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# COMPARISON OF ESTIMATORS EFFICIENCY FOR LINEAR REGRESSIONS WITH JOINT PRESENCE OF AUTOCORRELATION AND MULTICOLLINEARITY

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## ABSTRACT

This paper proposes a new estimator called Two stage K-L estimator by combining these two estimators previously proposed by Prais Winsten (1958) and Kibra with Lukman (2020) for autocorrelation and multicollinearity respectively and to derived the necessary and sufficient condition for its superiority over other competing estimators. Simulation study was used to ascertain the dominance of this new estimator using the finite sample properties of estimators in terms of the estimated mean squared error. The study findings shows that under severe autocorrelation and collinearity condition, the proposed Two stage K-L estimator appears to be having a similar performance with RMLE and MLE. Also, under severe autocorrelation and moderate collinearity condition, regardless of the sample size, the proposed Two stage K-L estimator is seen to outperform all other estimators and lastly, the Two stage K-L estimator appears to have an improved performance as the large sample sizes. The study recommends that when autocorrelation and multicollinearity level is at moderate to severe, the proposed Two stage K-L estimator will perform better regardless of the size of the data, and the degree of autocorrelation and multicollinearity should be considered while estimating parameters and thus applying an efficient estimator to avoid erroneous inferences.

**Keywords:** Autoregressive, Autocorrelation, K-L estimator, Multicollinearity, Regression.

## INTRODUCTION

Modern electronics and computers permit the collection of data from an ever-increasing variety of sources. As more and more data become available in many different fields, the size and complexity of typical data sets grows as well. With this growth comes the potential for more precise and accurate predictions. Regression analysis is a classic commonly used prediction tool. Regression analysis explore the relationship between a dependent variable (response variable) and one or more independent variable (explanatory variable). The general single-equation linear regression model can be represented as:

$$y = \beta_0 + \sum_{j=1}^{k} \beta_j X_j + U \quad (1)$$

where y is the dependent variable; $X_1, X_2, X_3, \ldots, X_k$ are the independent variables; $\beta_j, j = 0,1,2 \ldots k$ are the regression coefficients, U is the stochastic disturbance term or error term.

For a sample of n observations,

$$y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \ldots + \beta_k X_{ik} + U_i \quad (2)$$

where $i$ = 1, 2, … n.

Thus,

$$y_1 = \beta_0 + \beta_1 X_{11} + \beta_2 X_{12} + \ldots + \beta_k X_{1k} + U_1$$
$$y_2 = \beta_0 + \beta_1 X_{21} + \beta_2 X_{22} + \ldots + \beta_k X_{2k} + U_2$$
$$\vdots$$
$$y_n = \beta_0 + \beta_1 X_{n1} + \beta_2 X_{n2} + \ldots + \beta_k X_{nk} + U_n$$

In vector form

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ \cdot \\ \cdot \\ \cdot \\ y_n \end{bmatrix} = \begin{bmatrix} 1 & X_{11} & X_{12} & \ldots & X_{1k} \\ 1 & X_{21} & X_{22} & \ldots & X_{2k} \\ 1 & X_{31} & X_{32} & \ldots & X_{3k} \\ & & \cdot & & \\ & & \cdot & & \\ & & \cdot & & \\ 1 & X_{n1} & X_{n2} & \ldots & X_{nk} \end{bmatrix} \begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \cdot \\ \cdot \\ \cdot \\ \beta_k \end{bmatrix} + \begin{bmatrix} U_0 \\ U_1 \\ U_2 \\ \cdot \\ \cdot \\ \cdot \\ U_n \end{bmatrix}$$

$n \times 1 \qquad n \times (k+1) \qquad (k+1) \times 1 \qquad n \times 1$

The general form is:

$$y = X\beta + U \quad (3)$$

where $y$ is an (n × 1) vector of observations of the dependent variable, $X$ matrix is an $n \times (k+1)$ full rank matrix of explanatory variables, $\beta$ is a ((*k*+1) ×1 vector of unknown parameters to be estimated, U is (n × 1) vector of random error. The parameter $\beta$ in a linear regression model are commonly estimated using the Ordinary Least Squares Estimator (OLSE). The OLSE of $\beta$ is given as:

$$\hat{\beta}_{OLS} = (X'X)^{-1} X'y \quad (4)$$

The estimator is generally preferred if there is no violation in any of the assumptions of the linear regression model (Johnston, 1972; Ayinde et al., 2018).

The assumption of uncorrelated errors must be valid for the efficiency of the OLSE. Alternative estimators to the OLSE were proposed. Some researchers have worked on the methods for detecting the presence of autocorrelation and alternative estimators to estimate the parameters in the linear regression model with autocorrelation error. These include Aitken (1935), Cochran and Orcutt (1949), Durbin and Watson (1950), Hildreth and Lu (1960), Rao and Grilliches (1969), Beach and Mackinnon (1978), Kramer (1980), Busse et al. (1994), Kramer and Hassler (1998), Kleiber (2001), Kramer and Marmol (2002), Butte (2002), Nwabueze (2000), Nwabueze (2005), Olaomi (2004), Olaomi (2006), Olaomi and Ifederu (2006), Grochova and Strelec (2013). In time-series applications, there are many structures of autocorrelation (Olaomi and Ifederu, 2008).

Another popular assumption is that the independent (explanatory) variables are independent. However, in practice, there may be near to strong linear relationship among the explanatory variables which is referred to as multicollinearity. According to literature, the performance of OLSE drops when there is multicollinearity. The estimator possesses large variance and occasionally the regression coefficient will exhibit wrong sign (Gujarati, 1995; Ayinde et al., 2018; Lukman and Ayinde, 2017). Various methods of estimating the parameters in linear regression model with multicollinearity are available in the literature. Authors include Hoerl and Kennard (1970), McDonald and Galarneau (1975), Lawless and Wang (1976), Hocking, Speed and Lynn (1976), Dempster, Schatzoff and Wermuth (1977), Wichern and Churchill (1978), Gibbons (1981), Nordberg (1982), Saleh and Kibria (1993), Haq and Kibria (1996), Singh and Tracy (1999), Kibria (2003), Khalaf and Shukur (2005), Alkhamisi, Khalaf and Shukur (2006), Alkhamisi and Shukur (2008), Muniz and Kibria (2009), Dorugade and Kashid (2010), Mansson, Shukur and Kibria (2010), and recently Khalaf (2013), Ghadhan and Mohamed (2014), Dorugade (2014), Kibria and Shipra (2016), Ayinde et al. (2018), Lukman et al. (2017), Lukman et al. (2019a,b), Qasim et al. (2019), Kibria and Lukman (2020), Aslam and Ahmad (2020), Dawoud and Kibria (2020).

Literature have recently show that both problems can jointly exist in a linear regression model (Trenkler 1984; Bayhan and Bayhan, 1998; Ayinde et al., 2015; Lukman et al., 2015; Ozkale and Tugba, 2015; Tugba and Ozkale, 2019; Tugba, 2020). Trenkler (1984) proposed the generalized ridge estimator which takes the autocorrelation into account in the general linear regression model. Hussein and Zari (2012) combined the ridge ression estimator and the generalized least squares estimator to mitigate both problems. Recently, Eledum and Zahri (2013) proposed the feasible generalized ridge (FGR) estimator to deal with both the multicollinearity and autocorrelation problems. Dawoud and Kaçıranlar (2015) proposed the feasible generalized Liu (FGL) regression estimator by combining the Liu estimator and the feasible generalized least squares. Ozbay et al. (2016) combined the feasible generalized restricted ridge regression estimator to take account of both problems. Bello et al. (2017) also introduced feasible generalized Ridge Estimators as Alternatives to ridge and feasible generalized least squares estimator.

The Ordinary Least Square (OLS) estimator is popularly employed to estimate the regression parameter in the linear regression model (LRM). The estimator suffers setback in the presence of multicollinearity and/or autocorrelation. It produces inefficient estimates with large variance. Also, the two problems do exist jointly in LRM and in practice estimators to handle them together are rare. Thus, this research attempted to propose new estimators to handle both problems. This study considers the first-order Autoregressive structure AR(1). The proposed new estimator which is the Two stage K-L estimator. and performance compared with other existing five common estimators such as Prais Winsten method of estimation, Cochrane Orcutt method of estimation, Maximum Likelihood method and Restricted maximum likelihood. This study thus employs Monte Carlo simulation study approach to compare the finite estimators' property of the proposed estimator and the six (6) exiting ones formulated to correct for the problem of autocorrelation and multicollinearity in a regression model and thus, determine the best method in terms of mean squared error.

## MATERIALS AND METHODS

### The Proposed Estimator Derivation

Consider the Linear Regression Model with autoregressive of order 1, AR (1) given as:

$$y_t = \beta_0 + \beta_1 X_{t1} + \beta_2 X_{t2} + \cdots + \beta_p X_{tp} + U_t \qquad (5)$$

where $u_t = \rho u_{t-1} + \varepsilon_t$, ρ is the autocorrelation parameter ($|\rho| < 1$), $\varepsilon_t$ is a random term such that $\varepsilon_t \sim N(0, \sigma^2)$, $E(\varepsilon_i \varepsilon_j) = 0$ ($i \neq j$).
Equation (3.1) in matrix form is written as follows:

$$y = X\beta + U \qquad (6)$$

Pre-multiplying both sides of equation (5) by an $n \times n$ non-singular matrix P, we obtain

$$Py = PX\beta + PU \qquad (7)$$

The error term becomes PU with $E(PU) = 0$ and $E(PU'UP)' = \sigma^2 P\Omega P'$. Thus, if it is possible to specify P such that $P\Omega P' = I$ implying that $P'P = \Omega^{-\prime}$, then the OLS estimates of the transformed variable PY and PX in equation (7) have all the optimal properties of OLS and so the usual inferences could be valid. Re-defining equation (7) as

$$y^* = X^*\beta + U^* \qquad (8)$$

where $y^* = Py$, $X^* = PX$ and $U^* = PU$.
The generalized least squares estimator is obtained as follows:

$$\hat{\beta}_{GLS} = (X^{*\prime}X^*)^{-\prime} X^{*\prime}y^* = (X'P'PX)^{-\prime} X'P'Py$$
$$= (X'\Omega^{-\prime}X)^{-\prime} X'\Omega^{-\prime} y \qquad (9)$$

Ω is a known positive definite (p.d.) matrix. However, in practice, Ω is often unknown. A common practice is to use the estimated matrix of Ω in order to find the estimated generalized least square estimator (EGLSE) or Two Stages method estimator that is more efficient than the GLSE. We reform the Two Stages procedure as follows to propose the new estimator. From equation (9) where $E(U^*)=0$, $Cov(U^*)= \sigma^2 I$. Thus, the OLS estimator for model (9) is:

$$\hat{\beta}_{TS} = (X^{*\prime}X^*)^{-\prime} X^{*\prime}y^* \qquad (10)$$

where;

$$y^* = Py = \begin{bmatrix} (1-\rho^2)^{\frac{1}{2}} & 0 & 0 & \cdots & 0 & 0 \\ -\rho & 1 & 0 & \cdots & 0 & 0 \\ 0 & -\rho & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & -\rho & 1 \end{bmatrix} \begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \\ \cdot \\ \cdot \\ \cdot \\ Y_n \end{bmatrix}$$

$$X^* = PX = \begin{bmatrix} (1-\rho^2)^{\frac{1}{2}} & 0 & 0 & \cdots & 0 & 0 \\ -\rho & 1 & 0 & \cdots & 0 & 0 \\ 0 & -\rho & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & -\rho & 1 \end{bmatrix} \begin{bmatrix} 1 & X_{11} & X_{12} & \ldots & X_{1k} \\ 1 & X_{21} & X_{22} & \ldots & X_{2k} \\ 1 & X_{31} & X_{32} & \ldots & X_{3k} \\ & & \cdot & & \\ & & \cdot & & \\ & & \cdot & & \\ 1 & X_{n1} & X_{n2} & \ldots & X_{nk} \end{bmatrix}$$

Note that $X^{*\prime}X^* = X'P'PX = X'\Omega^{-1}X$ and $X^{*\prime}y^* = X'P'Py = X'\Omega^{-1}y$, where

$$\Omega^{-1} = \frac{1}{1-\rho^2}\begin{bmatrix} 1 & -\rho & 0 & \cdots & 0 & 0 \\ -\rho & 1+\rho^2 & -\rho & \cdots & 0 & 0 \\ 0 & -\rho & 1+\rho^2 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 1+\rho^2 & -\rho \\ 0 & 0 & 0 & \cdots & -\rho & 1 \end{bmatrix} \quad (11)$$

After estimating the $\rho$, we obtained $\Omega^{-1}$and then the two stage is given as Prais Winsten (1954):

$\hat{\beta}_{TS} = (X' \Omega^{-1} X)^{-1} X' \Omega^{-1} y$ (12)

*(to correct for autocorrelation)*

Kibria and Lukman (2020) proposed the K-L estimator to solve the problem of multicollinearity in the LRM.

$$\hat{\beta}_{KL} = (X'X + kI)^{-1}(X'X - kI)\hat{\beta}_{OLS} \quad (13)$$

*(to correct for multicollinearity)*

This paper therefore adopts these estimators (12) and (13) and proposed the Two stage K-L estimator as follows:

$$\hat{\beta}_{TKL} = (X'\Omega^{-1}X + kI)^{-1}(X'\Omega^{-1}X - kI_p)\hat{\beta}_{TS} \quad (14)$$

*(to correct for both problems)*

where

$$\hat{k} = \min\left[\frac{\hat{\sigma}^2}{2\hat{\beta}_i^2 + \frac{\hat{\sigma}^2}{\lambda_i}}\right].$$

**Simulation Studies Procedure**

To examine the proposed and existing estimators, consider a linear regression model of the form:

$$Y_t = \beta_0 + \beta_1 X_{t1} + \beta_2 X_{t2} + \cdots + \beta_p X_{tp} + U_t \quad (15)$$

$t = 1,2, \dots, n$ ; $p = 3, 7$

Were,

$\mu_t = \rho\mu_{t-1} + \varepsilon_t,\ |\rho| < 1,\quad t_1 1,2,, \dots \dots \dots n,\quad \varepsilon_t \sim N(0, \sigma^2)$.

The model was studied with fixed regressors, $X_{ti}$, t = 1, 2, ..., n; i = 1, 2, ..., p such that there exist different levels of multicollinearity among the regressors.

**Procedures for Generating the Error Term**

The error terms were generated by using the distributional properties of the autocorrelation error terms of AR (1) model given as:

$$u_t \sim N\left(0, \frac{\sigma_e^2}{(1-\rho^2)}\right) \quad (16)$$

Thus, assuming the model start from infinite past, the error terms were generated as follows:

$$u_1 = \frac{\varepsilon_1}{\sqrt{1-\rho^2}} \quad (17)$$

$$u_t = \rho u_{t-1} + \varepsilon_t,\quad t = 2,3,4 \dots . n \quad (18)$$

In this study, autocorrelation value $(\rho)$ will be varied from 0.6, 0.8, and 0.9.

**Procedures for Generating the Explanatory Variables**

The simulation procedure used by McDonald and Galarneau (1975), Wichern and Churchill (1978), Gibbons (1981), Kibria (2003), Lukman et al. (2019a,b), Lukman et al. (2020b) was also used to generate the explanatory variables in this study: This is given as:

$$X_{ti} = (1-\rho^2)^{\frac{1}{2}} Z_{ti} + \rho Z_{tp} \quad (19)$$

t=1, 2, 3, … , n; i=1, 2,…p.

where $Z_{ti}$ is independent standard normal distribution with mean zero and unit variance, $\rho$ is the correlation between any two explanatory variables and p is the number of explanatory variables. The values of $\rho$ were taken as 0.6, 0.8 and 0.9 respectively. Thus, the correlations between the variable are the same. In this study, the number of explanatory variable (p) was taken to be six (6).

**Procedures for Generating the Dependent Variable**

The true values of the regression coefficient of model (15) are taken as follows: $\beta_0$ was taken to be identically zero. When P=6, the values of $\beta$ were chosen to be: $\beta_1$=0.6, $\beta_2$=0.1, $\beta_3$=0.2, $\beta_4$=0.1, $\beta_5$=0.2, $\beta_6$=0.4. Sample sizes were varied between 25, 50, 100, 250 and 500. Three different values of σ: 0.5, 1 and 5 were also used. At a specified value of n, p and σ, the fixed Xs are first generated; followed by the U, and the values of Y are then obtained using the regression model. This process was done 1000 times.

**Criterion for Investigation and Performance of Ridge Parameters**

Several authors in literatures have used the Mean Square Error (MSE) to compare the performance of ridge regression estimator with the Ordinary Least Square estimator when there is multicollinearity and autocorrelation. These authors include Hoerl and Kennard (1970), Lawless and Wang (1976), Saleh and Kibria (1993), Kibria (2003), Khalaf and Shukur (2005), Alkhamisi and Shukur (2008), Mansson *et al*. (2010), Ozkale (2014), Dawoud and Kaciranlar (2015), Ozbay et al. (2016). For each replicate, the estimated MSE for each of the estimators $\alpha^*$ is obtained as follows:

$$MSE(\alpha^*) = \frac{1}{1000}\sum_{i=1}^{1000}(\alpha^* - \alpha)'(\alpha^* - \alpha),$$

(3.20)

where $\alpha^*$ would be any of the estimators earlier listed. The estimator with the smallest estimated MSE is considered best.

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**RESULTS**

**Table 3.1** Simulation Result of the Estimators Finite Properties

| n | Sig | r | mseOLS | msePW | mseCC | mseMLE | mseRMLE | mseRP | mseTSK |
|---|---|---|---|---|---|---|---|---|---|
| 25 | 0.6 | 0.6 | 1.765342 | 0.9216255 | 0.9552786 | 0.9189025 | 0.9002328 | 0.1895992 | 0.04949231 |
| 50 | 0.6 | 0.6 | 1.403258 | 0.6021851 | 0.6094751 | 0.6021621 | 0.6017794 | 0.3566228 | 0.2058936 |
| 100 | 0.6 | 0.6 | 1.080999 | 0.4354158 | 0.4349143 | 0.4353708 | 0.4353793 | 0.2509308 | 0.1057379 |
| 250 | 0.6 | 0.6 | 0.9064981 | 0.3599152 | 0.3600696 | 0.3599296 | 0.3599251 | 0.1989222 | 0.05630921 |
| 500 | 0.6 | 0.6 | 0.8682992 | 0.3337621 | 0.3338483 | 0.3337686 | 0.3337682 | 0.1786961 | 0.03667147 |
| 25 | 0.6 | 0.8 | 0.1784106 | 0.1114702 | 0.1132732 | 0.1120787 | 0.1111013 | 1.785956 | 1.799055 |
| 50 | 0.6 | 0.8 | 1.881733 | 1.024749 | 1.035084 | 1.024484 | 1.024364 | 0.8257871 | 0.6496964 |
| 100 | 0.6 | 0.8 | 1.40864 | 0.7548846 | 0.7539764 | 0.7547774 | 0.7548042 | 0.6525893 | 0.4925504 |
| 250 | 0.6 | 0.8 | 1.119396 | 0.629309 | 0.6294339 | 0.629329 | 0.6293293 | 0.5734614 | 0.4195086 |
| 500 | 0.6 | 0.8 | 1.047148 | 0.5909644 | 0.5911119 | 0.5909755 | 0.5909751 | 0.5470385 | 0.3947072 |
| 25 | 0.6 | 0.99 | 18.10109 | 11.94426 | 12.52129 | 11.93307 | 11.38448 | 7.084873 | 5.863368 |
| 50 | 0.6 | 0.99 | 16.89864 | 9.916169 | 10.04659 | 9.908455 | 9.914905 | 5.713724 | 5.104146 |
| 100 | 0.6 | 0.99 | 9.397394 | 5.008914 | 5.011178 | 5.007942 | 5.008922 | 2.882066 | 2.611191 |
| 250 | 0.6 | 0.99 | 4.441183 | 2.642572 | 2.644546 | 2.642557 | 2.642653 | 1.594143 | 1.41249 |
| 500 | 0.6 | 0.99 | 3.198046 | 1.989388 | 1.990154 | 1.989444 | 1.989448 | 1.272801 | 1.103113 |
| 25 | 0.8 | 0.6 | 0.03947308 | 0.01517657 | 0.01385783 | 0.01398733 | 0.0139804 | 1.95613 | 1.959766 |
| 50 | 0.8 | 0.6 | 1.898236 | 0.5830173 | 0.5861086 | 0.5831152 | 0.5830579 | 0.3944859 | 0.215662 |
| 100 | 0.8 | 0.6 | 1.399113 | 0.4351171 | 0.4355746 | 0.4350625 | 0.4351161 | 0.2159617 | 0.0558912 |
| 250 | 0.8 | 0.6 | 1.029742 | 0.3545254 | 0.3547274 | 0.3545202 | 0.3545262 | 0.1632184 | 0.008259548 |
| 500 | 0.8 | 0.6 | 0.909683 | 0.3342406 | 0.3342975 | 0.3342442 | 0.3342454 | 0.1290378 | 0.02285823 |
| 25 | 0.8 | 0.8 | 0.05638721 | 0.0243044 | 0.02222879 | 0.0226632 | 0.02268862 | 1.945945 | 1.950666 |
| 50 | 0.8 | 0.8 | 2.580897 | 0.9749083 | 0.9798055 | 0.9745946 | 0.9745825 | 0.9458319 | 0.7270531 |
| 100 | 0.8 | 0.8 | 1.801419 | 0.7477165 | 0.7482741 | 0.7475779 | 0.7476361 | 0.6746369 | 0.5038295 |
| 250 | 0.8 | 0.8 | 1.265281 | 0.6212664 | 0.6216583 | 0.621257 | 0.6212693 | 0.5891273 | 0.4337659 |
| 500 | 0.8 | 0.8 | 1.126614 | 0.5907184 | 0.5907927 | 0.5907245 | 0.5907261 | 0.5499454 | 0.397905 |
| 25 | 0.8 | 0.99 | 0.7076101 | 0.2854037 | 0.2575968 | 0.2736942 | 0.2742312 | 1.739748 | 1.792176 |
| 50 | 0.8 | 0.99 | 27.07308 | 8.340902 | 8.416762 | 8.33705 | 8.33804 | 8.757957 | 7.263274 |
| 100 | 0.8 | 0.99 | 14.3803 | 4.402461 | 4.411947 | 4.399927 | 4.400659 | 4.042587 | 3.595311 |
| 250 | 0.8 | 0.99 | 6.680919 | 2.475905 | 2.481586 | 2.475897 | 2.475993 | 2.037171 | 1.830955 |
| 500 | 0.8 | 0.99 | 4.274549 | 1.910301 | 1.910284 | 1.910339 | 1.910319 | 1.466288 | 1.29125 |
| 25 | 0.99 | 0.6 | 0.2945369 | 0.00607149 | 0.003705287 | 0.005545767 | 0.005028677 | 1.988992 | 1.989734 |
| 50 | 0.99 | 0.6 | 0.1016496 | 0.002810663 | 0.002255329 | 0.002229428 | 0.002233132 | 1.987827 | 1.989002 |
| 100 | 0.99 | 0.6 | 0.9445305 | 0.007852961 | 0.007844888 | 0.00767277 | 0.007695354 | 1.936978 | 1.944128 |
| 250 | 0.99 | 0.6 | 16.29794 | 0.2053679 | 0.2052671 | 0.2053562 | 0.2053515 | 0.4885477 | 0.6352601 |
| 500 | 0.99 | 0.6 | 7.63216 | 0.1316079 | 0.1316185 | 0.1316091 | 0.131608 | 1.063558 | 1.142964 |
| 25 | 0.99 | 0.8 | 0.3111623 | 0.008369842 | 0.005881484 | 0.006250723 | 0.006479785 | 1.982688 | 1.984997 |
| 50 | 0.99 | 0.8 | 0.1050946 | 0.004895734 | 0.003856986 | 0.00380672 | 0.003817653 | 1.984912 | 1.987325 |
| 100 | 0.99 | 0.8 | 0.972599 | 0.0135782 | 0.01357651 | 0.01329071 | 0.01333181 | 1.919205 | 1.933621 |
| 250 | 0.99 | 0.8 | 19.48769 | 0.4179911 | 0.4172226 | 0.4174324 | 0.4174017 | 0.1697942 | 0.08307402 |
| 500 | 0.99 | 0.8 | 17.79121 | 0.5827553 | 0.5827311 | 0.5827549 | 0.5827454 | 0.8542218 | 0.6341394 |
| 25 | 0.99 | 0.99 | 0.6487634 | 0.09806785 | 0.09462802 | 0.08131614 | 0.07956381 | 1.892854 | 1.939026 |
| 50 | 0.99 | 0.99 | 0.4545714 | 0.04321974 | 0.03614823 | 0.03552794 | 0.03578183 | 1.948407 | 1.957708 |
| 100 | 0.99 | 0.99 | 2.165703 | 0.07245178 | 0.07425715 | 0.0716985 | 0.07192039 | 1.667523 | 1.734786 |
| 250 | 0.99 | 0.99 | 2.492182 | 0.07202792 | 0.07000018 | 0.07016146 | 0.07010827 | 1.655592 | 1.67722 |
| 500 | 0.99 | 0.99 | 14.05857 | 0.4898829 | 0.4866271 | 0.4867259 | 0.4866702 | 0.467098 | 0.397045 |

***Source***: *Simulation study results extraction*

The table shows the finite performance properties of the proposed estimator and others to being compared

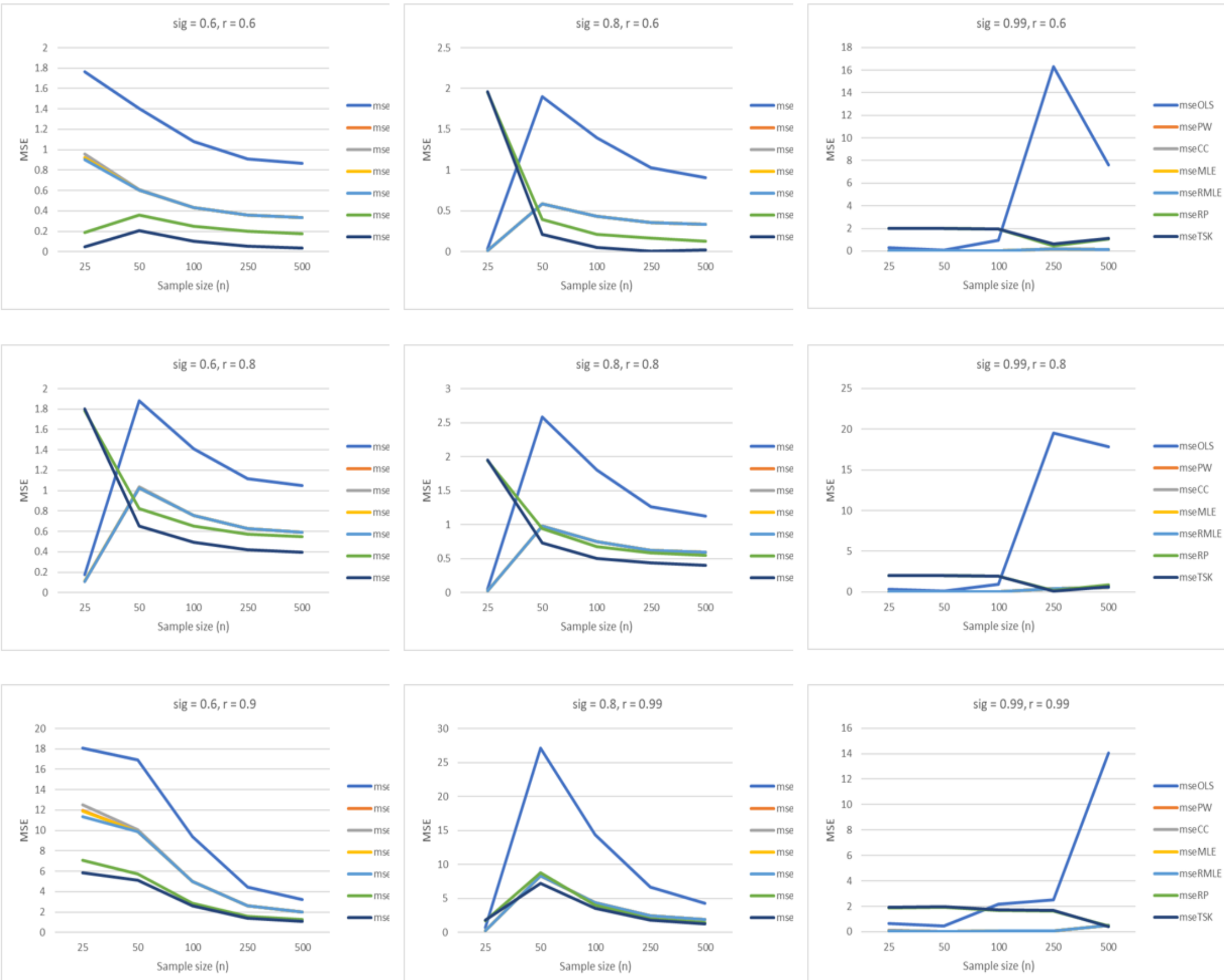


**Figure 3.1** Plots of the Estimators' Finite Properties at Different Sample Sizes and Degree of Assumption's Violation

**DISCUSSION**

As observed from the result of the simulation study, the six (6) other estimators in literature previously proposed to handle the autocorrelation and multicollinearity in a linear model was seen to be performing better than the Ordinary Least Square (OLS) is expected to only produce a robust result when all the classical assumptions are met and this corroborates the study of Mansson, Shukur and Kibria (2010), and recently Khalaf (2013), Ghadhan and Mohamed (2014), Dorugade (2014), Kibria and Shipra (2016), Ayinde et al. (2018), Lukman et al. (2017), Lukman et al. (2019a,b), Qasim et al. (2019), Kibria and Lukman (2020), Aslam and Ahmad (2020), Dawoud and Kibria (2020). However, the simulation study shows that the proposed Two stage K-L estimator out performed all existing estimators in literature at different samples sizes and various degree of the assumption violations. The Two stage K-L estimator is seen to have a minimum variance (MSE) when compared to the other estimators. This is thus expected since it involves the process of first correcting for autocorrelation in the model's error term and then transforming the X'X matrix with a suitable rho to remove the multicollinearity effect before estimating the model's parameter. This procedure is however alien and unique to other estimators which will just corrected of either of the two assumption's violations and not both simultaneously like the proposed two-stage K-L estimator.

**Conclusion and Recommendation**

Based on the general finding about the estimators presented and discussed earlier, the following conclusions are drawn:

- That under severe autocorrelation and multicollinearity condition, and as sample size is increasing, the Two stage K-L estimator proposed appears having a similar performance with RMLE and MLE.
- That under severe autocorrelation and moderate multicollinearity condition, regardless of the sample size, the

proposed Two stage K-L estimator is seen to be the best.

- That sample size has a significant effect on the performance of the estimators across all the autocorrelation and multicollinearity levels. However, the Two stage K-L estimator appears to have an improved performance as the sample size increase.

Based on the above findings, the following are recommended:

- That when autocorrelation and multicollinearity level between the predictors is moderate to severe, the proposed Two stage K-L estimator will perform better regardless of the size of the data.

That the degree of autocorrelation and multicollinearity between the variables should be considered while estimating parameters of Regression models so as to avoid erroneous inferences

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