UNSTEADY MAGNETO HYDRODYNAMIC POISEUILLE OSCILLATORY FLOW BETWEEN TWO INFINITE PARALLEL POROUS PLATES

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ABSTRACT

Alfred studied on the steady Magneto hydrodynamic (MHD) Poiseuille flow between two infinite parallel porous plates in an inclined magnetic field. The case of steady Poiseuille flow without oscillatory to extend the existing work. The study examines the unsteady MHD Poiseuille oscillatory flow between the two infinite parallel porous plates in a magnetic field. The motion of two dimensional unsteady oscillatory flow of viscous, electrically, conducting, incompressible fluid flowing between two infinite parallel plates at constant pressure gradient was examined. The analytical expression for the fluid velocity obtained was expressed in terms of Hartmann number. The effects of the magnetic inclinations, Hartmann number, suction/injection and pressure gradient to the velocity are presented graphically. It was discovered that the increase in the Hartmann number and suction/injection leads to the increase in the velocity.

Keywords: Magneto-hydrodynamic, Hartmann number, Pressure, Velocity

INTRODUCTION

Magneto hydrodynamic (MHD) is the study of the magnetic properties of electrically conducting fluid. Example of such magneto fluids includes plasma, liquid metals and salt or electrolytes. The word MHD is derived from Magneto-meaning magnetic field, Hydro-meaning water, and Dynamics meaning movement. The subject MHD was initiated by Swedish Electrical engineer Alfven in (1942). If an electrically conducting fluid is placed in a constant magnetic field, the motion of the fluid induces current which create forces on the fluid. The production of these currents leads to their use in the designing of electricity generation and production among other devices of MHD generators. Hassanien and Mansour (1990) discussed the magnetic flow through a porous medium between two infinite parallel plates.

In (2013) Alfred studied on the steady MHD Poiseuille flow between two infinite parallel porous plates in an inclined magnetic field. Alfred studied the case of steady Poiseuille flow without oscillatory to extend the work which examined the unsteady MHD Poiseuille oscillatory flow between two infinite parallel porous plates in a magnetic field.

Shercliff (1956) studied the steady motion of an electrically conducting fluid in pipes under transverse magnetic fields. Drake (1965) considered the flow in a channel due to periodic pressure gradient and solved the resulting equation by separation of

variables method. Singh and Ram (1978) studied laminar flow of an electrically conducting fluid through a channel in the presence of a transvers magnetic field under the influence of a periodic pressure gradient and solved the resulting differential equation by the method of Laplace transformation. Furthermore, Ram (1984) analyzed half effect on heat and mass transfer flow through porous media.

The magnetic flow through a porous medium between two infinite parallel plates was discussed by Hassanien and Mansour in (1990). Shimmomura (1991) discussed magneto hydrodynamics turbulent channel flow under a uniform transvers magnetic field. Singh (1993) considered steady magneto hydrodynamics fluid flow between two finite parallel plates. Al-Hadhrami (2003) determined the flow through horizontal channels of porous material and obtained the velocity expressions in terms of the Reynolds number. Ganesh (2007) studied the unsteady MHD stokes flow of a viscous fluid between two parallel porous plates. K D Singh and Reena Pathak (2010) discussed an analysis of an oscillatory rotary MHD Poiseuille flow with injection/suction and hall current.

Idowu and Olabode (2014) Studied unsteady MHD Poiseuille flow between two infinite parallel plates in an inclined magnetic field with heat transfer. Kuiry and Surva Bahadur (2015) discussed the effect of an inclined magnetic field on steady Poiseuille flow between two parallel porous plates in (2014). Gital and Abdulhameed (2013) studied mixed convection flow for unsteady oscillatory MHD second grade fluid in a porous channel with heat generation. It is assumed that the walls of the channel are porous so that the injection/suction may take place. Umavathi et al (2009) studied the problem of unsteady oscillatory flow and heat transfer in a horizontal composite porous medium. The flow is modelled using the Darcy –Brinkman equation. This present paper studied the slip effect on MHD oscillatory flow of fluid in a porous medium with heat and mass transfer and chemical reaction. The temperatures prescribed at the plates are uniform and asymmetric.

Oscillatory flow is a periodic flow that oscillates around a zero value. Oscillatory flow is a single swing or movement in one direction of an oscillating body. They are generally used in the literature to describe the flows in which velocity or pressure or both depend on time. Oscillatory flow is always important for it has many practical applications for example in the aerodynamics of helicopter rotor or in fluttering airfoil and also in a variety of bio –engineering problems.

MATERIALS AND METHODS

The governing and formation of MHD phenomena can simply be described as follows: Consider an electrically conducting fluid moving with velocity V. At right angles to this flow, we apply a magnetic field, the field strength of which is represented by the vector B. It shall be assumed that the fluid has attained steady state condition i.e. flow variables are independent of the time t. This condition is purely for analytic reasons so that no macroscopic charge density is being built up at any place in the system as well as all currents are constant in time. Because of the interaction of the two fields, namely, velocity and magnetic field, an electric field vector denoted E is induced at right angles to both V and B. This electric field is given by $E = V \times B$ (1)

Where \times stands for cross product of two vectors V and B. If assuming the conducting fluid is isotropic in spite of the magnetic field, we can denote the electrical conductivity of the fluid by a scalar. By Ohm's law, the density of the current induced in the conducting fluid denoted J is given by

$$J = \sigma(V \times B) \tag{2}$$

Simultaneously occurring with the induced current is the Lorentz force ${\sf F}$ given by

$$F = J \times B \tag{3}$$

This force conducting occurs because, as an electric generator, the conducting fluid cuts the lines of magnetic field. The vector F is the vector cross product of both J and B and is a vector perpendicular to the plane of both J and B. This induced force is parallel to V but in opposite direction. Laminar flow through a channel under uniform transverse magnetic field is important because of some of the uses such as the MHD generator, MHD pump and electromagnetic flow meter.

Considering an unsteady electrically conducting, viscous, incompressible fluid moving between two infinite parallel plates both kept at a constant distance 2h between them. Both plates of the channel are fixed with no motion. This is plane Poiseuille flow driven by a constant pressure gradient. The equations of motion are the continuity equation

$$\frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$
(4)
And the navier-strokes' equations

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial v}{\partial y} = \frac{1}{\rho} f_{Bx} - \frac{1}{\rho} \frac{\partial p}{\partial x} + \frac{\mu}{\rho} \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right)$$
(5)

$$\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = \frac{1}{\rho} f_{By} - \frac{1}{\rho} \frac{\partial p}{\partial y} + \frac{\mu}{\rho} \left(\frac{\partial^2 v}{\partial x} + \frac{\partial^2 v}{\partial y} \right)$$
(6)

Where ρ is the fluid density F_{Bx} , F_{By} , u, v are the components of the body force per unit mass of the fluid and the velocity in x and y directions respectively μ is the fluid viscosity and ρ is the pressure acting on the fluid. This flow is practically horizontal, in which we choose the axis of the channel formed by the two plates as the x-axis and assume that the flow is in this direction. Thus v=0 and u=u(y). Also assume that the flow is one dimensional. Also v=0 implies that the continuity equation collapse to

$$\frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} = 0$$

Hence, equations (5) and (6) becomes

$$\frac{\partial u}{\partial t} = \frac{1}{\rho} f_{Bx} - \frac{1}{\rho} \frac{\partial p}{\partial x} + \frac{\mu}{\rho} \frac{\partial^2 u}{\partial y^2}$$
(7)

$$0 = \frac{1}{\rho} f_{Bx} - \frac{1}{\rho} \frac{\partial p}{\partial y}$$
(8)

Let the pressure gradient be $-\frac{\partial p}{\partial x} = S$ Now since p = p(X), Equations (2.8) collapses and $\frac{\partial p}{\partial x} = \frac{dp}{dx} = -S$ (9)

Combining (7) and (9) we find

$$0 = \frac{1}{\rho} + f_{Bx} - \frac{s}{\rho} + \frac{\mu}{\rho} \frac{\delta^2 u}{\delta v^2}$$
(10)

The X – component of the Lorentz force in equation (10) can be expressed as follows

$$\frac{\partial u}{\partial t} + \frac{1}{\rho} = \frac{\sigma}{\rho} \left[(uiXj)XjB_0 \right] = -\frac{B_0}{\rho} \frac{\sigma}{\rho} ui.$$
(11)

Where B_0 is the Magnetic field strength component assumed to

be applied to a direction perpendicular to fluid motion. Equation (11) is achieved after using expressions of force and induced current in equations (2) and (3) together with the fact, that for any three vectors A,B and C it can be shown that (AXB)XC=(B.A)C-(B.C)A. In equation (11) i and j refer to unit rectangular vectors i and j. Hence equation (2.10) becomes;

$$\frac{\partial u}{\partial t} + \frac{d^2 u}{dy^2} - \frac{\sigma}{\mu^2} u B_0 + \frac{s}{\mu} = 0$$
(12)

Next is to investigate how magnetic inclination to the velocity influences the fluid flow. This behaviour is modelled by introducing an angle of inclination to the third term of equation (12) as follows;

$$\frac{\partial u}{\partial t} + \frac{d^2 u}{dy^2} - \frac{\sigma}{\mu} B_0^2 \operatorname{usin}^2 \theta + \frac{s}{\mu} = 0$$
(13)

Where θ is the angle between V and B in equation (13) it can be assumed that the two fields are inclined to each other at an angle θ lying in the range $0 \le \theta \le \frac{\pi}{2}$ and the equation is solved subject

to the boundary conditions u = 0 when $y = \pm h$

In order to non-dimensionalized the equation (13), the following non-dimensionalization quantities are introduced.

$$x^* = \frac{x}{l}, y^* = \frac{y}{l}, u^* = \frac{ul}{v}, P^* = \frac{pl^2}{\rho v}, t^* = \frac{tv}{l}$$
 (14)

Where *I* is the characteristic length of the plate and υ is the kinetic viscosity. Using these quantities into equation (13) and later dropping the asterisks the following are obtained;

$$\frac{\partial \left[\frac{u^* \upsilon}{L}\right]}{\partial \left[\frac{t^* L}{\upsilon}\right]} + \frac{\partial^2 \left[\frac{u^* \upsilon}{L}\right]}{\partial \left[y^* L\right]} - \frac{\sigma^2}{\mu L} \left(\frac{u^* \upsilon}{L}\right) \sin^2 \theta + \frac{s}{\mu} = 0$$

$$\frac{\upsilon^2}{L^2}\frac{\partial u^*}{\partial t^*} + \frac{\upsilon}{L^2}\frac{\partial^2 u^*}{\partial y^{*2}} - \frac{\sigma^2}{\mu L}B_0^2\sin^2\theta u + \frac{s}{\mu} = 0$$

$$\frac{\partial u^*}{\partial t^*} + \frac{\partial^2 u^*}{\partial y^{*2}} - \frac{\sigma^2}{\mu} B_0^2 L^2 u \sin^2 \theta + L^2 S = 0$$
(15)

Equation (15) may be written as

$$\frac{\partial u}{\partial t} + \frac{\partial^2 u}{\partial y^2} - M^2 u + L^2$$
(16)

Where M = M^{*} sin θ and M^{*} = IB⁰ $\sqrt{\frac{\sigma}{\mu}}$ = Ha

Where Ha is the Hartmann number defined by

$$\operatorname{Ha}^{2} = \frac{\left(\sigma_{B_{0}}^{2}L^{2}\right)}{\mu}$$

Since L and S are constants and letting c = L^2 S a constant, equation (16) becomes

$$\frac{\partial u}{\partial t} + \frac{\partial^2 u}{\partial y^2} + C = 0 \tag{17}$$

Mhd fluid flow between two infinite parallel plates one of which is porous. Suppose V_0 is a characteristic velocity moving perpendicular to the fluid flow at a constant given pressure gradient. For lower porous plate, this characteristic velocity is the one which will maintain an unsteady fluid flow against the suction and injection of the fluid in which it is moving perpendicular to the fluid flow. The origin is taken at the centre of the channel and the x, y coordinates axes are parallel and perpendicular to the channel walls respectively. The governing equation will be

$$\frac{\partial u}{\partial t} + V_0 \frac{\partial u}{\partial y} + \frac{s}{\rho} + \frac{\mu}{\rho} \frac{\partial^2 u}{\partial y^2}$$
(18)

This equation was derived directly from equation (5). For injection of flood into the channel, V_0 is taken as positive and for suction V_0 is negative. Since u is a function of y and t and earlier analysis, equation (17) follows. This may be written as:

$$\frac{\partial u}{\partial t} - \frac{\mu}{\rho} \frac{\partial^2 u''}{\partial y^2} + V_0 \frac{\partial u}{\partial y} + \frac{s}{\rho} = 0$$
(19)

The magnetic influence comes from the third term in equation (13).

To model this influence in equation (19) the term $(-M^2 u)$ was added which yielded

$$\frac{\partial u}{\partial t} - \frac{\mu}{\rho} \frac{\partial^2 u}{\partial y^2} + V_0 \frac{\partial u}{\partial y} + M^2 u + d = 0$$

$$\frac{\partial u}{\partial t} + \frac{\partial^2 u}{\partial y^2} + A \frac{\partial u}{\partial y} - M^2 u + d = 0$$
(20)
Where $A = \frac{V_0}{\mu}$

METHOD OF SOLUTION

In order to solve equation (16) and (20) closed form method of solution was used.

For purely oscillation, the following was assumed.

$$d = l = \ell^{i\omega t}$$

$$u(y,t) = {}^{u_0}(y) \ell^{i\omega t}$$
(21)

Considering equation (16)

$$\frac{\partial u}{\partial t} + \frac{\partial^{2} u}{\partial y^{2}} - M^{2} u + L^{2} = 0$$

$$\frac{\partial u}{\partial t} = iwtu_{0}(y)\ell^{iwt}$$

$$\frac{\partial u}{\partial y} = u_{0}(y)\ell^{iout}$$

$$\frac{\partial^{2} u}{\partial y^{2}} = u^{"}_{0}(y)\ell^{iwt}$$
(22)

$$i\omega t u_{o}(y)\ell^{i\omega t} + u_{0}(y)\ell^{i\omega t} - M^{2}u_{0}(y)\ell^{i\omega t} + \lambda^{2}\ell^{i\omega t} = 0$$

$$u_{0}(y) - (M^{2} - iw)u_{0}(y) = -\lambda^{2}$$

Solving for the homogenous part, i.e., complementary function (c.f)

$$\begin{split} M^2 - L_1 &= 0, \\ wereL_1 &= M^2 - i\omega \\ M^2 &= L_1 \\ M &= \pm \sqrt{L_1} \\ M_1 &= \sqrt{L_1} \\ M_2 &= \sqrt{L_1} \\ \text{Therefore, } u_0 \left(y \right) &= C_1 \ell^{m_1 y} + C_2 \ell^{m_2 y} \\ \text{For particular solution} \\ U_p &= K \end{split}$$

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 $0 - L_1 K = -\lambda^2$, $K = \frac{\lambda^2}{L_1}$

Therefore, the solution is

$$u_0(y) = C_1 \ell^{m_1 y} + C_2 \ell^{m_2 y} + \frac{\lambda^2}{L_1}$$

Applying the boundary condition u = 0 at $v = \pm 1$

$$0 = C_{1}\ell^{m_{1}} + C_{2}\ell^{m_{2}} + \frac{\lambda^{2}}{L_{1}}$$

$$C_{1}\ell^{M_{1}} + C_{2}\ell^{M_{2}} = \frac{\lambda^{2}}{L_{1}}$$

$$C_{1}\ell^{M_{1}} + C_{2}\ell^{M_{2}} = L_{2}$$
(25)
Where $L_{2} = \frac{-\lambda^{2}}{L_{1}}$

For
$$u = 0$$
 and $y = -1$, We have
 $C_1 \ell^{-m_1} + C_2^{-m_2} = L_2$ (26)
Solving (25) and (26) simultaneously as follows;
 $C_1 \ell^{m_1} + C_2 \ell^{m_2} = L_2 \times \ell^{-m_2}$
 $C_1 \ell^{-m_1} + C_2 \ell^{-m_2} = L_2 \times \ell^{m_2}$
 $C_1 \ell^{m_1 - m_2} + C_2 = L_2 \ell^{-m_2} - C_1 \ell^{m_2 - m_1} + C_2 = L_2 \ell^{m_2}$
 $C_1 \left[\ell^{m_1 - m_2} - \ell^{m_2 - m_1} \right] = L_2 \left(\ell^{-m_2} - \ell^{m_2} \right)$
 $C_1 = \frac{L_2 \left(\ell^{-m_2} - \ell^{m_2} \right)}{\ell^{m_1 - m_2} - \ell^{m_2 - m_1}}$

From (3.6) $C_2 \ell^{-m_2} = L_2 - C_1 \ell^{-m_1}$ $I = C_1 \ell^{-m_1}$

$$C_2 = \frac{L_2 - C_1 \ell^{-m_1}}{\ell^{-m_2}}$$

Therefore the general solution for equation (5) and (6) becomes

$$\frac{\partial u}{\partial t} + \frac{\partial^2 u}{\partial y^2} + A \frac{\partial u}{\partial y} - M^2 u + d = 0$$
(27)

$$C_3 \ell^{-m_3} + C_4 \ell^{-m_4} = -K_2 \tag{30}$$

Solving simultaneously,

$$C_{3} = \frac{-K_{2} \left(\ell^{m_{4}} - \ell^{-m_{4}} \right)}{\ell^{m_{3} - m_{4}} - \ell^{m_{4} - m_{3}}}$$
$$C_{4} = \frac{-K_{2} - C_{3} \ell^{-m_{3}}}{\ell^{-m_{4}}}$$

Therefore the solution of the equation (30) is

$$u(y, y) = \left(C_{3}\ell^{m_{3}y^{m_{4}}} + C_{4}\ell^{m_{4}y} + K_{2}\right)\ell^{ia}$$

RESULTS AND DISCUSSION



Figure 1: Effect of Hartman Number on Velocity Profile

In figure 1 the influence of velocity profile is presented where the Hartman number varies. It can be observed that increase in the Hartmann number lead to increase in velocity. Therefore, Hartmann number have significant impact on velocity profile of the distribution.



Figure 2: Effect of Variable Suction/Injection on Velocity Profile

Figure 2 shows effect of Hartmann number on variable suction/injection. For every change in Hartmann number have impact on variable suction/injection. It was observed that increase in Hartmann leads to increase in variable suction/injection which leads to small change on velocity profile.



Figure 3: Effect of Pressure gradient on velocity profile

In figure 3 the gradient of pressure varies on fluid flow velocity. There is a steady flow from the initial up to the point (-0.85, 0.68) when relative variation is observed which remain throughout the simulation. Hence pressure gradients have significant effect on velocity profile.



Figure 4: Effect Slip parameter on velocity profile

From figure 4 two points are critical to velocity profile when slip parameter are varies. The points are (-0.75, 0.8791) and (0.35,1.155). In the first point convergent point for any change in the slip parameter is observed before dispersed and later reconverted at the second point.

The unsteady MHD Poiseuille flow between two infinite parallel porous plate in a magnetic field was studied and the governing equation of the flow of fluid were solved using closed form technique, the effect of parameters is shown graphically against y using MATLAB. The results as shown on figures 1-4 are discussed based on the analysis on the graphs interpretations.

In figure 1 and 2 it was concluded that velocity is influence by Hartmann number and suction/injection. The increase in Hartmann number and suction/injection increases the velocity of flows. In the study, the motion of two dimensional unsteady oscillatory flow of viscous, electrically, conducting, incompressible fluid flowing between two infinite parallel plates one of which is porous and under the influence of a transvers magnetic field and constant pressure gradient was examined. The lower plate was assumed porous while the upper plate was not. The resulting

governing equations of motion are solved using closed form. The analytical expression for the fluid velocity obtained was expressed in terms of Hartmann number. The effects of the magnetic inclinations, Hartmann number, suction/injection and pressure gradient to the velocity are presented graphically and discussed. The equation was analyzed using closed form technique and the effect of the pertinent parameters on the fluid flow has been discussed with the aid of velocity profile.

Conclusion

Magneto hydrodynamic (MHD) Poiseuille Oscillatory flow is very important particularly in the fields of petroleum technology for the flow of oil through porous rocks, in chemical engineering for the purification and filtration processes. The principles of this subject are very useful in recovering the water for drinking and irrigation purposes. The knowledge of flows through magnetic field is also useful to study the movement of natural gas and water through the oil reservoirs. In conclusion to the research, it was discovered that the increase in the Hartmann number and suction/injection leads to the increase in the velocity.

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REFERENCES

Alfven, H., (1942). Existence of electromagnetic hydrodynamic wave: Nature, vol.150, issue 3805, pp. 405-406.

- Al-Hadhrami, A.K., Elliot, L., Ingham, M.D. and Wen,X., (2003). Flow through horizontal channel of porous metals: International Journal of Energy Research, vol.27,pp.875-889
- Alfred W. (2013). Unsteady MHD poiseuille flow between two infinite parallel porous plates in an inclined magnetic field.journal of computation and applied Mathematics.3(4):220-224
- Drake, D. G. (1965). On the flow in a channel due to a periodic pressure gradient: Quart.J. of mech. And Appl. Maths., vol. 18,No.1.

- Ganesh, S. and Krishnambal, S. (2007). Unsteady MHD stokes flow of viscous fluid between two parallel porous plates: Journal of Applied Science vol.7, pp.374-379.
- Gital, A.Y. and Abdulhameed M. (2013). Mixed Convection Flow for Unsteady Oscillatory MHD Second Grade Fluid in a Porous Channel with Heat generation, Int. J. Sci. Engr. Res. 4 (9) ISSN 2229 – 5518.
- Hassanien and Mansour (1990). Magnetic flow through a porous medium between two infinite parallel plates.
- Idowu A. S. (2014). Unsteady MHD poiseuille flow between two infinite parallel plates in an inclined magnetic field with heat transfer. Journal of Maths.vol.10, pp.47-53.
- Kuiry, D. R. and Surya (2015). Effect of an inclined magnetic field on stead poiseuille flow between two parallel porous plates. Journal of Mathematics. Vol.10,pp. 90-96.
- Ram, P. C., Singh, C.B. and Singh, U. (1984). Hall effect on heat and mass tramsfer flow through porous medium: Astrophysics space science,100, pp.45-51
- Shercliff, J. A., (1956). Entry of conducting and n0n-conducting fluid in pipes: Journal of Mathematical proc. Of the Cambridge philosophical soc., vol.52,pp.573-583.
- Shimmomura, Y., (1991). Large eddy simultation of magneto hydrodynamic turbulent channel flow under uniform magnetic field: phys. Fluid A, Vol, No.12, Pp. 3107-3116.
- Singh, C. B. and Ram, P. C., (1978). Unsteady magneto hydrodynamic fluid flow through a channel: journal of scientific research, vol.XXXVIII, No.2.
- Singh. K. D. And Reena Pathak (2011). Analysis of an oscillatory rotary MHD poiseuille flow with injection/suction and hall current: Department of Math proc Indian natn sci. Acad 76. N0.4 pp. 201-207.
- Umavathi, C., Chankha, A. J., Mateen A. and Al Mudhaf A. (2009). Unsteady Oscillatory Flow and Heat Transfer in a Horizontal Composite Porous Medium Channel, Nonlinear Analysis: Modeling and Control, 14 (3): 397 – 425