Science World Journal Vol. 15(No 2) 2020
www.scienceworldjournal.org
ISSN 1597-6343
Published by Faculty of Science, Kaduna State University

Full Length Research Article

MODELING AND SIMULATION OF TRANSIENT FLOW CHARACTERISTICS IN A PRODUCING GAS WELL

Baba Galadima Agaie[1], Jibrin Helma Mbaya[2], Abdullahi Ibrahim[1] and Usman Sani[1]

[1]Department of Mathematical Sciences, Federal University Dutse, Jigawa State, Nigeria
[2]Department of Mathematics and Statistics, Federal Polytechnics, Kaura Namoda PMB 1012, Zamfara State, Nigeria

Corresponding Author's Email Address: babaagaie@yahoo.com

ABSTRACT
Modeling transient flow characteristics in a producing gas well has been a problem long over in natural gas industry due to parameter changing during operation. In the past, authors concentrate on application of correlations and steady state approach, but most of their works does not give satisfactory results because the used applications neglect the transient aspect leading to premature closure of most wells. Many existing models are proposed to correct these anomalies using simplified governing equation ignoring the transient aspect of the flow characteristics. A one-dimensional transient compressible model comparing the conservation of mass, momentum and energy has been presented to investigate the transient behavior of flow characteristics in a producing gas well staged at different inclination due to environment change. The model is solved numerically using the implicit Steger-Warming flux vector splitting method (FSM). The work investigates flow characteristics along depth of the well using different wellbore diameter of 0.073m, 0.0883m, at different time and different thermal conductivities. It shows that transient occurs due to the temperature difference between the produced gas and the flow environment while gas pressure increases due to difference between wellbore pressure and reservoir pressure. The result of this work reflect gas flow law and the characteristics of heat transfer in formation.

Keywords: Modelling Transient Flow, Producing Gas Well, FSM

INTRODUCTION

Flow characteristics in a producing well are the major factors which highlight the ability of a well to produce. Problem always exist when an attempt to model the transient aspect because of their dependency on both space and time, however the technique appears to be superior in representing actual field conditions. It is observed that, for fluid that experience flow propagation, it always generates very large pressure surges and when field data are not available the boundary condition of such flow will be steady state solutions (Zhou and Adewumi, 1995). Solution to such problem is best obtain using scheme which is unconditionally stable. Finite scheme method (FSM), therefore, takes care of such flow propagation and has been applied in pipeline gas transportation (Bahbahni Najad and Shakeri, 2008). In gas well problem it is always good to mention the good work of some authors who contributed vigorously in finding solution to gas production.

Kirkpatrick (1959), presented a simple flowing temperature and pressure gradient that can be used to predict gas lift valves at the injection depth. Ramey (1962), present an analytical equation for wellbore temperature based on simplified heat balance. Jumping *et al* (2012), presents a model of coupled differential equations concerning pressure, temperature density and velocity in gas wells according to the conservation of mass, momentum and energy assuming the flow to be at steady state which is solved using fourth-order Runge Kutta method. Hasan and Kabir, (2012) developed a model by incorporating the kinetic energy term with Joule Thomson coefficient for the prediction of temperature of the produced and injected fluid in the tubing. Orodu *et al,* (2012), presented a predictive model based on analytical approach in order to predict gas flow in gas condensate reservoirs. Liu *et al* (2013), develop a model for the determination of wellhead and bottomhole pressure based on the principles of fluid dynamics, the conservation of mass and momentum were used in the development of the model. Jiuping *et al,* (2013), developed a couple system of partial differential equation for the variation of pressure, temperature, velocity and density at different time and depth in high pressure, high temperature well for two phase. Their solution follows the splitting techniques with Eulerian Generalized Reiman Problems (GRP) schemes. Tong *et al* (2014) presented a transient nonisothermal wellbore flow model for gas well testing. Their governing equation is based on depth and time dependent mass, momentum and gas state equation. Mbaya and Amin, (2015), presented isothermal model for unsteady flow of gas in the producing well without the energy equation that account for the heat changes in the wellbore.

In this work a one dimensional transient compressible model based on the conservation of mass, momentum and energy is presented and solved by Steger-Warming Flux Vector Splitting Scheme (FSM). Gas pressure, temperature, velocity and density along the depth of the well at different time are plotted with different productions temperature and different thermal conductivities, which reflect gas flow law and the characteristics of heat transfer in the information. The results can be used for dynamic analysis of gas production in a producing gas wells

MATERIALS AND METHODS

Gas well flow is governed by equations of motions and for accuracy numerical methods will be used in finding solutions to the nonlinear equations in the research implicit scheme method is considered. In this research continuity, momentum and heart equations are involved.

Governing Equations

The equation governing transient pressure, heat transfer and other parameters of a producing gas well are presented. Due to the transient nature during the inflow from the reservoir, a source term is incorporated in the governing equations.

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u)}{\partial x} = \frac{2\rho_I u_I}{R} \tag{1}$$

$$\frac{\partial(\rho u)}{\partial t}+\frac{\partial\left(a^2\rho+\alpha\rho u^2\right)}{\partial x}=-\alpha\frac{2}{R}\left(\rho u u_1\right)-\rho g\cos\theta+\frac{f\rho u|u|}{R} \quad (2)$$

$$\frac{\partial(\rho T_e)}{\partial t}+\frac{\partial(\rho u T_e)}{\partial x}=\frac{R\pi k_e r_{to}U_{to}\left(T_f-T_e\right)}{\rho_I u_1 C_p\left(k_e+r_{to}U_{to}f\left(t_D\right)\right)} \quad (3)$$

$$\rho=\frac{P}{ZRT} \quad (4)$$

where U_{to} is overall heat transfer coefficient, T_e is undisturbed formation (initial) temperature, T_f is the flowing fluid temperature, $f\left(t_D\right)$ is dimensionless transient heat conduction time function for formation, this parameter enter into wellbore heat loss calculations because heat flow in the surrounding formation varies with time, outer radius of tube (m), u_I inflow velocity, u local velocity, k_e thermal conductivity of earth, ρ_I inflow density, ρ fluid density, R radius of reservoir which is equal to wellbore radius, a sound wave, α is a momentum correction factor. U_{to} is calculated as:

$$U_{to}^{-1}=\left[\begin{array}{c}\frac{r_{to}}{r_{ti}h_f}+\frac{r_{to}\log\left(\frac{r_{to}}{r_{ti}}\right)}{k_{twb}}+\frac{r_{to}\log\left(\frac{r_{ins}}{r_{to}}\right)}{k_{ins}}+ \\ \frac{r_{to}}{r_{ins}\left(h_c+h_r\right)}+\frac{r_{to}\log\left(\frac{r_{co}}{r_{ci}}\right)}{k_{cas}}+\frac{r_{to}\log\left(\frac{r_h}{r_{co}}\right)}{k_{cem}}\end{array}\right] \quad (5)$$

In equation (5), r_{ti} inner radius of tubing, r_{cem} radius of cement, r_{cas} radius of casing, r_w wellbore radius, k_{an} thermal conductivity of annulus, k_{cem} thermal conductivity of cement ,h_r and h_c are heat transfer due to radiation and natural convection respectively Izge (2008).

$$h_r=\sigma F_{tci}\left(T_{to}^{*2}+T_{tci}^{*2}\right)\left(T_{to}^{*}+T_{tci}^{*}\right) \quad (6)$$

$$h_c=\frac{k_{hc}}{r_{to}\log\left(\frac{r_{ci}}{r_{to}}\right)} \quad (7)$$

In equation (6), the asterisk refers to absolute temperature in ^oR (T+460) and σ is the Stefan Botzmann constant (1.713 x10^{-10} Btu/sqft hour ^oR), F_{tci} is the view factor representing the fraction of the radiation emitted from the external surface area of tubing, which is intercepted by the inner casing surface area. The term relates the geometry of the wellbore and the emitting properties of the tubing and casing surface to the radiant heat flux.

For estimating flowing fluid temperature, the formation temperature and its spatial derivative at the wellbore/formation interface are needed. According to (; Izgec, 2008), the value of $t_D>1.5$ is acceptable for engineering accuracy because at $t_D=1.5$ a maximum error is generated. $f\left(t_D\right)$ Can be calculated as:

$$f\left(t_D\right)=\begin{cases}1.128\sqrt{t_D}\left(1-0.3\sqrt{t_D}\right) & t_D<1.5 \\ \left(0.4063+0.5\log t_D\right)\left(1+\frac{0.6}{t_D}\right) & t_D>1.5\end{cases} \quad (8)$$

Where t_D function time dimensionless. Ramey defined an equation for calculating the transient time function and the thermal conductivity of the earth as; $t_D=\frac{\alpha_i t}{r_{wb}^2}$, α_i is calculated using equation (9)

$$\alpha_i=\left[1+m-mT_{pr}^{0.5}\right]^2 \quad (9)$$

$$m=0.48+1.574\omega-0.176\omega^2 \quad (10)$$

Where m is dimensionless and ω is Pitzer acentric factor, dimensionless (Zakia and Housam, 2009), calculated as $w=-\log_{10}\left(P^{sat}\right)-1$, at $T_r=0.7$. It is an important factor that is used in proper characterization of any single pure component

Finite Difference Scheme

To solve equations (1), (2) and (3), numerical procedure must to be followed which starts by writing the equations in a flux vector form as:

$$\frac{\partial Q}{\partial t}+\frac{\partial E(Q)}{\partial x}=H(Q) \quad (11)$$

Where $Q=\begin{bmatrix}\rho \\ \rho u \\ \rho T\end{bmatrix}$, $E(Q)=\begin{bmatrix}\rho u \\ \alpha\rho u^2+\rho a^2 \\ \rho u T\end{bmatrix}$,

$$H(Q)=\begin{bmatrix}\frac{2\rho u u_I}{R} \\ \frac{2\alpha\rho u u_I}{R}+\rho a^2 \\ \frac{2\pi k_e r U\left(T_{ei}-T\right)}{\rho u_I C_p\left(k_e+rUf(t)\right)}\end{bmatrix} \quad (12a,b,c)$$

Equation (11) in terms of *q* can be written as follows:

$$Q=\begin{bmatrix}q_1 \\ q_2 \\ q_3\end{bmatrix},\quad E(Q)=\begin{bmatrix}q_2 \\ \frac{\alpha q_2^2}{q_1}+a^2 q_1 \\ \frac{q_2 q_3}{q_1}\end{bmatrix}$$

Science World Journal Vol. 15(No 2) 2020
www.scienceworldjournal.org
ISSN 1597-6343
Published by Faculty of Science, Kaduna State University

$$H(Q) = \begin{bmatrix} \dfrac{2q_1u_I}{R} \\ \dfrac{2\alpha q_2 u_1}{R} + q_1 g\cos\theta \\ \dfrac{R\pi k_e rU(T_{ei} - T_e)}{q_2 C_p(k_e + rUf(t))} \end{bmatrix} \quad (13a,b,c)$$

From equation (12) A and B are expressed as follows:

$A = \dfrac{\partial E(Q)}{\partial Q}$ and $B = \dfrac{\partial H(Q)}{\partial Q}$ are obtained as follows:

$$A = \begin{bmatrix} 0 & 1 & 0 \\ -\alpha u^2 + a^2 & 2\alpha u & 0 \\ -uT & T & u \end{bmatrix}$$

$$B = \begin{bmatrix} 2u_I & 0 \\ g\cos\theta & \dfrac{2\alpha u_I}{R} \\ 0 & -\dfrac{R\pi k_e rU(T_{ei} - T)}{q_2^2 C_p(k_e + rUf(t))} \end{bmatrix} \quad (14a,b)$$

Where A and B are Jacobian matrix of $E(Q)$ and $H(Q)$ fluxes respectively. The state equation for gas defined in (4) is used for the calculation of pressure and density.

$$A^+ = \frac{u}{2a}\begin{bmatrix} a-u & \dfrac{a+u}{u} & \dfrac{T(a-u)}{au} \\ -(u^2+a^2) & \dfrac{(u+a)^2}{u} & \dfrac{T(-u^2+2ua+a^2)}{au} \\ \dfrac{-a^2(u+a)}{T} & \dfrac{a^2(u-a)}{uT} & \dfrac{a(u+a)}{u} \end{bmatrix}$$

$$A^- = \frac{u}{2a}\begin{bmatrix} u-a & \dfrac{-(u-a)}{u} & \dfrac{T(u-a)}{au} \\ (u-a)^2 & \dfrac{-(u-a)^2}{u} & \dfrac{T(u-a)^2}{au} \\ \dfrac{a^2(u-a)}{T} & \dfrac{-a^2(u-a)}{uT} & \dfrac{a(u-a)}{u} \end{bmatrix} \quad (15a,b)$$

$$E^+ = \begin{bmatrix} \dfrac{\rho(u+a)}{2} \\ \dfrac{\rho(u+a)^2}{2} \\ \dfrac{\rho a^2(u+a)}{2T} \end{bmatrix}$$

$$E^- = \begin{bmatrix} \dfrac{\rho(u-a)}{2} \\ \dfrac{\rho(u-a)^2}{2} \\ \dfrac{\rho a^2(u-a)}{2T} \end{bmatrix} \quad (16a,b)$$

Implicit Steger-Warming Scheme

Equation (11) can be solved numerically using a suitable scheme. Here an implicit Steger-Warming flux vector splitting scheme is used because it is unconditionally stable as compared to other methods (Toro, 2009). Using the implicit scheme, the time derivative is approximated by a first order backward difference approximation to obtain equation (17)

$$\left[I + \frac{\Delta t}{\Delta x}\left(A_j^{n(+)} - A_j^{n(-)}\right) - \Delta t B_j^n\right]\Delta Q_j - \left(\frac{\Delta t}{\Delta x}A_{j-1}^{n(+)}\right)\Delta Q_{j-1} + \left(\frac{\Delta t}{\Delta x}A_{j+1}^{n(-)}\right)$$

$$= -\frac{\Delta t}{\Delta x}\left[E_j^{n(+)} - E_{j-1}^{n(+)} + E_{j+1}^{n(-)} - E_j^{n(-)}\right] + \Delta t H_j^n \quad (17)$$

Initial Conditions

To obtain the initial conditions, we refer to equations (1), (2) and (3) and assumed the flow to be in steady state condition, hence $\dfrac{\partial\rho}{\partial t}$, $\dfrac{\partial(\rho u)}{\partial t}$ and $\dfrac{\partial\rho T}{\partial t}$ are all zeros therefore:

$$\frac{\partial\rho u(x,0)}{\partial x} = \frac{2\rho_1 u_1(x,0)}{R} \quad (18)$$

$$\frac{\partial\rho u(x,0)}{\partial x} = -a^2\frac{\partial\rho}{\partial x} - f\frac{\rho u^2(x,0)}{R} - \rho g\cos\theta - \alpha\frac{2\rho u u_1(x,0)}{R} \quad (19)$$

$$\frac{\partial\rho u T_e(x,0)}{\partial x} = \frac{R\pi k_e r_{to} U_{to}\left(T_f(x,0) - T_e\right)}{\rho_1 u_1(x,0) C_p\left(k_e + r_{to} U_{to} f(t_D)\right)} \quad (20)$$

Boundary Condition

The flow boundary depends on production time and the distance.

$$\rho(0,t) = \rho_0(t) \quad (21)$$

$$\frac{\partial u(0,t)}{\partial x} = u_0(t) \quad (22)$$

Science World Journal Vol. 15(No 2) 2020
www.scienceworldjournal.org
ISSN 1597-6343
Published by Faculty of Science, Kaduna State University

$$T(0,t) = T_0(t) \tag{23}$$

$$\frac{dT_f(0,t)}{dx} = \frac{\dot{Q}_0}{(k_e)_0} \tag{24}$$

$$P(0,t) = P_0(t) \tag{25}$$

where ρ, x, T_f, k_e, T_0 and $\dot{Q}_0$ are the inlet gas density, depth (m), is the temperature of the flowing fluid, thermal conductivity of the formation, temperature of flow environment (tubing) and $\dot{Q}_0$ heat flux of the undisturbed formation.

RESULTS AND DISCUSSION

Temperature and pressure at different time and space has been investigated. The tubing in the well is divided into two, the upper part with diameter of 0.0889*m*; and lower part with diameter of 0.073*m*, well diameter is 0.125*m*, depth of the well is 7100 (ft) i.e. (2164 meters), well roughness is 0.001, other variables used are ground temperature 160 ^0C, ground thermal conductivity is 2.06, ground temperature gradient 0.00218 $^oC/m$, friction coefficient 1.2, density is 1000 kg/m^3 and bottom hole pressure is 70 *MPa.* Time variation is 5 minutes, 15 minutes, 20 minutes and 1 hour. The result obtained using this model is first compared with the work of (Jiuping *et al,* 2013) and are shown to be in good agreement. When the tube outside diameter is 0.073*m* and initial pressure is 70 *Mpa*, temperature is plotted as shown in Fig. 1 at different depth. When we consider upper part with diameter 0.0889*m*, the flow is kept constant and the temperature increases with the increasing depth of the well and keeping the depth constant, the temperature increases with the increasing time. Similarly, when the depth is kept constant, the pressure is plotted as in Fig. 2; as time increases, pressure increases. At the upper part when the diameter is 0.0889m and the flow rate is kept constant the pressure increases with the increasing depth of the well. The reason is that as time increases, the flow from production also increases and the frictional head leads to increase in pressure. The results are in good agreement with (Jiuping*et al*., 2013) obtained by using Generalized Reiman Problem (GRP). Table 1 shows the comparison between (Jiuping*et al*, 2013) and the present model.

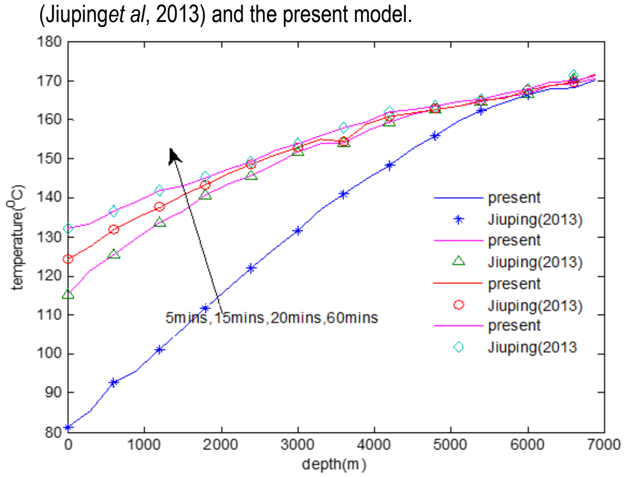

Figure 1. Temperature distribution at different depth and time

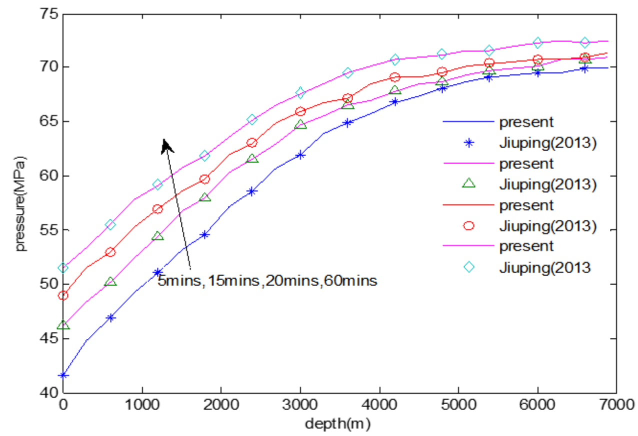

Figure 2. Pressure distribution at different depth and time

Other results obtained includes; the result of different thermal conductivities of the earth at different time which shows that they influence the distribution of temperature between the formation and the tubing as shown in Fig. 3. Thermal conductivity of cement influences temperature distribution. This can be seen in Fig. 4, where increase in cement thermal conductivity causes a reduction in temperature distribution in the well with time because of the exchange of heat between the formation and the cement. The tubing thermal conductivity effect has less impact on temperature distribution inside the tube as in Fig. 5. Fig. 6 shows how heat transfer influences velocity distribution. It is observed that decrease in heat transfer causes velocity to increase because as production continues fluid becomes lighter and lighter making velocity increase. Generally, the study of heat transfer process from well to the surrounding can be done adequately if the geometry, thermal resistance of annulus and cement are properly considered.

Effects of inclination on the flow characteristics are also presented. At pressure 70 Mpa and various inclination angles of 0^0, 15^o, 30^o, 45^o, 60^o and well of depth 7000 ft pressure is plotted and it increases with increasing of inclination as shown in Fig. 7. Similarly, at same values of inclination, the velocity and density are plotted as Fig. 8 and 9. Under these values of angles the velocity is found to increase as the angle increases while the case is different with density profile which decreases as the inclination increases. Sound wave at upper part of the well and lower part has been plotted to determine the behavior of this parameter at different levels and at different tubing radius. It is observed that towards the wellhead and with tubing radius of 0.083 m and at depth of 100 m, the result is plotted in Fig. 10 which shows that it is less heard than when the depth is 500 m and at radius of 0.073 m as shown in Fig. 11

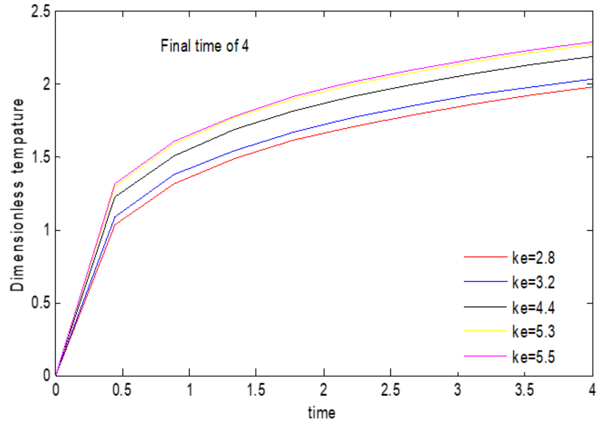

Figure 3. Earth thermal conductivities at different time

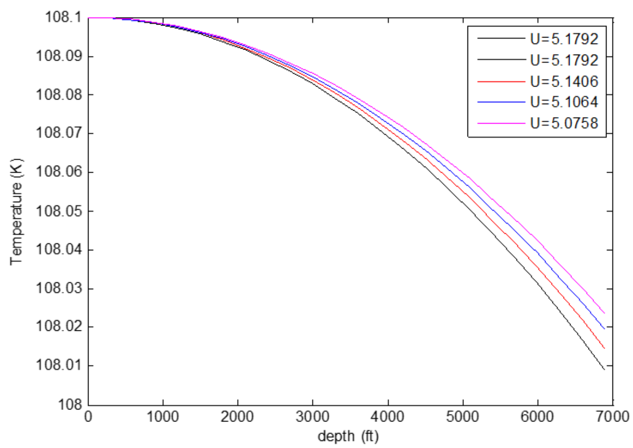

Figure 4. Influence of overall heat transfer

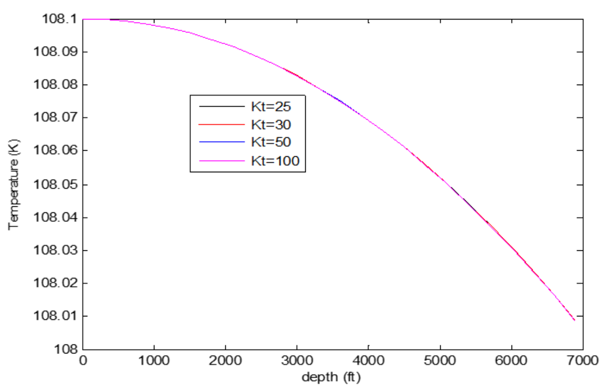

Figure 5. Thermal conductivity of cement on temperature distribution

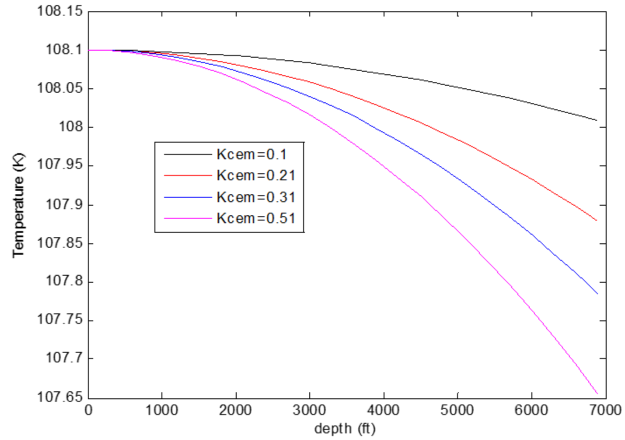

Figure 6. Thermal conductivity of tube

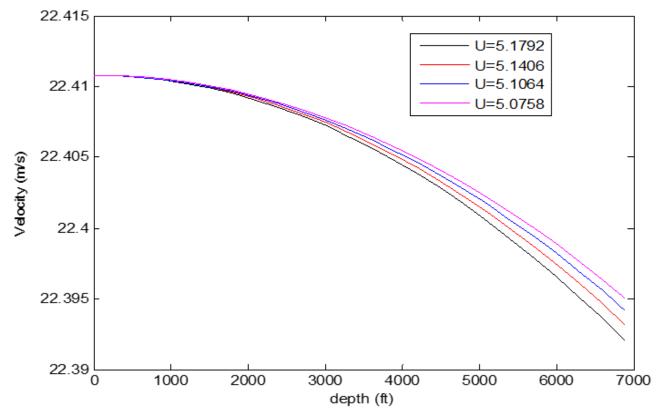

Figure 7. Overall heat transfer on velocity

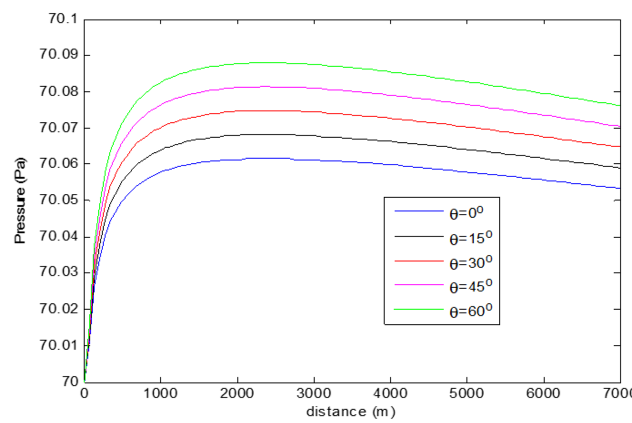

Figure 8. Pressure at different inclination angle

Science World Journal Vol. 15(No 2) 2020
www.scienceworldjournal.org
ISSN 1597-6343
Published by Faculty of Science, Kaduna State University

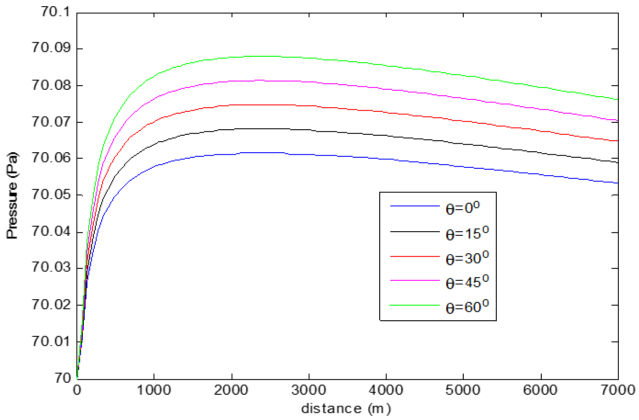

Figure 9. Velocity at different inclination angle

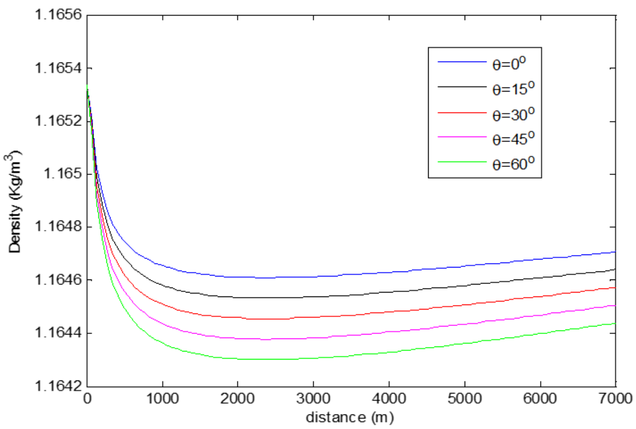

Figure 10. Density at different inclination angle

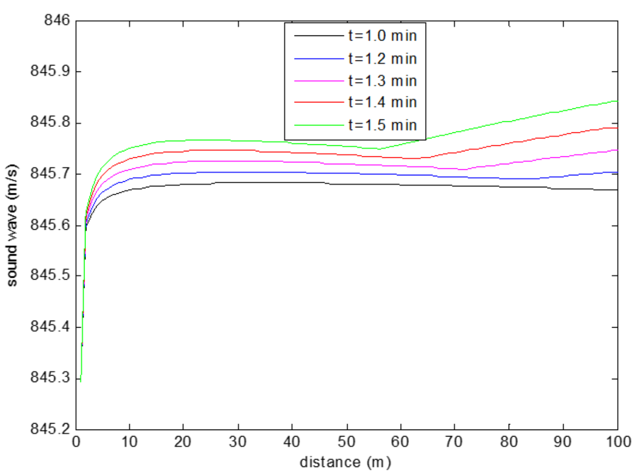

Figure 11. Sound wave profiles at the upper part of the well at different times (min)

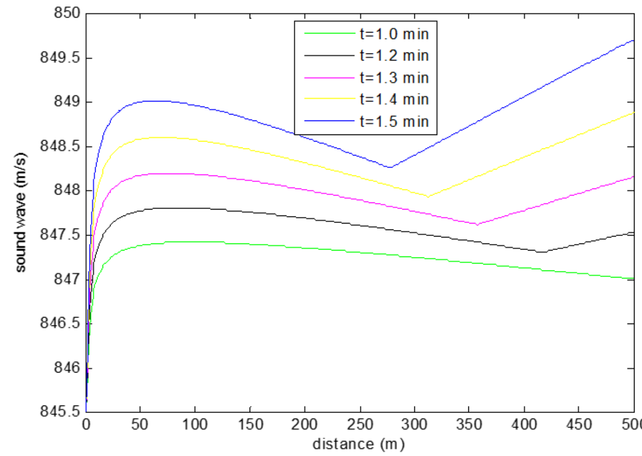

Figure 12. Sound wave profiles at the lower part the well at different times (min).

Table 1: Compares the result of wellbore temperature at time 20 minutes shows a relative percentage error between the calculated results of measurement and Steger-warming as 4.62% and with GRP scheme as 5.12% and by Lax Fredrick method as 6.70%. The relative error between the results in pressure prediction at the same time calculated by Steger-Warming method is 7.44% and GRP is 8.81% while Lax Fredrick method is 9.92%. This means that the prediction of the temperature and pressure is more accurate in actual calculation using Steger-Warming than GRP and LxF method.

Table 1: Comparative results with respect to wellhead temperature and pressure

Producing Well	Temp	Rel. error %	Pressure	Rel. error %
Measured Expt. Juiping et al, 2013)	180.65		76.10	
SW method	172.67	7.98 (4.62%)	70.83	5.27(7.44%)
GRP method	171.78	8.87 (5.12%)	69.92	6.18(8.81%)
Lax F. method	169.30	11.35 (6.70%)	69.36	6.74(9.72%)

Conclusion

The paper considers the variation of pressure, temperature, velocity and density at different times and depths in the well. The gas pressure and temperature curves along the depth of the well are plotted, and the curves reflect the gas flow law and the behavior of heat transfer in formation. It was observed that other flow parameters such as heat transfer and thermal conductivities change with both space and time describing a measure of the ability of heat flow on the flowing gas. The result of the thermal conductivities of flow characteristics will help in determining the feasibility of utilizing a geologic unit to generate industrial geothermal power. It will help well completion engineers in designing casing materials, tubing and the types of cement that can be used in order to regulate the temperature profile of flowing gas and to avoid crack of well or leakage. Furthermore, the work in this paper can increase the safety and reliability of deep completion test, yield notable economic and social benefits and prevent or reduce accidents caused by improper use of well materials.

Acknowledgement
Financial support provided by **TETFund,** Refernce number **TETFUND/DAST&D/POLY/ZAMFARA STATE/CA/2017/VOL.1/2018** is highly appreciated.

REFERENCES

Behbahani Nejad and Bagheri, A. (2008). A Matlab Simulinks Library for Transient Flow Simulation of Gas Network. World *Acedamy of Science Engineering and Technology,*

Hasan, A. R., and Kabir, C. S. (2012). Wellbore Heat Transfer Modeling and Applications. *Journal of Petroleum Science and Engineering.* 86-87, pp. 127-136.

Izgec Bulent. (2008). Transient Fluid and Heat Flow Modelling in Coupled Wellbore/Reservoirs". *Dissertation-Texas A and M University. Petroleum Engineering.*

Jiuping Xu, Wu, Z., Wang, S., Qi, B., Chen, K., Li, X. and Zhao, X.(2012). Prediction of Temperature, Pressure, Density,Velocity Distribution in H-T-H-P Gas Wells" *The Canadian Journal of Chemical Engineering.*

Jiuping Xu, Wu, Z., Wang, S., Qi, B., Chen, K., Li, X. and Zhao, X.(2013). A direct Eulurian GRP Schemes for the Prediction of Gas liquid two phase in HPHT transient gas wells" *Journal of Abstruct and Applied Analysi.,* (2013).

Kirkpatrick, C. V. (1959). Advances in Gas-Lift Technology" Drill Prodution. Practical. 59:24–60. *Journal of Chemical Engineering Science*.

Liu Mingze, Bai Bing, Li Xiao-chun, Shi Lu, Li Qi (2012). A Fast Finite Difference Method for Determination of Wellhead Injection Pressure. Journal of *Central South University, Press and Springer-Verlag Berlin Heidelberg*.

Mbaya, J.H and Amin, N (2016). Modeling Unsteady Flow of Gas and Heat Transfer in Producing Wells" *Journal of Advanced Research in Fluid Mechanics and Thermal Sciences*. Vol. 6 pg 19-33 ISSN (online) 2289-7879.

Orodu O. D (2012). Well Deliverability Prediction of Gas Flow in a Gas Condensate Reservoirs. Modeling near Two Phase in one Dimension. *Brazilian Journal of Petroleum Aand Gas. A Publication of the ABPG.*

Ramey, H. J.(1962). Wellbore Heat Transmission.*Journal of Petroleum Technology and* Science, vol. 225, pp. 427–435.

Tong, L., Hai-quam Zhong, Ying-Chuan Li.(2014). Transient Simulation of Wellbore Pressure and Temperature during Gas Well "*Testing. Journal of Engineering Research and Technology.*

Zhou, J., and Adewumi, M. A. (1995) Simulation of Transient Flow in Natural Gas Pipelines using TVD Scheme. *International Journal of Numerical Method.*Vertical and Slanted Wells. *Dissertation, Louisiana State University, Baton Rouge, LA,*