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# ANALYSIS OF MAGNETOHYDRODYNAMICS EFFECTS ON CONVECTIVE FLOW OF DUSTY VISCOUS FLUID

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## ABSTRACT
This paper presents a finite difference analysis of Magnetohydrodynamics (MHD) effects on convective flow of dusty viscous fluid incorporating viscous dissipation with respect to fluid and dusty particle. The criterion for the existence and unique solution is established and the properties of solution are examined. The dimensionless governing coupled non-linear partial differential equations governing the phenomenon were solved numerically using finite difference scheme. The effects of various parameters on the velocities are shown graphically and discussed. It is observed that both the velocity of the fluid $(\phi)$ and dust particles $(\psi)$ increases with an increase in Grashof number. Velocity of the fluid decreases with an increase in Magnetic parameter. Increase in Porosity parameter has little effects on the velocity of the fluid and that of the particle. Slight increase in volume faction of dust particles increases the velocity of the particles rapidly and increases moderately the velocity of the fluid. Increase in Mass concentration of dust particles decreases the velocity of both the fluid and particles. Also, considerable effect was noticed on the temperature $(\theta)$ by increasing the Eckert number, temperature also increased. Increase in the Prandtl number decreases the temperature of the fluid.

**Keywords:** Convective Flow, Dusty Viscous Fluid, MHD, Viscous Dissipation.

## INTRODUCTION
The word Magnetohydrodynamics (MHD) is derived from Magneto- meaning magnetic field and Hydro- meaning liquid, and Dynamics- meaning movement. Momentum and heat transfer of dusty fluids have tremendous application in engineering and sciences. In the past few decades researchers have been focusing on analysing the heat and mass transfer characteristics of dusty fluids through different channels. In the present study we discuss the MHD effects on convective flow of viscous fluid embedding with conducting dust particles. The convective flow of dusty viscous fluid has a variety of applications like waste-water treatment, combustion and petroleum transport, power plant (Saidu *et al.*, 2010).

The concept of heat transfer of a dusty fluid has a wide range of applications in air conditioning, refrigeration, pumps, accelerations, nuclear reactor space, heating, power generation, chemical processing, filtration and geothermal systems. Nowadays electromagnetic pumps and their modifications are widely used in metallurgy and materials processing in order to transport and dose (exact batching) melting metal (Ivlev *et al.*, 1993). The experimental and theoretical works on MHD flow with chemical reaction have been done extensively in various areas i.e. sustain plasma confinement for controlled thermo nuclear fusion, liquid metal cooling of nuclear reactions and electromagnetic casting of metals, keeping the above facts in mind many authors have been attracted in this field of study of heat and mass transfer through dusty fluids.

Makinde and Aziz (2011) studied boundary layer flow of a nanofluid over a stretching surface. Attia and Aboul-Hassan (2002) studied the flow of a conducting, viscoelastic fluid between two horizontal porous plates in the presence of transverse magnetic field. Attia (2006) investigated the time varying couette flow with heat transfer of a dusty viscous incompressible, electrically conducting fluid under the influence of a constant pressure gradient is studied without neglecting the Hall Effect.

Siddiqa S, *et al.* (2017) studied these problems involving natural convection flow of a two-phase dusty non-Newtonian fluid along a vertical surface and solved it analytically and numerically to compute velocities and friction factors under influences of magnetic and porous medium resistances. Ghadikolaei, *et al.* (2018) studied steady two dimensional dusty nano fluid flow over radiating surfaces. Saidu, *et al*. (2010) studied the laminar convective flow of a dusty viscous fluid through a porous medium of non-conducting walls in the presence of uniform transverse magnetic field with volume fraction and considering porous parameter.

In this paper we discuss a laminar convective flow of a dusty viscous fluid through a porous medium of non-conducting walls in the presence of uniform transverse magnetic field incorporating viscous heat dissipation is investigated. A similar problem but with negligible viscous dissipation is considered in Saidu et al. (2010).

## MODEL FORMULATION
In formulating our model, the following assumptions were made:

(i) The pressure gradient $\frac{\partial p}{\partial x}$ is negligible because is already taken care of in $g\beta\left(T-T_0\right)$

(ii) Viscous heat dissipation is considered based.

Based on the above assumptions and following Saidu et al. (2010), the equations governing this phenomenon are;

$$(1-\varphi)\frac{\partial u_f}{\partial t}=(1-\varphi)\left[\upsilon\frac{\partial^2 u_f}{\partial y^2}+g\beta\left(T-T_0\right)\right]+\frac{KN_0}{\rho}\left(u_p-u_f\right)+\frac{\sigma\mu_c^2H_0^2}{\rho}u_f-\frac{\mu}{K_1}u_f \quad (1)$$

$$N_0 m\frac{\partial u_p}{\partial t}=\varphi\left[\mu\frac{\partial^2 u_p}{\partial y^2}+\rho g\beta\left(T-T_0\right)\right]-KN_0\left(u_p-u_f\right) \quad (2)$$

$$\frac{\partial T}{\partial t}=\frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y^2}+\frac{\upsilon}{C_p}\left[\left(\frac{\partial u_f}{\partial y}\right)^2+\left(\frac{\partial u_p}{\partial y}\right)^2\right] \quad (3)$$

Subject to the initial and boundary conditions

$$\left.\begin{array}{lll} u_f(y,0)=0, & u_f(0,t)=0, & u_f(h,t)=U \\ u_p(y,0)=0, & u_p(0,t)=0, & u_p(h,t)=U \end{array}\right\} \quad (4)$$

$$T(y,o)=T_0,\quad T(0,t)=T_0,\quad T(h,t)=T_1$$

Where,

$\upsilon$ = kinematic viscosity, $u_f$ = The velocity of the fluid, $u_p$ = Velocity of the dust particles, $m$ = mass of each dust particles, $N_0$ = number density of dust particles, $T$ = temperature of the fluid, $T_0$ = initial temperature fluid and wall, $\beta$ = volumetric coefficient of thermal expansion, $C_p$ = specific heat at constant pressure, $\varphi$ = volume fraction of dust particles, $K$ = stokes resistance coefficient, $H_0$ = magnetic field induction, $\mu_c$ = magnetic permeability, $\sigma$ = electric conductivity of the liquid, $k$ = thermal conductivity

$K_1$ = porous parameter.

## MATERIAL AND METHODS

### Non-Dimensionalization

Here, we non-dimensionalize equation (1)-(4) in order to reduce the number parameter and make the equations dimensionless, using the following dimensionless variables:

$$t'=\frac{\upsilon t}{h^2},\ \psi=\frac{u_p h}{\upsilon},\ y'=\frac{y}{h},\ \theta=\frac{T-T_0}{T_1-T_0},\ \phi=\frac{u_f h}{\upsilon}, \quad (5)$$

Using (5), and after dropping the prime, (1) – (4) becomes

$$\frac{\partial\phi}{\partial t}=\frac{\partial^2\phi}{\partial y^2}+Gr\theta+\varepsilon_1(\psi-\phi)+\varepsilon_2 M\phi-\varepsilon_3\phi \quad (6)$$

$$f\frac{\partial\psi}{\partial t}=\varphi\frac{\partial^2\psi}{\partial y^2}+\varphi Gr\theta-\alpha(\psi-\phi) \quad (7)$$

$$\frac{\partial\theta}{\partial t}=\frac{1}{Pr}\frac{\partial^2\theta}{\partial y^2}+E_c\left[\left(\frac{\partial\phi}{\partial y}\right)^2+\left(\frac{\partial\psi}{\partial y}\right)^2\right] \quad (8)$$

$$\left.\begin{array}{lll} \phi(y,0)=0; & \phi(0,t)=0; & \phi(1,t)=b_1 \\ \psi(y,0)=0; & \psi(0,t)=0; & \psi(1,t)=b_2 \\ \theta(y,o)=0; & \theta(0,t)=0; & \theta(1,t)=1 \end{array}\right\} \quad (9)$$

Where,

$M=\dfrac{h^2\sigma\mu_c^2 H_0^2\upsilon}{\mu}$ (Magnetic Parameter),

$\varepsilon_1=\dfrac{h^2 K N_0\upsilon}{\mu(1-\varphi)};\ \varepsilon_2=\dfrac{1}{(1-\varphi)};\ \varepsilon_3=\dfrac{h^2\mu}{K_1\upsilon(1-\varphi)}$ (Porous parameter), $f=\dfrac{\upsilon N_0 m}{\mu}$ (Mass concentration of dust particles), $Gr=\dfrac{h^3 g\beta(T_1-T_0)}{\upsilon^2}$ (Grashof number), $\alpha=\dfrac{Kh^2}{m\upsilon}$; (Concentration resistance ratio), $\Pr=\dfrac{\mu c_p}{k}$ (Prandtl number); $E_c=\dfrac{\upsilon^2}{h^2 C_p(T_1-T_0)}$ (Eckert number).

### Existence and Uniqueness of solution

Our proof of existence of unique solution of the system of parabolic equations (6)-(9) will be analogous to Ayeni's (1978) proof.

**Theorem 1:** There exists a unique solution $\phi(y,t),\psi(y,t)$ and $\theta(y,t)$ of equations (6) – (8) which satisfy the initial and boundary conditions (9).

**Proof:** Rewrite the equations (6)-(8) as

$$\frac{\partial\phi}{\partial t}=\frac{\partial^2\phi}{\partial y^2}+f(y,t,\phi,\psi,\theta),\ y\in R^n,\ t>0 \quad (10)$$

$$\frac{\partial\varphi}{\partial t}=\frac{\varphi}{f}\frac{\partial^2\psi}{\partial y^2}+g(y,t,\phi,\psi,\theta),\ y\in R^n\ t>0 \quad (11)$$

$$\frac{\partial v}{\partial t}=\frac{1}{\Pr}\frac{\partial^2\theta}{\partial y^2}+h(y,t,\phi,\psi,\theta),\ y\in R^n \quad t>0 \quad (12)$$

Where

$$f(y,t,\phi,\psi,\theta)=Gr\theta+\varepsilon_1(\psi-\phi)+\varepsilon_2 M\phi-\varepsilon_3\phi$$

$$g(y,t,\phi,\psi,\theta)=\frac{\varphi}{f}Gr\theta-\frac{\alpha}{f}(\psi-\phi)$$

$$h(y,t,\phi,\psi,\theta)=E_c\left[\left(\frac{\partial\phi}{\partial y}\right)^2+\left(\frac{\partial\psi}{\partial y}\right)^2\right]$$

Ignoring the second term at the right hand side, the fundamental solutions of equations (10), (11), and (12) are; (see Toki and Toki's (2007)).

$$F(y,t)=\frac{y}{2\pi^{\frac{1}{2}}t^{\frac{3}{2}}}\exp\left(-\frac{y^2}{4t}\right) \quad (13)$$

$$G(y,t)=\frac{y}{2\left(\frac{\varphi}{f}\right)^{\frac{1}{2}}\pi^{\frac{1}{2}}t^{\frac{3}{2}}}\exp\left(-\frac{y^2}{4\frac{\varphi}{f}t}\right) \quad (14)$$

$$H(y,t)=\frac{y}{2\left(\frac{1}{\Pr}\right)^{\frac{1}{2}}\pi^{\frac{1}{2}}t^{\frac{3}{2}}}\exp\left(-\frac{y^2}{4\frac{1}{\Pr}t}\right) \quad (15)$$

respectively.

Clearly,

$$f\left(y,t,\phi,\psi,\theta\right)=Gr\theta+\varepsilon_1\left(\psi-\phi\right)+\varepsilon_2M\phi-\varepsilon_3\phi$$

, $g\left(y,t,\phi,\psi,\theta\right)=\dfrac{\varphi}{f}Gr\theta-\dfrac{\alpha}{f}\left(\psi-\phi\right)$,

$h\left(y,t,\phi,\psi,\theta\right)=E_c\left[\left(\dfrac{\partial\phi}{\partial y}\right)^2+\left(\dfrac{\partial\psi}{\partial y}\right)^2\right]$ are

Lipchitz continuous. Hence, this completes the proof.

**Properties of solution**

**Theorem 2:** Let

$$Gr=\varepsilon_1=\varepsilon_2=\varepsilon_3=\varphi=f=\alpha=\Pr=E_c=1$$

in equations (6), (7) and (9). Then $\dfrac{\partial\phi}{\partial t}\geq 0,\ \dfrac{\partial\psi}{\partial t}\geq 0$ , and

$$\frac{\partial\theta}{\partial t}\geq 0$$

In the proof, we shall make use of the following lemma of kolodner and Pederson (1966).

**Lemma (Kolodner and Pederson (1966))**

Let $u\left(x,t\right)=o\left(e^{\alpha|x|^2}\right)$ be a solution $\mathbb{R}^n\times\left[0,t\right)$ of the differential inequality

$$\frac{\partial u}{\partial t}-\Delta u+k\left(x,t\right)\quad u\geq 0$$

Where k is bounded from below. If $u(x,0)\geq 0$, then $u(x,t)\geq 0$ for all $\left(x,t\right)\in\mathbb{R}^n\times\left[0,t_0\right)$.

**Proof of Theorem 2**

Given

$$\frac{\partial\phi}{\partial t}-\frac{\partial^2\phi}{\partial y^2}=\theta+\psi-\phi+\phi-\phi \quad (16)$$

$$\frac{\partial\psi}{\partial t}-\frac{\partial^2\psi}{\partial y^2}=\theta-\psi+\phi \quad (17)$$

$$\frac{\partial\theta}{\partial t}-\frac{\partial^2\theta}{\partial y^2}=\left(\frac{\partial\phi}{\partial y}\right)^2+\left(\frac{\partial\psi}{\partial y}\right)^2 \quad (18)$$

Differentiating with respect to t, we have

$$\frac{\partial}{\partial t}\left(\frac{\partial\phi}{\partial t}\right)-\frac{\partial}{\partial t}\left(\frac{\partial^2\phi}{\partial y^2}\right)=\frac{\partial\theta}{\partial t}+\frac{\partial\psi}{\partial t}-\frac{\partial\phi}{\partial t} \quad (19)$$

$$\frac{\partial}{\partial t}\left(\frac{\partial\psi}{\partial t}\right)-\frac{\partial}{\partial t}\left(\frac{\partial^2\psi}{\partial y^2}\right)=\frac{\partial\theta}{\partial t}-\frac{\partial\psi}{\partial t}+\frac{\partial\phi}{\partial t} \quad (20)$$

$$\frac{\partial}{\partial t}\left(\frac{\partial\theta}{\partial t}\right)-\frac{\partial}{\partial t}\left(\frac{\partial^2\theta}{\partial y^2}\right)=\frac{\partial}{\partial t}\left(\frac{\partial\phi}{\partial y}\right)^2+\frac{\partial}{\partial t}\left(\frac{\partial\psi}{\partial y}\right)^2=2\frac{\partial\phi}{\partial t}\frac{\partial}{\partial y}\left(\frac{\partial\phi}{\partial t}\right)=2\frac{\partial\psi}{\partial t}\frac{\partial}{\partial y}\left(\frac{\partial\psi}{\partial t}\right) \quad (21)$$

Let $p=\dfrac{\partial\phi}{\partial t},v=\dfrac{\partial\psi}{\partial t}$ and $w=\dfrac{\partial\theta}{\partial t}$. Then, equations (19), (20) and (21) becomes

$$\frac{\partial p}{\partial t}-\frac{\partial^2 p}{\partial y^2}+p\geq 0 \text{ since } w\geq 0,\ v\geq 0 \quad (22)$$

$$\frac{\partial v}{\partial t}-\frac{\partial^2 v}{\partial y^2}+v\geq 0 \text{ Since } w\geq 0,\ p\geq 0 \quad (23)$$

$$\frac{\partial w}{\partial t}-\frac{\partial^2 w}{\partial y^2}+0w\geq 0 \text{ Since } 2p\frac{\partial p}{\partial y}\geq 0,\ 2v\frac{\partial v}{\partial y}\geq 0 \quad (24)$$

This can be written as

$$\frac{\partial p}{\partial t}-\frac{\partial^2 p}{\partial y^2}+K\left(y,t\right)p\geq 0 \quad (25)$$

$$\frac{\partial v}{\partial t}-\frac{\partial^2 v}{\partial y^2}+K_1\left(y,t\right)v\geq 0 \quad (26)$$

$$\frac{\partial w}{\partial t}-\frac{\partial^2 w}{\partial y^2}+K_2\left(y,t\right)w\geq 0 \quad (27)$$

Where, $K=1$, $K_1=1$ and $K_2=0$. Clearly, $K\ and\ K_1$ are bounded from below and $K_2$ is bounded everywhere. Hence, by Kolodner and Pederson's lemma $p(y,t)\geq 0$, $v(y,t)\geq 0$ and $w(y,t)\geq 0$ i.e. $\dfrac{\partial\phi}{\partial t}\geq 0,\ \dfrac{\partial\psi}{\partial t}\geq 0$ and $\dfrac{\partial\theta}{\partial t}\geq 0$. This completes the proof.

**Numerical Solution by Finite Difference Scheme**

Using explicit finite difference scheme, i.e. by using forward finite difference for the order time derivative and central difference for the second order partial derivatives, we transform equation (6) – (9) as thus:

$$\phi_{i,j+1}=\phi_{i,j}+\frac{k}{h^2}\left(\phi_{i+1,j}-2\phi_{i,j}+\phi_{i-1,j}\right)+kGr\theta_{i,j}+ k\varepsilon_1\left(\psi_{i,j}-\phi_{i,j}\right)+k\varepsilon_2M\phi_{i,j}-k\varepsilon_3\phi_{i,j} \quad (28)$$

$$\psi_{i,j+1}=\psi_{i,j}+\frac{k\varphi}{fh^2}\left(\psi_{i+1,j}-2\psi_{i,j}+\psi_{i-1,j}\right)+ \frac{k\varphi}{f}Gr\theta_{i,j}-\frac{k\alpha}{f}\left(\psi_{i,j}-\phi_{i,j}\right) \quad (29)$$

$$\theta_{i,j+1}=\theta_{i,j}+\frac{k}{h^2P_r}\left(\theta_{i+1,j}-2\theta_{i,j}+\theta_{i-1,j}\right)+ \frac{kE_c}{h^2}\left(\left(\phi_{i+1,j}-\phi_{i,j}\right)^2+\left(\psi_{i+1,j}-\psi_{i,j}\right)^2\right) \quad (30)$$

With the initial and boundary conditions as follows

$$\left.\begin{array}{lll} \phi_{i,0}=0 & \phi_{0,j}=0 & \phi_{1,j}=b_1 \\ \psi_{i,0}=0 & \psi_{0,j}=0 & \psi_{1,j}=b_2 \\ \theta_{i,0}=0 & \theta_{0,j}=0 & \theta_{1,j}=1 \end{array}\right\} \quad (31)$$

The stability condition is $\varepsilon \le \frac{1}{2}$. Here we set $h=\frac{1}{10}=0.1$ and $k=\frac{1}{250}=0.004$ so that $\varepsilon=\frac{k}{h^2}=\frac{2}{5}<\frac{1}{2}$. A computer program in Pascal codes was written to perform the iterative computations.

## RESULTS AND DISCUSSION

The results accruing from the iterations were analyzed to obtain the various effects of the parameter on the both the velocity of the fluid and that of the particles as well as temperature of the dusty fluid. The results are presented in the tables and graph below.

**Table 1**: Velocity of the fluid with various values of Grashof number

| x | Gr=5 | Gr=10 | Gr=15 |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0.1 | 0.0085 | 0.0119 | 0.0168 |
| 0.2 | 0.0183 | 0.0256 | 0.0358 |
| 0.3 | 0.0309 | 0.0426 | 0.0589 |
| 0.4 | 0.0474 | 0.0642 | 0.0873 |
| 0.5 | 0.0685 | 0.0907 | 0.1208 |
| 0.6 | 0.0941 | 0.1211 | 0.1571 |
| I0.7 | 0.1232 | 0.1526 | 0.1911 |
| 0.8 | 0.1533 | 0.1806 | 0.2156 |
| 0.9 | 0.1805 | 0.1987 | 0.2214 |
| 1.0 | 0.2 | 0.2 | 0.2 |

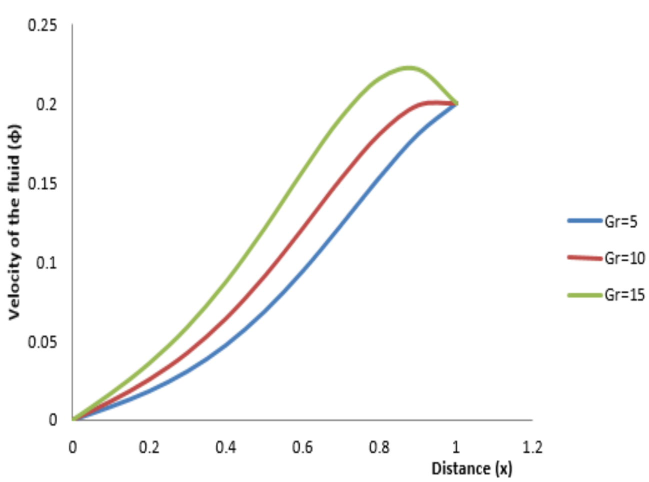


**Figure 1**: The effect of Velocity of the fluid φ(y,t) with various values of Grashof number

Figure1 shows the effect of Velocity of the fluid $\phi(y,t)$ with various values of Grashof number (Gr) when t = 0.1, M=5, Ec=1, f=0.5 from Table1. It is observed that the velocity of the fluid $\phi(y,t)$ increases with an increase in Grashof number (Gr) at a steady time. Higher Grashof number means high buoyancy which means higher flow movement. It is the ratio of buoyancy to viscous forces; if it's large then there is a strong convective current.

**Table 2:** Velocity of the fluid $\phi(y,t)$ with various values $M$ when t = 0.1

| x | M=1 | M=3 | M=5 |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0.1 | 0.0111 | 0.0097 | 0.0085 |
| 0.2 | 0.0238 | 0.0209 | 0.0183 |
| 0.3 | 0.0398 | 0.0351 | 0.0309 |
| 0.4 | 0.0601 | 0.0534 | 0.0474 |
| 0.5 | 0.0853 | 0.0764 | 0.0685 |
| 0.6 | 0.1145 | 0.1037 | 0.0941 |
| I0.7 | 0.1454 | 0.1337 | 0.1232 |
| 0.8 | 0.1739 | 0.1631 | 0.1533 |
| 0.9 | 0.1943 | 0.1871 | 0.1805 |
| 1.0 | 0.2 | 0.2 | 0.2 |

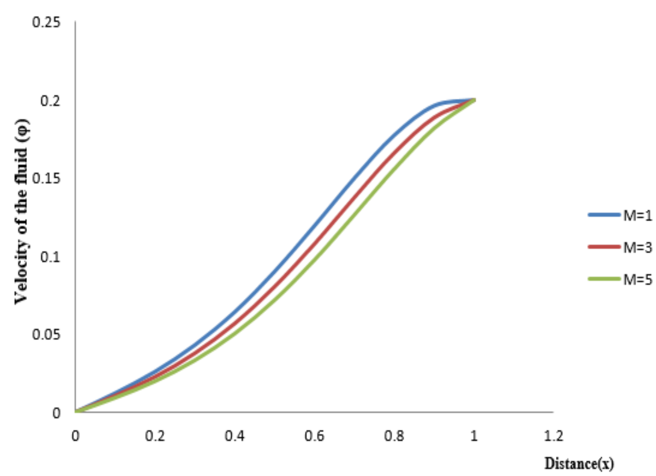


**Figure 2**: The effect of various values of Magnetic Parameter (M) on the velocity of the fluid

Figure 2 shows the effect of various values of Magnetic Parameter (M) on the velocity of the fluid $(\phi)$ when t = 0.1, Gr=5, f=0.5, Pr=1 from Table 2. It is observed that the velocity of the fluid decreases with an increase in the Magnetic Parameter at a steady time. Increase in the magnetic parameter results in depreciation of velocity phases of the fluid and dust particles. Physically, this justified due to the fact than an increase in magnetic field sets a drag-force called Lorentz force, which results in retarding effects on the velocity field i.e an increase in magnetic field parameter produces opposite force to flow called the Lorentz force which has a tendency to slow down the flow and hence fluid and dust velocity diminishes.

**Table3:** Comparison of solution for temperature of the system when Pr = 1, Ec = 0 with Saidu et al (2010).

| X | 0 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| ***Saidu et al*** | 0 | 0.0324 | 0.0708 | 0.1218 | 0.1884 | 0.2775 | 0.3903 | 0.5261 | 0.6791 | 0.8396 | 1 |
| FDM | 0 | 0.0320 | 0.0696 | 0.1181 | 0.1827 | 0.2674 | 0.3749 | 0.5055 | 0.6570 | 0.8242 | 1 |

**Conclusion**

An analysis of the effect of Magnetohydrodynamics on convective flow of dusty viscous fluid has been carried out. The specific governing equations where solved numerically using explicit finite difference scheme and the results have been analysed. In this study, the various parameter were analysed and the effects where observed. It is seen that velocity of the fluid and dust particles increases and decreases with increase in Grashof number and Magnetic parameter respectively. However, due to lack of equipment, no experimental results have been obtained hence the only comparison done was between the numerical values and values obtained from result of Saidu et al. (2010).

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