

FITTING PLACKETT-BURMAN DESIGN ON CHILD DISEASE EXPERIMENT

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ABSTRACT

A child health experiment was designed to achieve the maximum positive variables through the screening of different disease variables, using the Plackett-Burman design. Eleven variables of the children disease were identified: Acute Bronchial Meningitis, Respiratory tract infection, chronic liver disease, Congestive cardiac failure, upper respiratory tract infection, Urinary tract infection, Ante-partum Haemorrhage, Post-partum Haemorrhage, Sickle cell disease, Neonatal Jandice, and Benign prostatic hyperplasia. The selected variables were evaluated through statistical analysis, based on their significance, coefficient value and standard effect plot. The results suggested that three variables, namely, upper respiratory tract infection, urinary tract infection, Ante-Partum Haemorrhage and the combination of urinary tract infection/Ante-Partum Haemorrhage had influence with high confidence levels, while the remaining eight variables did not show significant effect on the children age. The coefficient of determination value R^2 (63.42%) also showed the model used for prediction to be significant (p less than 0.05). The plot for the standard effect for each component and its traits provided accurate data by which to select well-suited variables and further statistical optimization of medium and process parameters was explored using stepwise regression methodology.

Keywords: Disease, Factorials, Fractional, Plackett-Burman, Stepwise Regression.

INTRODUCTION

Child health has improved greatly in the past decade, thanks to research that has quantified health problems and identified strategies for improving child health. The working group on women and child health reviews the major advances in this field in developing countries since 1990 and argues that research is fundamental to further improvements in child health, Child mortality (before age 5) has shown a relative decrease of 15% since 1990 but remains above 100 per 1000 live births in more than 40 countries.

Nigeria has one of the largest populations of youth in the world (NCDC, 2013). Nigeria is the most populous country in Africa with more than 170 million people (NCDC, 2013). It is a nation made of more than 250 ethnic groups, 380 languages, and a diverse range of cultural and religious beliefs and practices. Health problems in Nigeria are challenging, but addressing them using public health principles is necessary to support stability in this important area of the world.

The inadequate programs designed to address the numerous health problems in Nigeria have led to the little improvement in our health status. Overall life expectancy at birth is 54 years; infant

mortality rate is 86 per 1000 live birth while maternal mortality ratio is 840 per 100,000 live births.⁴ Besides the continued neglect of the importance of addressing public health issues would make matters worse for poor Nigerians most of who are at the receiving end.

Nigerians will continue to die unnecessarily from preventable conditions and disease if there are no proper programs designed to address each of these problems. The first WHO Global Status Report on non-communicable disease listed Nigeria and other developing countries as the worst hit with deaths from non-communicable diseases. These diseases with a rising burden in Nigeria include cardiovascular disease, cancer, diabetes, chronic respiratory diseases, sickle cell disease, asthma, coronary heart disease, obesity, stroke, hypertension, road traffic injuries and mental disorders.

The Plackett-Burman statistical design is very frequently used to study the effects of child diseases on children between zero to five years old. It is a two factorial (i.e. -1 and $+1$) design that locates significant variables for the children by screening "n" variables in "n+1" experiments. All eleven factors chosen in the present investigation were tested at these two levels, based on the Plackett-Burman matrix design.

Plackett-Burman designs are saturated orthogonal arrays of strength two because all degrees of freedom are utilized to estimate main effects. Orthogonal arrays of strength two allow all the main effects to be estimated independently and they are universally optimal for the main effects model (Cheng, 1980). Orthogonal arrays include both regular and non-regular designs. For regular designs, the concepts of strength and resolution are equivalent because a regular design of resolution R is an orthogonal array of strength $t = R - 1$. For a regular design of resolution R, the projection onto any R factors must be either a full factorial or copies of a half-replicate of a full factorial. The projection for non-regular designs is more complicated.

The Plackett-Burman (PB) experimental design approach is one of the most commonly used two-level experimental designs (Shek, 2012), which was adopted to determine the disease(s) with the best performance from the observations, and to evaluate the individual significance of each factor.

Plackett-Burman (PB) designed by Plackett and Burman (1946) is a special case of fractional factorial design. For the case of two levels, Plackett-Burman design is constructed from Hadamard matrix, whose rows are mutually orthogonal with elements of either -1 or $+1$. PB design can be mainly classified as regular design and

irregular design (Plackett, et al, 1946). A two-level regular PB design has run size of a power of 2 and its aliasing structure is always simple. On the other hand, a two-level irregular PB design (i.e., 12, 20 and 24 runs) has a complex aliasing structure, that is, there exist effects that are partially aliased, (Montgomery, 2000). Irregular fractional factorial designs such as Plackett-Burman designs and other orthogonal arrays are widely used in various screening experiments for their run size economy and flexibility, (Hamada, et al, 2000).

Selection of significant child disease is an uphill task from the viewpoint of variable size. A classical method for screening large variables is the use of Plackett-Burman design (PBD). It is a small-sized two-level factorial experimental design programmed to identify critical disease parameters from *N* number of variables in *N*+1 experiments without recourse to the interaction effects between and among the variables. Since the sample size is traditionally small, the interaction effects are completely shrouded in the main effects. PBD therefore simply screens the design space to detect large main effects, (Ekpenyong, et al, 2015). The objective of the present study was to identify the most prevalent diseases using PBD to improve treatment of these diseases.

MATERIALS AND METHOD

Producing the Diseases

The bacteria that produces; Acute Bronchial Meningitis, Respiratory tract infection, Chronic liver disease, Congestive cardiac failure, upper respiratory tract infection, Urinary tract infection, Ante-partum Haemorrhage, Post-partum Haemorrhage, Sickle cell disease, Neonatal Jaundice, and Benigu prostral hyperplasia were identified from stools, urine and blood samples of patients from the Plateau State hospitals management board as collated from the various general hospitals in each of the local government areas.

Disease Experiments

Screening: The PBD incorporated into MINITAB 17 statistical software was used to screen 11 diseases in 18 randomized experimental runs. The experiments were carried out in the Plateau State specialist hospital laboratory and included Acute Bronchial Meningitis, Respiratory tract infection, Chronic liver disease, Congestive cardiac failure, upper respiratory tract infection, Urinary tract infection, Ante-partum Haemorrhage, Post-partum Haemorrhage, Sickle cell disease, Neonatal Jaundice, and Benigu prostral hyperplasia. Each was tested only at two levels, low and high. Data were analyzed using the same statistical software that generated the design and significant diseases were selected.

Plackett-Burman Design and the Experimentation

The experimental design employed to fit the multiple regression models of the child disease study was a 2⁵⁻¹ half-fractional factorial central composite rotatable design. Results of preliminary screening for the effects of increasing concentrations of tissues on the growth of the bacterium were also considered in the selection of factor levels of significant diseases from PBD for disease formation. Actual levels of each factor were calculated using the equation of Myers and Montgomery, (Ekpenyong, et al, 2015).

$$Coded_Value = actual_level - \frac{high_level + low_level}{2} \div \frac{high_level - low_level}{2}$$

Table 1: Child Disease Experiment: The Factors and Levels

SYMBOL	FACTOR (NAME)	FACTOR LEVEL	
A	Acute Bronchial Meningitis	-1	+1
B	Respiratory tract infection	-1	+1
C	Chronic liver disease	-1	+1
D	Congestive cardiac failure	-1	+1
E	upper respiratory tract infection	-1	+1
F	Urinary tract infection	-1	+1
G	Ante-partum Haemorrhage	-1	+1
H	Post-partum Haemorrhage	-1	+1
I	Sickle cell disease	-1	+1
J	Neonatal Jaundice	-1	+1
K	Benigu prostral hyperplasia	-1	+1

Plackett-Burman Design

Factors: 11 Replicates: 1
 Base runs: 18 Total runs: 18
 Base blocks: 1 Total blocks: 1
 Center points: 6

Table 2: Design Table (randomized):

Plackett-Burman design matrix (randomized) for bacteria contribution to child disease formation in coded units

Run	A	B	C	D	E	F	G	H	J	K	L	AGE
1	1	-1	1	1	-1	1	-1	-1	-1	1	1	2.3
2	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	1.9
3	1	-1	-1	-1	1	1	1	-1	1	1	-1	2.8
4	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	3.6
5	-1	-1	-1	1	1	1	-1	1	1	-1	1	3.1
6	1	1	1	1	-1	-1	-1	1	-1	-1	1	0.8
7	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	0.7
8	-1	1	1	-1	1	-1	-1	-1	1	1	1	2.8
9	1	1	1	1	-1	1	1	1	-1	-1	1	3.4
10	-1	1	-1	-1	-1	1	1	1	-1	1	1	1.6
11	1	1	-1	1	1	-1	1	-1	-1	-1	1	4.1
12	1	-1	1	-1	-1	-1	1	1	1	-1	1	3.2
13	-1	1	-1	1	-1	1	-1	1	-1	-1	1	2.7
14	-1	1	1	1	-1	1	1	-1	1	-1	-1	0.8
15	1	1	-1	1	-1	-1	-1	1	1	1	-1	0.9
16	1	1	1	-1	1	1	-1	1	-1	-1	-1	3.2
17	1	1	-1	1	-1	-1	-1	1	-1	1	1	1.2
18	-1	-1	1	1	1	-1	1	1	-1	1	-1	3.7

A, Acute Bronchial Meningitis; B, Respiratory tract infection; C, Chronic liver disease; D, Congestive cardiac failure; E, Upper respiratory tract infection; F, Urinary tract infection; G, Ante-partum Haemorrhage; H, Post-partum Haemorrhage; I, Sickle cell disease; J, Neonatal Jaundice; K, Benigu prostral hyperplasia; '1' high; '-1' low.

Statistical Analysis

All data generated from the factorial experiment were subjected to multiple regression analysis using least squares to build the regression models. A second-order (quadratic) function was used to fit the data generated. Experimental design, data analysis and interaction plotting were done with MINITAB 17 statistical software, also Statgraphics 18 statistical software was used for confirmation of model fits where predicted responses were plotted against experimentally-derived data. All hypotheses were tested at 95% confidence level.

RESULTS AND DISCUSSION

A Plackett-Burman design was created, which studied the effects of 11 factors in 18 runs. The design is to be run in a single block. The order of the experiments has been fully randomized. This will provide protection against the effects of lurking variables. The table 4 shows 2 experimental designs capable of estimating effects to within +/-1.0 with 95.0% confidence when the experimental error sigma equals 1.0. All of the designs have at least 6 centerpoints in each block.

In this paper, Plackett-Burman design was successfully applied in analyzing irregular fractional factorial designs to model the child disease experiment data and the stepwise regression was obtained. According to the numerical results obtained, it was observed that the Plackett-Burman design fits the data and the best model generated $Y=E+F+G+FG$ as the best.

Stepwise Selection of Terms

α to enter = 0.15, α to remove = 0.15

The stepwise procedure added terms during the procedure in order to maintain a hierarchical model at each step.

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	4	13.8204	3.4551	5.64	0.007
Linear	3	5.4871	1.8290	2.98	0.070
E	1	4.4204	4.4204	7.21	0.019
F	1	0.2133	0.2133	0.35	0.565
G	1	0.8533	0.8533	1.39	0.259
2-Way Interactions	1	4.0837	4.0837	6.66	0.023
F*G	1	4.0837	4.0837	6.66	0.023
Error	13	7.9707	0.6131		
Lack-of-Fit	8	1.2974	0.1622	0.12	0.995
Pure Error	5	6.6733	1.3347		
Total	17	21.7911			

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
0.783026	63.42%	52.17%	48.28%

Coded Coefficients

Term	Effect	Coef	SE Coef	T-Value	P-Value
Constant		2.378	0.185	12.88	0.000
E	1.288	0.644	0.240	2.69	0.019
F	-0.267	-0.133	0.226	-0.59	0.565
G	0.533	0.267	0.226	1.18	0.259
F*G	-1.237	-0.619	0.240	-2.58	0.023

Regression Equation in Uncoded Units

$$AGE = 2.378 + 0.644 E - 0.133 F + 0.267 G - 0.619 F*G$$

Alias Structure (up to order 2)

Factor	Name
A	A
B	B
C	C
D	D
E	E
F	F
G	G
H	H
J	J
K	K
L	L

Aliases

I
 E - 0.12 A + 0.13 B - 0.13 C - 0.12 D - 0.12 H + 0.13 J + 0.13 K - 0.12 L + 0.25 AB - 0.50 AC
 - 0.25 AD + 0.12 AE + 0.37 AF + 0.38 AG - 0.25 AH - 0.25 AJ - 0.25 AK - 0.50 AL
 + 0.50 BC - 0.25 BD - 0.13 BE - 0.38 BF - 0.37 BG - 0.25 BH - 0.25 BJ - 0.25 BK
 + 0.50 BL - 0.25 CD + 0.12 CE - 0.38 CF - 0.38 CG + 0.25 CH - 0.25 CJ + 0.25 CK
 - 0.25 CL - 0.12 DE - 0.38 DF + 0.38 DG + 0.50 DH - 0.25 DJ - 0.50 DK + 0.25 DL
 - 0.12 EH + 0.12 EJ + 0.12 EK + 0.12 EL + 0.38 FH + 0.38 FJ - 0.38 FK - 0.37 FL
 - 0.38 GH - 0.38 GJ + 0.38 GK - 0.38 GL - 0.50 HJ - 0.25 HK - 0.25 HL + 0.50 JK
 + 0.25 JL - 0.25 KL
 F - 0.33 AB + 0.33 AC - 0.33 AD + 0.33 AE - 0.33 AG - 0.33 AH - 0.33 AJ + 0.33 AK
 - 0.33 AL
 + 0.33 BC - 0.33 BD - 0.33 BE + 0.33 BG + 0.33 BH - 0.33 BJ - 0.33 BK - 0.33 BL
 + 0.33 CD - 0.33 CE - 0.33 CG - 0.33 CH - 0.33 CJ - 0.33 CK - 0.33 CL - 0.33 DE
 - 0.33 DG - 0.33 DH + 0.33 DJ - 0.33 DK + 0.33 DL - 0.33 EG + 0.33 EH + 0.33 EJ
 - 0.33 EK - 0.33 EL - 0.33 GH + 0.33 GJ + 0.33 GK - 0.33 GL - 0.33 HJ - 0.33 HK
 + 0.33 HL - 0.33 JK - 0.33 JL + 0.33 KL
 G - 0.33 AB - 0.33 AC - 0.33 AD + 0.33 AE - 0.33 AF - 0.33 AH + 0.33 AJ - 0.33 AK
 + 0.33 AL
 - 0.33 BC + 0.33 BD - 0.33 BE + 0.33 BF - 0.33 BH - 0.33 BJ - 0.33 BK + 0.33 BL
 + 0.33 CD - 0.33 CE - 0.33 CF + 0.33 CH + 0.33 CJ - 0.33 CK - 0.33 CL + 0.33 DE
 - 0.33 DF - 0.33 DH - 0.33 DJ - 0.33 DK - 0.33 DL - 0.33 EF - 0.33 EH - 0.33 EJ
 + 0.33 EK - 0.33 EL - 0.33 FH + 0.33 FJ + 0.33 FK - 0.33 FL - 0.33 HJ + 0.33 HK
 + 0.33 HL - 0.33 JK - 0.33 JL - 0.33 KL
 FG - 0.37 A + 0.37 B - 0.37 C - 0.37 D - 0.37 H + 0.37 J + 0.37 K - 0.37 L - 0.25 AB
 - 0.50 AC + 0.25 AD + 0.37 AE + 0.12 AF + 0.12 AG + 0.25 AH + 0.25 AJ + 0.25 AK
 - 0.50 AL + 0.50 BC + 0.25 BD - 0.37 BE - 0.13 BF - 0.12 BG + 0.25 BH + 0.25 BJ
 + 0.25 BK + 0.50 BL + 0.25 CD + 0.37 CE - 0.13 CF - 0.12 CG - 0.25 CH + 0.25 CJ
 - 0.25 CK + 0.25 CL - 0.37 DE - 0.12 DF + 0.12 DG + 0.50 DH + 0.25 DJ - 0.50 DK
 - 0.25 DL - 0.37 EH + 0.37 EJ + 0.37 EK + 0.37 EL + 0.13 FH + 0.12 FJ - 0.12 FK
 - 0.12 FL - 0.12 GH - 0.13 GJ + 0.12 GK - 0.12 GL - 0.50 HJ + 0.25 HK + 0.25 HL
 + 0.50 JK - 0.25 JL + 0.25 KL

Fits and Diagnostics for Unusual Observations

Obs	AGE	Fit	Resid	Std Resid
6	0.800	2.378	-1.578	-2.07 R

R Large residual

Means

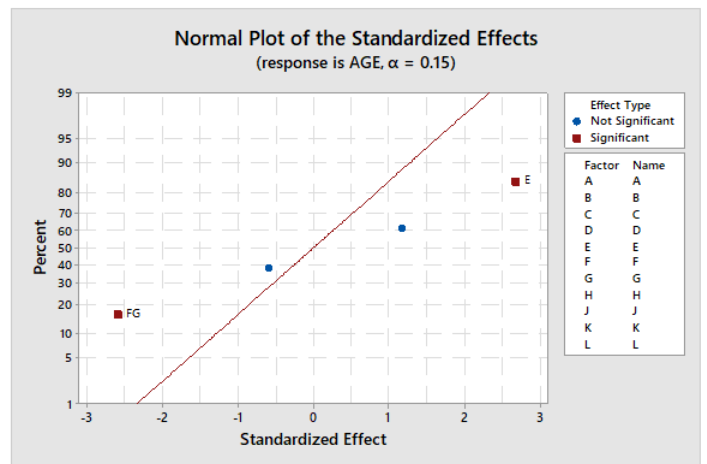


Fig. 1: Normal plot effects showing the distribution of the factors that are significant

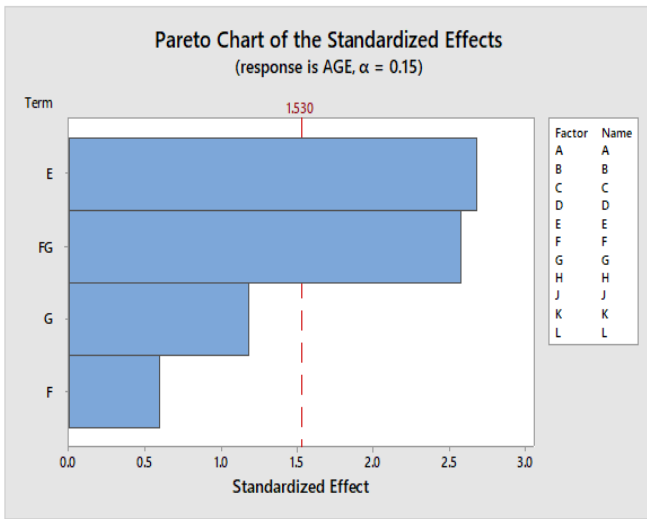


Fig. 2: Pareto chart of the standardized effects of the significant factors.

Table 3: Input Factors

Number of Factors	Min. Center points per Block	Max. Runs per Block	Exp. Error Sigma	Absolute Error	Confidence Level
11	6		1.0	1.0	95.0%

Table 4: Selected Designs

Design	Runs	Resol.	Corner Points	Center Points	Error D.F.	Reps.	Blocks	Tolerance
Sixteenth fraction 2^{11-4}	134	V	128	6	67	1	1	0.352848
Plackett-Burman $2^{11*3/512}$	36	III	24	12	24	2	1	0.842585

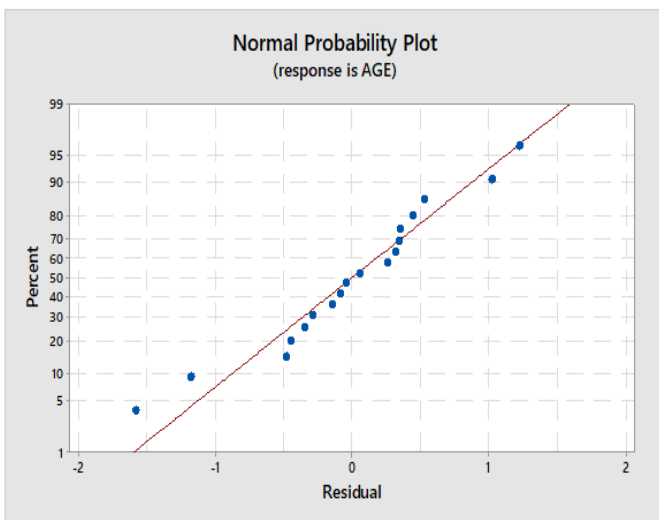


Fig. 3: Normal probability plot showing the residuals and percentage of the response variable

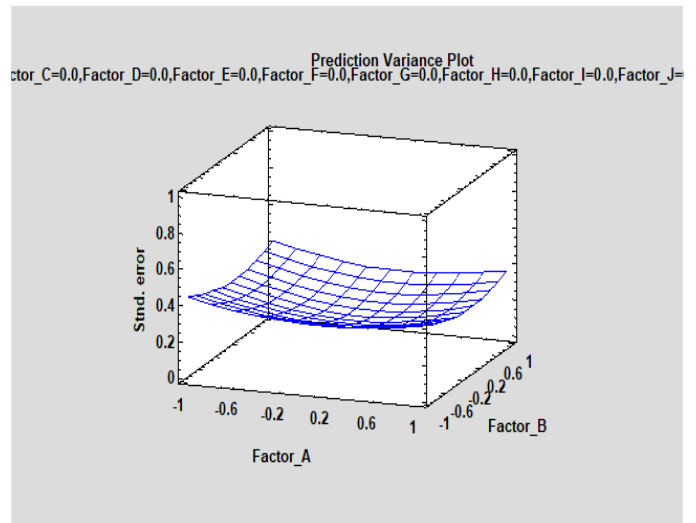


Fig. 4: This plot shows the standard error of the predicted response as a function of Factor A and Factor B, with all other factors held constant at the indicated levels.

The standard error indicates how precisely the response can be predicted at a specific location within the experimental region

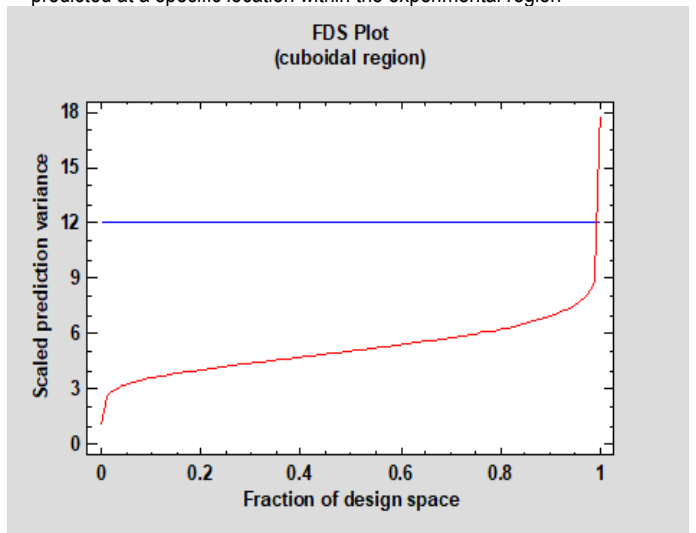


Fig. 5: This plot summarizes the prediction performance of the selected experimental design over the experimental region.

Good designs have small and relatively constant variance over a large fraction of the region.

Table 5: Table of Predictions

StdOrder_1	RunOrder_1	Blocks_1	PFITS1	PSEFITS1	CLIM1	CLIM2	PLIM1	PLIM2
1	1	1	1.95278	0.53808	0.79032	3.11524	-0.0998	4.00531
2	2	1	2.37778	0.18456	1.97906	2.7785	0.6398	4.11576
3	3	1	2.53811	0.53808	1.37365	3.69857	0.48357	4.58865
4	4	1	2.37778	0.18456	1.97906	2.7785	0.6398	4.11576
5	5	1	3.24028	0.4614	2.24348	4.23708	1.27681	5.20375
6	6	1	2.37778	0.18456	1.97906	2.7785	0.6398	4.11576
7	7	1	0.98194	0.4614	-0.0149	1.97874	-0.9815	2.94541
8	8	1	2.26944	0.53808	1.10699	3.4319	0.21691	4.32198
9	9	1	2.37778	0.18456	1.97906	2.7785	0.6398	4.11576
10	10	1	1.24861	0.4614	0.25181	2.24541	-0.7149	3.21208
11	11	1	4.04028	0.4614	3.04348	5.03708	2.07681	6.00375
12	12	1	2.75278	0.53808	1.59032	3.91524	0.70024	4.80531
13	13	1	2.37778	0.18456	1.97906	2.7785	0.6398	4.11576
14	14	1	1.24861	0.4614	0.25181	2.24541	-0.7149	3.21208
15	15	1	0.98194	0.4614	-0.0149	1.97874	-0.9815	2.94541
16	16	1	3.24028	0.4614	2.24348	4.23708	1.27681	5.20375
17	17	1	2.37778	0.18456	1.97906	2.7785	0.6398	4.11576
18	18	1	4.04028	0.4614	3.04348	5.03708	2.07681	6.00375

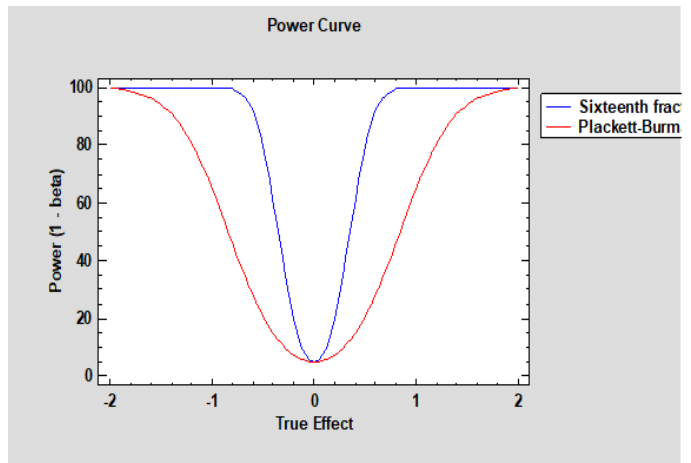


Fig. 7: The power curve showing difference in designs.

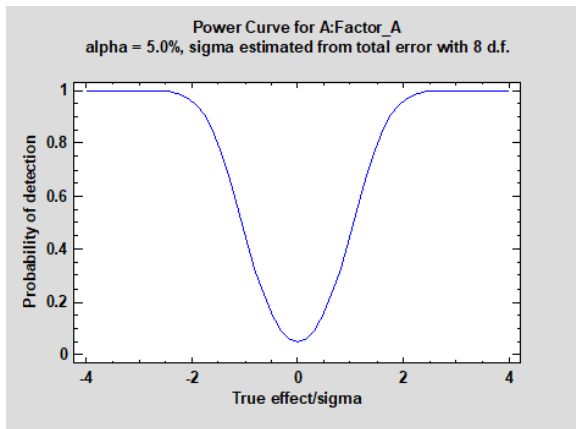


Fig. 6: The power curve of factor A.

The power curve shows the probability that the statistical tests in the Analyze Data step will identify a factor as having a significant effect as a function of the magnitude of that effect. The effect is defined as the difference between the response at the high level of that factor and the response at the low level of that factor. It is plotted here as a function of the ratio δ/σ , where δ is the effect and σ is the standard deviation of the experimental error. At a ratio of 4, the power equals 1.0. This means that if the true effect is 4 times the background sigma, there is a 100.0% chance that it will be called statistically significant at the 95.0% confidence level. Usually, we would like this power to be close to 90% or higher

The power curve shows the probability that the statistical tests in the Analyze Data step will identify a factor as having a significant effect as a function of the magnitude of that effect. The effect is defined as the difference between the response at the high level of that factor and the response at the low level of that factor. It is plotted here as a function of the ratio δ/σ , where δ is the effect and σ is the standard deviation of the experimental error. At a ratio of 2, the power equals 1.0. This means that if the true effect is 2 times the background sigma, there is a 100.0% chance that it will be called statistically significant at the 95.0% confidence level. Usually, we would like this power to be close to 90% or higher. Comparing the two designs, we can visibly see that the Plackett-Burman design is a better function.

Conclusion

The paper considered several factors that contribute to children health aged zero to five years old, fitted a Plackett-Burman design and made use of forward stepwise selection procedure and concluded that the model $Y=E+F+G+FG$ is the best model that fit the data. By using the new methodology, very good predictions and inferences for the parameters of interest were observed. From the fitted model, the outcome variable is the children age from zero to five years old while the exposure variables the different diseases that were identified from sample tests. Based on the model obtained, it was identified that upper respiratory tract infection, urinary tract infection, Ante-Partum Haemorrhage and the combination of urinary tract infection/Ante-Partum Haemorrhage contribute more in resulting to children health challenges from birth to five years old.

Area for Further Study

The study revealed that there is a need for the exploration of other children diseases. It is necessary for such a study to be conducted in all state general hospital, teaching hospitals and other health centers across the federation.

Conflict of Interest

Ali, H., Akanihu, C. N., Nahum, H. E. & Makut, A. B. write to declare that there is no conflict of interest on this article

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