level since the corresponding p-value is less than 5% level of significance, while other lags are not significant. While, in the second regime, the following lags are significant (NGN/GBP(-1, -2, -4, -5) since their corresponding p-values are less than 5% level of significance while, only the third lag (i.e. NGN/GBP(-3)) is statistically insignificant as its p-value is greater than 5% level of significance. The R-squared adjusted shows that about 99.2% variability in exchange rate of Naira to one US Dollar is explained by changes in its lags. While, the F-statistics which shows the general significance of the model is given as (2248.38) with probability value of 0.0000 indicating that the model is statistically significant at 5% level of significance. So, the estimates are valid for statistical explanation and policy inferences.

Thus, the fitted SETAR model established for Naira to one Pounds Sterling exchange rate is given as SETAR(2;5,4) and its process can be written as:

$$Y_t = \begin{cases} -11.55 + 1.31Y_{t-1} - 0.21Y_{t-2} - 0.17Y_{t-3} + 0.10Y_{t-4} \\ \quad + 0.06Y_{t-5} \quad \text{if } Y_{t-4} < 359 \\ 80.91 + 0.99Y_{t-1} - 0.29Y_{t-2} + 0.05Y_{t-3} + 0.40Y_{t-4} \\ \quad - 0.36Y_{t-5} \quad \text{if } 359 \leq Y_{t-4} \end{cases} \quad (12)$$

**Model Diagnostic Test on SETAR Models**

In order to authenticate the suitability and forecasting ability of SETAR (3;5,3) model for NGN/GBP and SETAR (2;5,4) model for NGN/USD, a serial correlation test is performed utilizing Breusch-Godfrey Serial Correlation LM Test. While, heteroscedasticity is tested utilizing the autoregressive conditional heteroscedasticity (ARCH) test. The results of the aforementioned diagnostic tests for NGN/GBP and NGN/USD are presented in Tables 9 and 10,

respectively.

**Table 9:** Breusch-Godfrey Serial Correlation LM Test on Monthly Exchange Rate of NGN/GBP and NGN/USD

Models	F-Statistic	Prob.	Chi-Square Statistic	Prob.
SETAR(3;5,3)	0.713165	0.4916	1.582694	0.4532
SETAR(2;5,4)	1.42452	0.6782	31.37012	0.9871

\* Significant at the 0.05 level. H<sub>0</sub>: There is no serial correlation, H<sub>1</sub>: There is serial correlation

The result of SETAR(3;5,3) for NGN/GBP showed that the F-statistics (0.713165) with p-value of 0.4916 and Chi-square value of 1.582694 with corresponding p-value of 0.4532 are greater than 5% level of significance. By implication, the null hypothesis of no serial correlation is not rejected and the model is free from serial correlation and adequate for prediction. The result as displayed in Table 9 also showed that the SETAR(2;5,4) model for NGN/USD is free from serial correlation since both p-values corresponding to the obtained F-statistics and Chi-square statistics are greater than 5% level of significance.

**Table 10:** ARCH Heteroscedasticity Test on Monthly Exchange Rate of NGN/GBP and NGN/USD

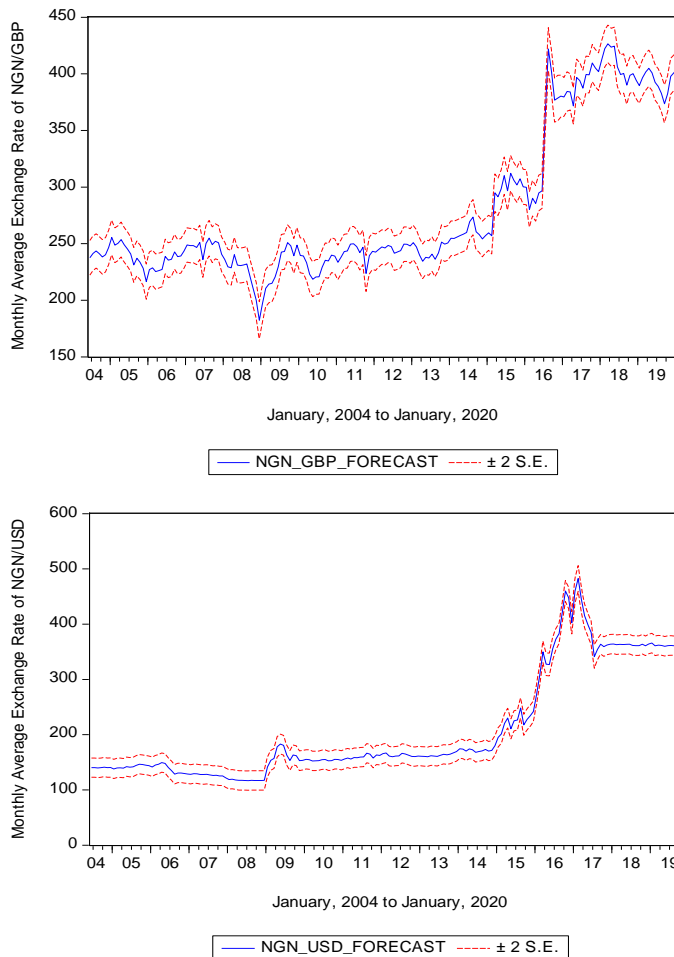
Models	F-Statistic	Prob.	Chi-Square Statistic	Prob.
SETAR(3;5,3)	0.713165	0.4916	1.582694	0.4532
SETAR(2;5,4)	1.42452	0.6782	31.37012	0.9871

\* Significant at the 0.05 level. H<sub>0</sub>: There is no Heteroscedasticity, H<sub>1</sub>: There is Heteroscedasticity

The results of the ARCH test for heteroscedasticity are summarized in Table 10. As indicated in Table 10, the value of the computed F-statistics and the Chi-square statistics are not sufficiently large in the case of both models to reject the null hypothesis, which indicates that there is no heteroscedasticity in both models. Therefore, SETAR(3;5,3) and SETAR(2;5,4) models are free from a heteroscedasticity problem and adequate for forecasting NGN/GBP and NGN/USD exchange rates series.

**Forecasting**

After determining the adequacy of SETAR (3;5,3) and SETAR (2;5,4) models for NGN/GBP (£1.00) and NGN/USD (\$1.00) exchange rates, respectively. The static forecasting method is employed to perform a series of one-step ahead forecasts taking into account the entire observations of each series as presented in Figure 3 below.



**Figure 3:** One-Step Ahead Forecast of SETAR (3;5,3) and SETAR (2;5,4) Models

**DISCUSSION**

In testing for structural break and also, to fit a nonlinear time series model, it is vital to investigate whether the series under consideration follows a nonlinear process. So, in testing for nonlinearity we employed the Brock, Dechert and Scheinkman (BDS) nonlinearity test. The BDS test as shown in Table 1 assists this study to establish that the exchange rates (i.e. NGN/GBP and NGN/USD) for the period under review are both nonlinear. That is, both series are independently and identically distributed (*iid*); The hypothesis that the series is *iid* for the exchange rates is rejected since the resulted reported that the z-statistic is high (and the probability of z-statistics is less than 0.05 percent level of significance). Thus, our findings suggests that both series under study may exhibits structural breaks, regimes or instability and also, nonlinear models are predictable to have superior efficiency than linear models.

When a series is stationary, then we can ascertain that fairly good estimates of parameters of time series are obtained, a stationarity investigation using Augmented Dickey Fuller (ADF) test was conducted on NGN/GBP and NGN/USD so as to determine these assertions. The results as shown in Table 2 shows that both series were non-stationary at all levels. Then, they became stationary after both series were differenced. By implication, the

results specify that NGN/GBP and NGN/USD exchange rate datasets are both integrated of order one (i.e.  $I(1)$ ).

Vogelsang and Perron (1998) points out that structural change and unit roots are closely related, and researchers should bear in mind that conventional unit root tests are biased towards a false unit root null when the data are trend stationary with a structural break. Based on the results obtained in Table 3, the structural break point test reveals that only one significant break date in the exchange rate of NGN/GBP and NGN/USD exist, respectively; which is evident in Figure 2 and, the break date identified is 2015M11 and 2016M05 for NGN/USD and NGN/GBP, respectively; The reason in the difference between the two break dates maybe attributed to the three segments of the foreign exchange market (i.e. Retail Dutch Auction System (RDAS), Interbank and Bureau De Change (BDC)) as all these segments have different rates. But in mid-February 2015, the CBN closed down the RDAS window so as to limit the depreciation of the Naira and went on to adopt the interbank rate as the approved transaction. In-line to these effect, the CBN in June 2016 decided to adopt a market driven flexible exchange rate regime to curtail the continuous rise in the exchange rate of the Naira to USD and other foreign currencies. Another reason to the different dates is the nominal effect exchange rate (NEER) and real effective exchange rate (REER) index because an increase in REER implies higher inflation in the domestic economy. These break dates/breakpoints/structural instability can be linked to economic instability such as free fall of oil price that is, since Nigeria depends solely on the revenue generated from oil exportation, our economy is affected anytime there is fall in oil prices in the world market; Higher importation and lower exportation is not good for our economy as Nigeria imports about anything possible to the country which increases the demand for US Dollar and Pounds Sterling which in-turn depreciate the Naira; Inability of the government to diversify the economy and develop other sectors such as agriculture, IT industries, chemical industries etc. will continue to see the Naira losing its value against other foreign currencies not just the US Dollar and Pounds Sterling alone. Also, the structural break dates results obtained seem to point to asymmetric transmission of Nigeria's political instability and other political related shocks. In addition, another domestic shock are the new policies (i.e. new administration) era which might be the explanation for the break date in November, 2015 and May 2016. Note that, the structural break date identified in this study is not independent of the sample period or of the data frequency. Also, the results as shown in Table 5 follows the methodology described in Zivot and Andrews (1992) that treats the breakpoint as endogenous and specify only a single break point which was later extended by Vogelsang and Perron (1998) that uses a multiple breakpoint test in identifying structural break in a series. Our results showed that the ADF unit root tests in Table 2 is biased towards the non-rejection of non-stationarity in both series. While, Table 3 shows that NGN/GBP and NGN/USD exchange rates follows an  $I(0)$  process as against the ADF test of  $I(1)$  in Table 2. The results in Table 3 indicates that it is imperative to account for structural break in the time series for monthly exchange rates of Naira to pounds sterling and Naira to US Dollar. But, more importantly the results as shown in Table 3, suggests that unit root test should be investigated more carefully if the series under consideration exhibit structural break.

The result as presented in Table 5 showed that, NGN/GBP exchange rate (i.e. SETAR (3;5,3)) entails two threshold values



which includes 259.9599 and 388.92999 there-by prompting the series into a three (3) regime process. The R-squared adjusted shows that about 98.8% variability in exchange rate of Naira to one Pounds Sterling is explained by changes in its lags. While, the F-statistics which shows the general significance of the model is given as (859.306) with probability value of 0.0000 indicating that the model or regressors are jointly significant at 5% level of significance. While, for NGN/USD exchange rate (i.e. SETAR (2;5,4)), the result as presented in Table 8 showed that the series generated a single threshold value of 359 which led to two (2) regime. The R-squared adjusted shows that about 99.2% variability in exchange rate of Naira to one US Dollar is explained by changes in its lags. While, the F-statistics which shows the general significance of the model is given as (2248.38) with probability value of 0.0000 indicating that the model is statistically significant at 5% level of significance. Therefore, the estimates for both models are valid for statistical explanation and policy inferences.

Diagnostic tests were applied to the models to confirm that they are suitable for forecast. In order to authenticate the suitability of SETAR(3;5,3) and SETAR(2;5,4) models, a correlation test is performed utilizing the Breusch-Godfrey correlation LM test. Heteroscedasticity is tested utilizing the ARCH test. The results of the aforementioned diagnostic tests for NGN/GBP and NGN/USD exchange rates are presented in Tables 9 and 10. Table 9 shows the estimated Breusch-Godfrey correlation F-statistics and Chi-square statistics of SETAR(3;5,3) and SETAR(2;5,4) models for NGN/GBP and NGN/USD, respectively. Based on the results obtained, the null hypothesis of no serial correlation is not rejected at 5% level of significance, this results further indicates that there is no correlation problem within the models. Therefore, based upon the correlation test, the constructed models are deemed to be accurate. Also, the results of the ARCH test for heteroscedasticity are summarized in Table 10. As indicated in Table 10, the value of the computed F-statistics and the Chi-square statistics are not sufficiently large in the case of both models to reject the null hypothesis, which indicates that there is no heteroscedasticity in the models. Therefore, SETAR(3;5,3) and SETAR(2;5,4) models are free from a heteroscedasticity problem and adequate for forecasting NGN/GBP and NGN/USD exchange rates.

Finally, Figure 3 shows the one-step ahead forecast of the dependent variable after determining the adequacy of SETAR(2;5,2) model for NGN/GBP exchange rate and SETAR(3;5,3) model for NGN/USD exchange rate. The static forecasting technique is used to perform a series of one-step ahead forecasts taking into account the whole observations of each series.

## 1. Conclusion

This study focuses on investigating endogenously structural breaks in the datasets under consideration and since structural break can be observed more closely with low frequency data the monthly data of Naira to Euro exchange rates from January, 2004 to March, 2019 was applied. Test for nonlinearity of Nigerian monthly exchange rate of the Naira to £1.00 and \$1.00 was investigated before the modelling approach was applied, the approach employed to investigate nonlinearity is the BDS nonlinearity test. Based on the result obtained, the test indicated that, both exchange rates showed that non-linearity exist in both series. We used the Vogelsang and Perron (1998) and Bai and

Perron (1998) frame work, and provide evidence of structural break which are associated with unstable political, economic and trade liberalization. Our findings showed that the identified break dates (i.e. November, 2015 and May, 2016) is susceptible to internal shocks and some policy implications, which may be linked to the 2015 general elections in Nigeria and policy change towards financial institution. Also, the date break is associated with external shocks and this identified break date might be linked to international commodity price and global financial crises. The findings of structural break in the series are useful for future empirical studies using exchange rates in Nigeria. The study also revealed that testing for unit root in a series with breakpoints should be handled carefully as unit root and structural break are closely related. Our findings revealed that, the conventional unit root test applied (i.e. ADF test) is biased towards the non-rejection of the null hypothesis of unit root in both exchange rates series. The result showed that, the structural break unit root test rejected the null hypothesis that is, there is no unit root in both series.

Self-exciting threshold autoregressive models have been used in order to be able to model the volatile changes in currency market, since nonlinear time series models gives better fit in-wake of structural breaks in the time series data. In performing the modelling procedures two SETAR models were developed. After performing the modelling procedures, SETAR(3;5,3) and SETAR(2;5,4) were generated and applied in estimating Nigerian monthly exchange rate of the Naira to GBP and Naira to USD, respectively. The models generated were statistically significant as shown by their respective F-statistics. Based on the following diagnostic test; Breusch-Godfrey correlation LM test and ARCH test, it can be seen that both SETAR models developed for modelling Nigeria monthly exchange rates of NGN/GBP and NGN/USD are free from both serial correlation and heteroscedasticity. Therefore, the diagnostic test revealed that both models are adequate and suitable for prediction

## REFERENCES

- Bai, J. and Perron, P. (1998). Estimating and testing linear models with multiple structural changes. *Econometrica Journal*, 66(1): 47-78.
- Bakare A.S (2011). The Consequences of Foreign Exchange Rate Reforms on the Performances of Private Domestic Investment in Nigeria. *International Journal of Economics and Management Sciences*, 1(1): 25-31.
- Bauwens, L., Preminger, A. and Rombouts, J. (2010). Theory and inference for a Markov-switching GARCH model. *Econometrics Journal*, 13, 218-244.
- Brock, W., Davis, D., Sheinkman, J. and LeBaron, B. (1996). A Test for Independence Based on the Correlation Dimension. *Econometric Reviews*, 15(3): 197-235.
- Central Bank of Nigeria, CBN (2010). Monetary Policy Transmission Mechanism in Nigeria, Abuja: Central bank of Nigeria.
- Central Bank of Nigeria, CBN (2013). Central Bank of Nigeria Annual Report, Abuja: Central Bank of Nigeria.
- Central Bank of Nigeria, CBN (2015). Central Bank of Nigeria Annual Report, Abuja: Central Bank of Nigeria.
- Central Bank of Nigeria, CBN (2016). Central Bank of Nigeria Annual Report, Abuja: Central Bank of Nigeria.

- Central Bank of Nigeria, CBN. (2019): Annual Statistical Bulletin. CBN, Nigeria: URL: [www.cbn.gov.ng/rates/exrate.asp](http://www.cbn.gov.ng/rates/exrate.asp). Accessed on 04<sup>th</sup> December, 2019.
- Central Bank of Nigeria, CBN. (2020): Annual Statistical Bulletin. CBN, Nigeria: URL: [www.cbn.gov.ng/rates/exrate.asp](http://www.cbn.gov.ng/rates/exrate.asp). Accessed on 14<sup>th</sup> April, 2020.
- Chukwu, J. O., Agu, C.C. and Onah, F. E. (2010). Cointegration and structural Breaks in Nigerian Long-run Money demand Function. *International Research Journal of Finance and Economics*, 3(38): 7-15.
- Dickey, D. A. and Fuller, W. A. (1979). Distribution of the Estimators for Autoregressive Time Series with a Unit Root, *Journal of the American Statistical Association*, 74, 427-431.
- Engle, R. F. (1982). Autoregressive Conditional Heteroskedasticity with Estimates of the Variance of U.K. Inflation. *Econometrica Journal*, 50, 987-1008.
- Godfrey, L. G. (1988). *Specification Tests in Econometrics: The Lagrange Multiplier Principle and Other Approaches*. Cambridge University Press, London.
- Leybourne, J. and Newbold, J. (2003). Multiple trend breaks and the unit-root hypothesis. *Review of Economics and Statistics*, 79, 212-218.
- Paresh, K. N. and Stephan, P. (2010). A New Unit Root Test with Two Structural Breaks in Level and Slope at Unknown Time. *Journal of Applied Statistics*, 37(9): 1425-1438.
- Perron, P. (1989). The Great Crash, the Oil Price Shock and the Unit Root Hypothesis. *Econometrica*, 57, 1361-1401.
- Perron, P. (1997). Further Evidence on Breaking Trend Functions in Macroeconomic Variables, *Journal of Economics*, 80(2): 355-385.
- Perron, P. (2018). Unit Root and Structural Breaks. *Econometrics*, 1, 1-169.
- Roberto, D. (2015). A Threshold Model of the US Current Account. *Economic Modelling*, 48 (7): 270-280.
- Solarin, S. and Anoruo, E. (2015). Nonlinearity and Unit Root Hypothesis for African per Capita Real GDP. *International Economic Journal*, 29(4): 617-630.
- Tong, H. (1990). *Non-Linear Time Series: A Dynamical System Approach*. Oxford University Press, London.
- Tong, H. and Lim, K. S. (1980). Thresholds Autoregression, Limit Cycles and Cyclical Data. *Journal of the Royal Statistical Society*, 42, 245-292.
- Vogelsang, T. J. and Perron, P. (1998). Additional Tests for a Unit Root Allowing for a Break in the Trend at an Unknown Time. *International Economic Review*, 39(5): 1073-1100.