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VIBRATION OF SINGLE – WALLED CARBON NANOTUBE (SWCNT) ON A WINKLER FOUNDATION WITH MAGNETIC FIELD EFFECT

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ABSTRACT
In this study, vibrational analysis of a single – walled carbon nanotube (CNT) incorporating longitudinal magnetic field based on Euler -Bernoulli Beam theory (EBBT) and Eringen's Non – local Elasticity theory is used. One parameter finite element method together with Newmark time integration method is used to study effects of non -local parameter, magnetic field and fluid velocity on the deflection of the pipe. MATLAB software is used to simulate the system response. From the result obtain, its shows that fluid velocity affects most, the deflection of the pipe than non –local parameter and magnetic field strength.

Keywords: Vibration, Dynamics, Nanotube, Finite, Galerkin, Magnetic.

INTRODUCTION

The study of vibration of various structures has been of great interest to the engineering and scientific worlds for many years. These structures have multitude of application in almost every field. The technology of conveying fluid such as petroleum liquids and waters through long or slender pipelines, which cover different types of foundation, has evolved over the years. The speed of this liquid or fluid in the pipelines has imparted energy to the pipelines making it to vibrate. The vibration of the conveying pipe can put pressures on the walls of the pipe resulting on the pipe to deflect. This deflection of the conveying pipe as a result of the moving fluid may lead to structural instability.

Surface energy is usually consider small and negligible in classical mechanics but when material and fluid conveying pipes shrinks to nano-meters, it plays a critical role in their static or dynamical behaviour due to high specific surface area of nano-materials (Zhen, 2016).In recent years, the subject area of nanotechnology has become the focus of attention of industries, scientists and researchers. Among the nano-materials, carbon nanotubes (CNTs) have received the highest amount of attentions owing to their novel mechanical, chemical, thermal, and electrical properties as shown in the works of Baughman *et al* (2002), Li *et al* (2008), Gibson *et al* (2007). Carbon nanotubes are named based on their unique sizes. A CNT has a diameter of few nano-meters while its length can be up to several millimeters. In an article in "nano letter", Wang *et al* (2009) show that a single walled carbon nanotube can have a length to diameter ratio up to 132,000,000:1. A CNT can be constructed by folding a sheet of graphene into a cylinder. Wang *et al* (1998) used Hamilton principle to derive the element matrices and find the influence of fluid pressure and coupled damping on the natural frequency. It shows that natural frequency changes less with increase on pressure and fundamental frequency decreases with increase in fluid velocity.

Zhou *et al* (2021) studied the geometric imperfection on stability and dynamics of fluid conveying pipe. They use extended Lagrange method to model the non – linear differential equation based on Euler – Bernoulli Beam Theory. The results shows that geometric imperfection in the form of half sinusoidal wave increase the flow velocity with change in the amplitude without change its base shapes

He *et al* (2013) elaborate on the impressive applicability of carbon nanotube and structural, mechanical and electronic properties of it. Carbon nano tubes are not only used for drug delivery into body cells without metabolism but also for tissue regeneration, bio- senor diagnosis and antioxidant. Shalk *et al* (2016) analysed the instability and vibration of straight pipe with clamped end condition using Hamilton principle and Euler – Bernoulli Beam Theory to model the system. Finite element analysis is used for simulation of the system to generate the natural frequencies. It was observed that the frequency of an empty pie decrease as with weldment compare to without weldment. Other literatures include *Jiya et al* (2018), Mustafa, (2017), Benjamin *et al* (2020). Hosseinpour and Makkiabadi (2020).

The study of dynamic behaviour of carbon nanotube has been investigated with the aids of experimental measurements, density functional theory, molecular dynamics simulations, and continuum mechanics. However, there are difficulties in performing experiment at the nano-scale level. Over the years, the classical continuum theories (which do not consider the small scale effects) have been widely used to small scale structures. The demerit of such classical continuum theories is witnessed in their scale free models, as they cannot incorporate the small scale effects in their formulations. Therefore, in other to correct the inadequacy in the classical continuum model, Eringen developed non-local continuum mechanics based on non-local elasticity theory. The non-local elasticity theory considers the stress state at a given point to be a function of the strain field at all points in the body
Therefore in this work, Eringen's non-local elasticity theory coupled with Euler – Bernoulli Beam theory is used to analyse Single – walled Carbon nano-tube on Winkler foundation incorporating longitudinal magnetic field effect

Finite Element Method

Finite element analysis is one of the popular methods for numerical simulation in research. During the recent years, high speed computers are developed with high storage, different numerical integration algorithms are implemented in finite element code for solving problem of dynamics. It is a dominant discretization technique in structural mechanics. The basic concept in the physical interpretation of the FEM is the subdivision of the mathematical model into disjoint (no overlapping) components of simple geometry called finite elements or elements for short. The response of each element is expressed in terms of a finite number of degrees of freedom characterized as the value of an unknown function, or functions, at a set of nodal points.

Problem Formulation

The problem under consideration is Single – Walled Carbon Nanotube on a Winkler foundation. The problem is modelled based on Eringen's Non local Elasticity theory and Euler – Bernoulli Beam Theory. The longitudinal magnetic field is given by Maxwell's equation in Arani, *et al* (2017) as

$$\bar{J} = \nabla \times \bar{h}, \nabla \times e = -\eta\left(\frac{\partial h}{\partial t}\right),$$
$$\nabla . \bar{h} = 0, e = -\eta\left(\frac{\partial h}{\partial t} \times \bar{H}\right), \bar{h} = \nabla \times \left(\bar{D} \times \bar{H}\right) \quad (1)$$

Where J is current density

h is distributing vectors of magnetic field

η is magnetic field permeability

e is Electric field Strength.

$\bar{H}\left(H_x, 0, 0\right)$ is Displacement vector

$\bar{D}\left(u_x, u_y, u_z\right)$ is magnetic field vector

By using equation (1) h and J are describe as

$$\bar{h} = \nabla \times \left(D \times \bar{H}\right) = H_x \frac{\partial w}{\partial x} k$$

and (2)

$$\bar{J} = \nabla \times \bar{h} = -H_x \frac{\partial^2 w}{\partial x^2}$$

The Lorentz force in the three directions is

$$\bar{f} = \left(f_x, f_y, f_z\right) = \left(0, 0, \eta H_x^{\ 2} \frac{\partial^2 w}{\partial x^2}\right) \quad (3)$$

The Physical model is given below

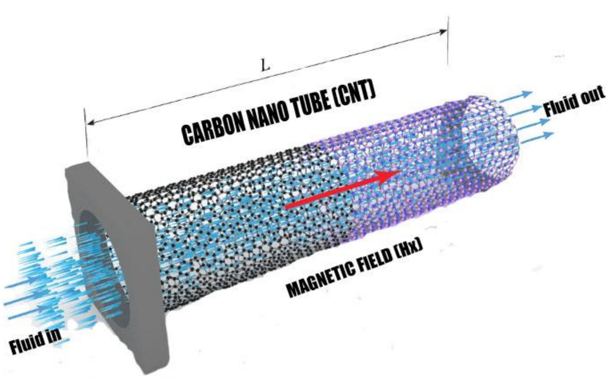

Figure 1: Clamped - Free Single – Walled Carbon Nanotube

The governing partial differential equation of the system is given by

$$\left.\begin{array}{l} EI\frac{\partial^4 w}{\partial x^4} + m\frac{\partial^2 w}{\partial t^2} + m_f\left(u^2\frac{\partial^2 w}{\partial x^2} + 2u\frac{\partial^2 w}{\partial x \partial t} + \frac{\partial^2 w}{\partial t^2}\right) - \\ \eta H_x A\frac{\partial^2 w}{\partial x^2} + kw - \left(e_0 a\right)^2 \frac{\partial^2}{\partial x^2} \\ \left(m\frac{\partial^2 w}{\partial t^2} + m_f\left(u^2\frac{\partial^2 w}{\partial x^2} + 2u\frac{\partial^2 w}{\partial x \partial t} + \frac{\partial^2 w}{\partial t^2}\right) - \eta H_x A\frac{\partial^2 w}{\partial x^2} + kw\right) = 0 \end{array}\right\} \quad (4)$$

Where

I	Second moment of inertia
E	Young modulus of elasticity
m	pipe mass per unit length
m_f	Fluid mass per unit length
u	fluid velocity
η	Magnetic field permeability
H_x	magnetic field strength
A	cross sectional area
k	foundation stiffness
e_0	non – local material constant
a	internal characteristic length of the nanotube
w	displacement
L	length of the pipe
t	time
x	cartesian axis

Initial condition

$$w(x,0) = \frac{\partial w(x,0)}{\partial t} = 0 \quad (5)$$

Boundary condition

$$w(0,t) = w''(L,t) = 0 \quad (6)$$
$$w'(0,t) = w'''(L,t) = 0 \quad (7)$$

Finite Element Discretization

The entire pipe system is discretized into beam element. Each element is a two – node clamped – clamped beam element with each having two (2) degree of freedom. The beam element has a total of four (4) degree of freedom per element. The displacement function is defined as:

$$w\left(x,t\right) = \sum_{i=1}^{4} N_i\left(x\right) U_i\left(t\right) \quad (8)$$

Where $N_i\left(x\right)$ the shape is function of the bending and $U_i\left(t\right)$ is the unknown function of time

We use Cubic interpolating function called Hermite cubic polynomial for our shape function as

$$\left.\begin{aligned} & EI\int_0^l [N]^T \frac{\partial^4 w}{\partial x^4}dx + m\int_0^l [N]^T \frac{\partial^2 w}{\partial t^2}dx + \\ & m_f\int_0^l [N]^T \left(u^2\frac{\partial^2 w}{\partial x^2} + 2u\frac{\partial^2 w}{\partial x\partial t} + \frac{\partial^2 w}{\partial t^2}\right)dx - \\ & \eta H_x A\int_0^l [N]^T \frac{\partial^2 w}{\partial x^2}dx + k\int_0^l [N]^T w dx - \\ & (e_0a)^2\frac{\partial^2}{\partial x^2}\left(\begin{aligned} & m\int_0^l [N]^T \frac{\partial^2 w}{\partial t^2}dx + m_f\int_0^l [N]^T \\ & \left(u^2\frac{\partial^2 w}{\partial x^2} + 2u\frac{\partial^2 w}{\partial x\partial t} + \frac{\partial^2 w}{\partial t^2}\right)dx - \\ & \eta H_x A\int_0^l [N]^T \frac{\partial^2 w}{\partial x^2}dx + k\int_0^l [N]^T w dx \end{aligned}\right) = 0 \end{aligned}\right\} \quad (11)$$

After simplifying and applying the boundary condition, we have

$$\left.\begin{aligned} & EI\int_0^l [N'']^T \frac{\partial^2 w}{\partial x^2}dx + (m + m_f) \\ & \int_0^l [N]^T \frac{\partial^2 w}{\partial t^2}dx + (m_f u^2 - \eta H_x A)\int_0^l [N]^T \frac{\partial^2 w}{\partial x^2}dx + 2m_f u\int_0^l [N]^T \frac{\partial^2 w}{\partial x\partial t}dx \\ & +k\int_0^l [N]^T w dx - (e_0a)^2 (m + m_f) \\ & \int_0^l [N]^T \frac{\partial^4 w}{\partial x^2\partial t^2}dx - (e_0a)^2 (m_f u^2 - \eta H_x A)\int_0^l [N'']^T \frac{\partial^2 w}{\partial x^2}dx \\ & +2(e_0a)^2 m_f u\int_0^l [N']^T \frac{\partial^3 w}{\partial x^2\partial t}dx - k(e_0a)^2\int_0^l [N]^T \frac{\partial^2 w}{\partial x^2}dx = 0 \end{aligned}\right\} \quad (12)$$

After simplifying, we have

$$[M]\{\ddot{U}\} + [G]\{\dot{U}\} + [K]\{U\} = 0 \quad (13)$$

Where

$$[M] = (m + m_f)\int_0^l (N)^T N dx - (e_0a)^2 (m + m_f)\int_0^l (N)^T N'' dx \quad (14)$$

$$[G] = 2m_f u\int_0^l [N]^T N' dx + 2(e_0a)^2 m_f u\int_0^l [N']^T N'' dx \quad (15)$$

$$\left.\begin{aligned} & [K] = EI\int_0^l [N'']^T N'' dx + (m_f u^2 - \eta H_x A)\int_0^l [N]^T N'' dx + k\int_0^l [N]^T N dx + \\ & (e_0a)^2 (m_f u^2 - \eta H_x A)\int_0^l [N'']^T N'' dx - (e_0a)^2 k\int_0^l [N]^T N'' dx \end{aligned}\right\} \quad (16)$$

After performing differentiation and integration on Equation (14) - (16), we have

Mass Matrix $[M] = [M]_1 - [M]_2$

$$[M]_1 = \frac{(m_f + m_p)}{420}\begin{bmatrix} 156 & 22l & 54 & -13l \\ 22l & 4l^2 & 13l & -3l^2 \\ 54 & 13l & 156 & -22l \\ -13l & -3l^2 & -22l & 4l^2 \end{bmatrix} \quad (17)$$

$$[M]_2 = \frac{(e_0a)^2 (m + m_f)}{30l}\begin{bmatrix} 36 & 3l & -36 & 3l \\ 3l & 4l^2 & -3l & -l^2 \\ -36 & -3l & 36 & -3l \\ 3l & -l^2 & -3l & 4l^2 \end{bmatrix} \quad (18)$$

The Damping Matrix $[G] = [G]_1 + [G]_2$

$$[G]_1 = \frac{2m_f u}{30}\begin{bmatrix} 30 & 6l & 30 & -6l \\ -6l & 0 & 6l & -l^2 \\ -30 & -6l & -30 & 6l \\ 6l & l^2 & -6l & 0 \end{bmatrix} \quad (19)$$

$$[G]_2 = \frac{2(e_0a)^2 m_f u}{30l}\begin{bmatrix} 36 & 3l & -36 & 3l \\ 3l & 4l^2 & -3l & -l^2 \\ -36 & -3l & 36 & -3l \\ 3l & -l^2 & -3l & 4l^2 \end{bmatrix} \quad (20)$$

Stiffness Matrix is

$$[K] = [K]_1 + [K]_2 + [K]_3 + [K]_4 - [K]_5$$

$$[K]_1 = \frac{2EI}{l^3}\begin{bmatrix} 6 & 3l & -6 & 3l \\ 3l & 2l^2 & -3l & l^2 \\ -6 & -3l & 6 & -3l \\ 3l & l^2 & -3l & 2l^2 \end{bmatrix} \quad (21)$$

$$[K]_2 = \frac{(m_f u^2 - \eta H_x A)}{30l}\begin{bmatrix} 36 & 3l & -36 & 3l \\ 3l & 4l^2 & -3l & -l^2 \\ -36 & -3l & 36 & -3l \\ 3l & -l^2 & -3l & 4l^2 \end{bmatrix} \quad (22)$$

$$[K]_3 = \frac{k}{420}\begin{bmatrix} 156 & 22l & 54 & -13l \\ 22l & 4l^2 & 13l & -3l^2 \\ 54 & 13l & 156 & -22l \\ -13l & -3l^2 & -22l & 4l^2 \end{bmatrix} \quad (23)$$

$$[K]_4 = \frac{2EI(e_0a)^2(m_f u^2 - \eta H_x A)}{l^3}\begin{bmatrix} 6 & 3l & -6 & 3l \\ 3l & 2l^2 & -3l & l^2 \\ -6 & -3l & 6 & -3l \\ 3l & l^2 & -3l & 2l^2 \end{bmatrix} \quad (24)$$

$$[K]_5 = \frac{(e_0a)^2 k}{30l}\begin{bmatrix} 36 & 3l & -36 & 3l \\ 3l & 4l^2 & -3l & -l^2 \\ -36 & -3l & 36 & -3l \\ 3l & -l^2 & -3l & 4l^2 \end{bmatrix} \quad (25)$$

RESULTS AND DISCUSSION

Matlab software is used to obtain the Global Mass Matrix, Global damping matrix and Stiffness matrix. Free vibration of equation of motion is analyse using Newmark time integration method written in Matlab software to investigate effect of non – local parameter, magnetic strength, fluid velocity on pipe vibration. The following values were used for simulation of forced vibration

$$L = 50m, n = 20, E = 2.065\times10^{11} N/m^2,$$
$$d = 5m,\ pipe\,density = 7580kg/m^3,$$
$$fluid\ density = 1000kg/m^3,$$
$$e_0 = 0.31nm, a = 0.142N/M,$$
$$u = 10m/s,\ \eta = 4\pi\times10^{-7} H/m,$$
$$H_x = 1.5\times10^8 A/m,\ k = 203.6\times10^6 kn/m^3$$

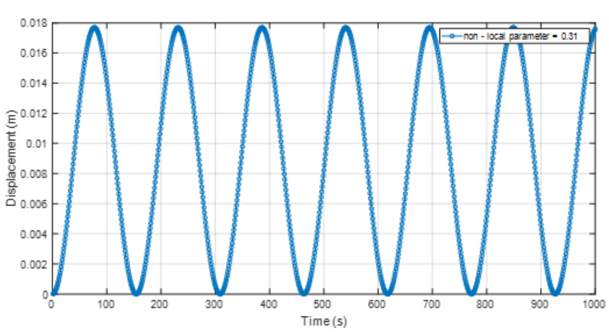

Figure 2: Effect of Non -local parameter at 0.31nm on deflection of the pipe

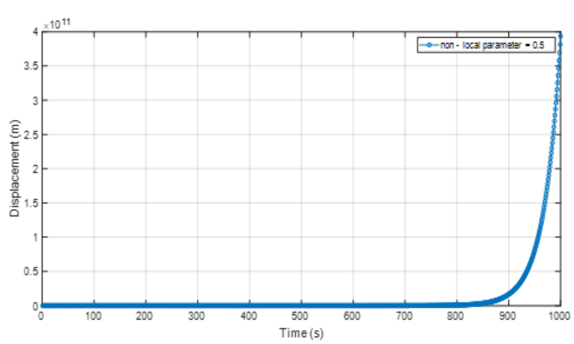

Figure 3: Effect of Non -local parameter at 0.5nm on deflection of the pipe

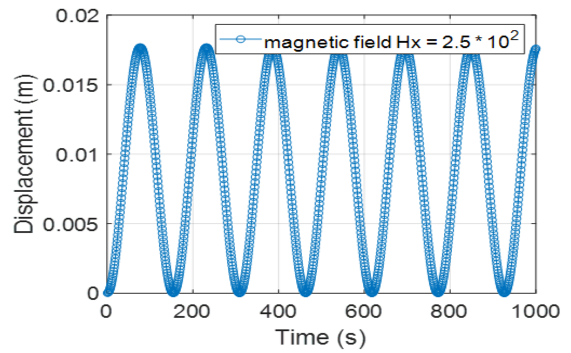

Figure 4: Effect of magnetic field at 2.5 $\times 10^2 A/m$ on deflection of the pipe

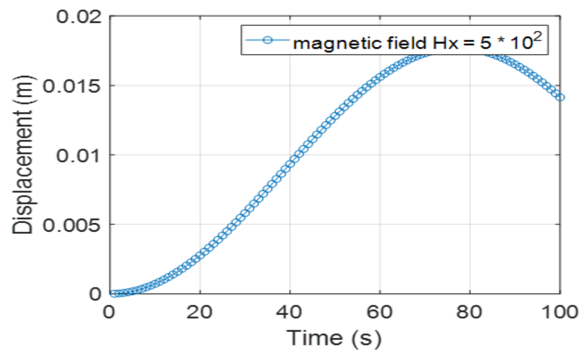

Figure 5: Effect of magnetic field at 5 $\times 10^2 A/m$ on deflection of the pipe

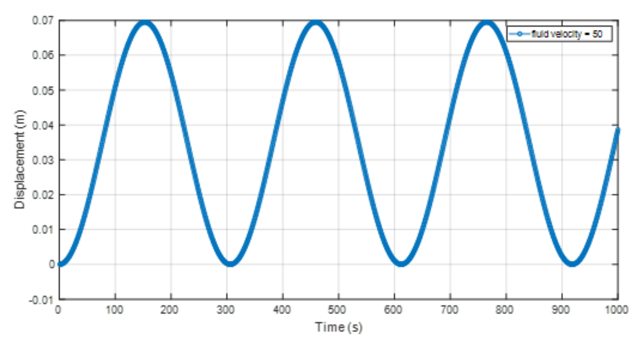

Figure 6: Effect of fluid velocity at 50m/s on deflection of the pipe

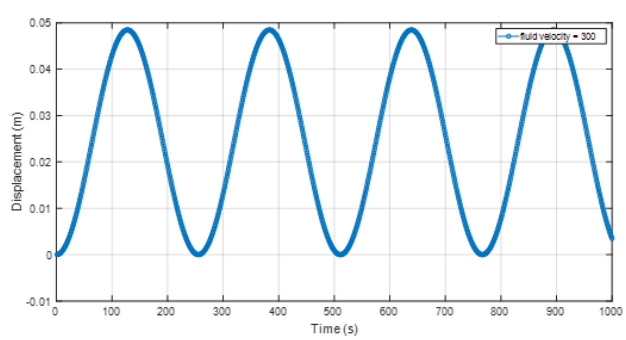

Figure 7: Effect of fluid velocity at 300m/s on deflection of the pipe

The results obtained from Newmark time integration method of free vibration of free – clamped support system of a single – walled carbon nanotube is discuses in details here. Figure 2 and Figure 3 shows the effect of non -local parameter at 0.31nm and 0.5 nm on the deflection of the single – walled carbon nanotube (CNT). At when e_0=0.31nm, the frequency of the deflection was oscillating on the positive axis over time while at e_0=0.5nm, the frequency of the deflection was very small until around 900sec it's rapidly increase.

Figure 4 and Figure 5 shows effect of magnetic field on the deflection of the pipe. When the value of the magnetic frequency is $H_x = 2.5 \times 10^2 A/m$ the frequency of deflection was oscillating on a regular interval on the positive axis while at $H_x = 5 \times 10^2 A/m$ the frequency of the deflection gradually increase before its start to decrease again.

Figure 6 and Figure 7 shows the effect of fluid velocity of the deflection of the pipe. At when the fluid velocity was at u = 50m/s, the pipe oscillation was slower than when the fluid velocity was at u = 300m/s. That means as the fluid velocity increases the pipe vibrate faster.

Conclusion

We have studied the non-local vibration of a single – walled carbon nanotube (CNT) using Galerkin finite element method and Newmark time integration method. From the study the following conclusion are obtained:

1. The non – local parameter affect the deflection of the pipe at small value than when the value is increase.
2. That magnetic field also affect the deflection of the pipe as the value is increase.
3. That the effect of fluid velocity mostly affects the deflection of the pipe as the value is increase.

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