Science World Journal Vol. 16(No 4) 2021
www.scienceworldjournal.org
ISSN: 1597-6343 (Online), ISSN: 2756-391X (Print)
Published by Faculty of Science, Kaduna State University

Full Length Research Article

HEAT TRANSFER ON UNSTEADY MHD FLOW OF FOURTH-GRADE FLUID IN A HORIZONTAL INFINITE PARALLEL PLATES WITH SUCTION EFFECTS

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ABSTRACT

This work examines the heat transfer on unstable MHD flow of fourth grade fluid in horizontal parallel plates. The plates are arranged so that the upper plate oscillates and moves while the lower plate is stationary. The temperatures prescribed by the plates are uniform and asymmetric. Dimensionless parameters are set to convert the governing equations to dimensionless form. Solutions for momentum and energy equations are explicitly obtained through the He-Laplace scheme. The effect of various flow parameters on velocity profile and temperature distribution is discussed using graphs. Significant results of this study show that velocity profile and temperature distribution increase with the increase in thermal radiation parameter, while the velocity and temperature distribution decrease with the increase in suction parameter. The results of this work have significant application in refrigeration industry for refrigeration, energy transfer in cooling tower, evaporation, etc.

Keywords: Heat transfer, Thermal radiation, MHD, Fourth-grade fluid, Suction, He-Laplace.

INTRODUCTION

The study of heat transfer is of realistic significance to engineers and scientists because of its universal incidence in many branches of science and engineering. This phenomenon plays a significant role in chemical industry, power and cooling industry for dying, evaporation, energy transfer in cooling tower and flow in s desert cooler, etc Satya et.al (2015).

Heat transfer in the other hand can be said to be the transmission of energy from one region to another as a result of increase in temperature difference between them. While mass transfer can also be said to be the transportation or movement of materials or substances from a region of higher concentration to a region of lower concentration through a semi permeable membrane. Heat transfer occurs when there is temperature difference between two or more regions or surroundings. Much of the understanding of plasma came from the study of Magneto hydrodynamics (MHD) as it is the study of interactions of electrically conducting fluids and electromagnetic fields. When fluid such as ionized gases (plasma, mercury and molten iron, electrolytes which are only but a few electrically conducting fluids, moves through a magnetic field, consequently a current is induced, and in turn the current interacts with the magnetic field to produce a body force on the fluid. Such forces (current) generated has been used in the generation of electricity by the help of (MHD) designed generators. The analysis of suction effect on fluids through magneto hydro dynamics has attracted many researches due to its application in geothermal and oil reservoir , where it deals with the behaviour of fluids under rest and motion, the phenomenon of unsteady Magneto-hydrodynamics (MHD) flow with suction effect have become very popular and a subject of growing interest due to its application in many engineering and geophysical processes such as cooling of nuclear reactor, geothermal reservoirs, underground energy transport, MHD pump ,MHD power generators, etc.

The fundamental concept behind MHD is that magnetic fields can induce currents in a moving conductive fluid, which in turn polarizes the fluid and reciprocally changes the magnetic field itself which as some many application in the field of engineering and science and as such as been a field of studies which have been attracting a considerable attention of engineers and scientists all over the world but it was Hannes Alfven (1942), a Swedish electrical engineer first initiated the study of MHD.

Fourth-grade fluid is an important subclass of differential type that is capable of describing shear thinning and shear thickening effects. It is also non-Newtonian. Examples are ketchup, blood, paint, cream, nail polish, etc.

These are non – Newtonian fluids as such, many empirical and semi-empirical non-Newtonian models or constitutive equations have been proposed. Rehan *et al*. (2010) considered the steady flow of a fourth grade fluid, between two parallel plates. They analyzed four types of flows: Couette flow, plug flow, Poiseuille flow and generalized Couette flow. The nonlinear differential equation describing the velocity field was solved by optimal homotopy asymptotic method (OHAM). They observed that the OHAM was more efficient and flexible than the perturbation and Homotopy analyses method. Islam *et al*. (2011) considered the steady flow of a non-Newtonian fluid with slippage between the plate and the fluid. The constitutive equations of the fluids were modelled for fourth-grade non-Newtonian fluid with partial slip. They employed homotopy perturbation and optimal homotopy asymptotic methods to solve the non-linear differential equation. Shehzad *et al*. (2018) reported the electro-osmotic Couette-Poiseuille flow of power law Al2O3- PVC nanofluid through a channel, in which upper wall is moving with constant velocity. The influences of magnetic field, mixed convection, joule heating, and viscous dissipation were also incorporated. The flow was generated because of constant pressure gradient in axial direction. The resulting flow problem was coupled nonlinear ordinary differential equations, which were at first modeled and then transform into dimensionless form through appropriate transformation. Analytical solution of the governing equation was carried out.

Fenuga *et al* (2020) investigated the mathematical model and solution for an unsteady MHD fourth grade fluid flow over a vertical plate in a porous medium with the effects of the magnetic field and suction/injection parameters using Homotopy Perturbation Method. They displayed graphically and discussed the impact of dimensionless second, third and fourth grade parameters with the effects of magnetic field and suction/injection parameters on the velocity field. They found out that increase in suction parameter decreases the momentum boundary layer thickness while injection parameter enhances velocity distribution in the boundary layer. Magnetic field reduces velocity throughout the boundary layer because the Lorentz force which acts as retarding force reduces the boundary layer thickness.

Yurusoy (2020) investigated the time dependent boundary layer flow of a modified power-law fluid of fourth grade on a stretched surface with an injection or suction boundary condition. The fluid model is a mixture of fourth grade and power-law fluids in which the fluid may display shear thickening, shear thinning or normal stress textures. They the scaling and translation transformations which is a type of Lie Group transformation, time dependent boundary layer equations were reduced into two alternative ordinary differential equations systems (ODEs) with boundary conditions. He found out that the boundary layer thickness decreases as the power-law index value increases. And also, the fourth-grade fluid parameter, as the parameter increases, the boundary layer thickness decreases while the velocity in the y direction increases.

Priyadarsan and Panda (2020) carried out a numerical investigation to study the unsteady flow of incompressible and electrically conducting fourth-grade fluid through a porous medium between two infinite parallel plates under transverse magnetic field with time-dependent suction. The lower plate is at rest and the upper plate is moving and oscillating in its own plane about a constant mean velocity with time-dependent suction. The basic equations governing the flow and heat transfer are reduced to a set of non-linear partial differential equations. The governing equations are simplified using perturbation method with respect to time and the resulting sixth-order non-linear differential equations are solved numerically using Runge-Kutta method in association with the multi-shooting technique. Their investigation revealed that the higher-grade fluid parameters influence significantly the fluid temperature. Khan *et al*. (2018) discussed the unsteady flow of non- Newtonian fluid with the properties of heat/sink in the presence of thermal radiation through a binary mixture embedded in a porous. Santhosha *et al*. (2017) studied the radiation and chemical combined effects on MHD free convective heat and mass transfer flow of viscous, incompressible, conducting elastic fluid through porous medium finite by a porous plate within the presence of heat generation. The momentum, energy and mass diffusion equation were coupled non-linear partial differential equations. They employed two term perturbation method. Joseph et al. (2021) investigated the unsteady MHD flow of a fourth-grade fluid in a horizontal parallel plate's channel. They considered the upper plate to be oscillating and moving while the bottom plate is stationary. Solutions for momentum, energy and concentration equations are obtained by the He-Laplace scheme. They found that velocity and temperature fields increase with the increase in the thermal radiation parameter, while velocity and concentric fields decrease with an increase in the chemical reaction parameter. Furthermore, velocity, temperature and concentric fields decrease with an increase in the suction parameter.

Taza *et al*. (2016) studied the unsteady thin film flow of a fourth grade fluid over a moving and oscillating vertical belt. They employed adomian decomposition method (ADM) and optimal homotopy asymptotic method (OHAM) to find the solution of the non- linear differential equations that governed the flow. Hayat *et al*. (2007) presented the exact solution for four types of flows between two parallel plates, viz; Couette flow, plug flow, Poiseuille flow and generalized Couette flow. The nonlinear second-order differential equation for the velocity field was solved exactly in each case. The nonlinear differential equation describing the velocity field was solved by optimal homotopy asymptotic method (OHAM). They observed that the OHAM is more efficient and flexible than the perturbation and Homotopy analyses method.

Idowu and Sani (2019) carried out an analysis for unsteady magnetohydrodynamic (MHD) flow of a generalized third grade fluid between two parallel plates. The fluid flow was as a result of the plate oscillating, moving and pressure gradient. Three flow problems were investigated, namely: Couette, Poiseuille and Couette-Poiseuille flows and a number of nonlinear partial differential equations were obtained which were solved using the He-Laplace method. Expressions for the velocity field, temperature and concentration fields were given for each case and finally, effects of physical parameters on the fluid motion, temperature and concentration were plotted and discussed. They found that an increase in the thermal radiation parameter increases the temperature of the fluid and hence reduces the viscosity of the fluid while the concentration of the fluid reduces as the chemical reaction parameter increases.

In the above aforementioned investigations however, the heat transfer on unsteady MHD flow of fourth-grade fluid in a horizontal infinite parallel plates with suction effect have not been given more attention. Suction is the act or process of sucking. A force that causes a fluid or solid to be drawn into interior space or to adhere to surface because of the difference between the external and internal pressure. This is considered due to the porosity of the channel plates.

FORMULATION OF THE PROBLEM

We consider the unsteady flow of an electrically conducting incompressible fourth grade fluid between two horizontal parallel plates channel as shown in figure 1 below. The fluid is subjected to a uniform transverse magnetic field. We assumed the bottom plate is fixed (stationary) and the top plates is moving with constant velocity.

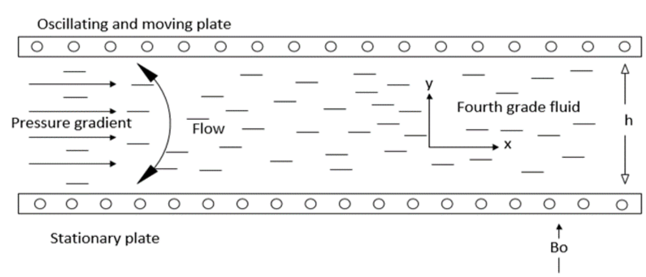

Figure 1: Physical Schematic of the Flow Configuration

The state of this fluid is determined by the history of the deformation gradient without a preferred reference configuration. Its constitute equation can be written as

$$T(x,t) = -PI + f_{s=0}^{\infty}\left(F_t^t(s)\right) \qquad (1)$$

Where PI is the undetermined part of the stress – tensor, F is the deformation gradient and f is the functional.

Coleman and Noll (1960) prescribed different sort of incompressible fluid category n as viscous fluid agreeing on Hayat et al. (2007). Incompressible fluid of differential type of grade n is the simple fluid obeying the constitutive equation

$$T = -PI + \sum_{j=1}^{n} S_j \quad (2)$$

obtained by asymptotic expansion of the functional in equation (1) through a retardation parameter α. For $n = 4$ as Hayat et al. (2005, 2007), the first four (4) tensors S_j are given by

$$S_1 = \mu A_1 \quad (3)$$

$$S_2 = \alpha_1 A_2 + \alpha_2 A_1^2 \quad (4)$$

$$S_3 = \beta_1 A_3 + \beta_2(A_1A_2 + A_2A_1) + \beta_3(trA_1^2)A_1 \quad (5)$$

$$S_4 = \gamma_1 A_4 + \gamma_2(A_3A_1 + A_1A_3) + \gamma_3 A_2^2 + \gamma_4(A_2A_1^2 + A_1^2A_2) + \gamma_5(trA_2)A_2 + \gamma_6(trA_2)A_1^2 + [\gamma_7 trA_3 + \gamma_8 tr(A_2A_1)]A_1 \quad (6)$$

Where, μ is the coefficient of shear viscosity, $\alpha_i (i = 1,2)$, $\beta_i (i = 1,2,3)$ and $\gamma_i (i = 1(1)8)$ are material constants. The A_n are the Rivlin – Ericksen tensors defined by the recursion relation

$$A_n = \frac{d}{dt}A_{n-1} + A_{n-1}L + L^T A_{n-1}, \; n>1 \quad (7)$$

$$A_1 = L + L^T \quad (8)$$

where $L = \nabla V$, $\frac{d}{dt}$ is the material time derivative and $V$is the velocity.

We note that when $\gamma_i = 0$, the fourth grade model reduces to the third grade model. When $\beta_i = 0$, the third grade model reduces to second grade model. When $\alpha_i = 0$, $\beta_i = 0$ and $\gamma_i = 0$ then the model reduces to classical Navier – Stoke fluid.

The thermally radiative flow is heading x – direction along infinite porous plate with heat generation. Here, U_0 is the uniform velocity and T_∞ isthe fluid temperature.

Under the above consideration, the equations that described the physical circumstances are

$$\frac{\partial \mathrm{v}}{\partial y} = 0 \quad (9)$$

$$\frac{\partial u}{\partial t} + \mathrm{v}\frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \mathrm{v}\frac{\partial^2 u}{\partial y^2} + \frac{\alpha_1 \nu}{\rho}\frac{\partial^3 u}{\partial y^2 \partial t} + \frac{\beta_1 \nu^2}{\rho}\frac{\partial^4 u}{\partial y^2 \partial t^2} + \frac{6(\beta_2+\beta_3)}{\rho}\left(\frac{\partial u}{\partial y}\right)^2 \frac{\partial^2 u}{\partial y^2} + \frac{\gamma_1 \nu^3}{\rho}\frac{\partial^5 u}{\partial y^2 \partial t^3} + \frac{2\mathrm{v}(3\gamma_2+\gamma_3+\gamma_4+\gamma_5+\gamma_7+\gamma_8)}{\rho C_p}\left[2\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2}\frac{\partial^2 u}{\partial t \partial y} + \left(\frac{\partial u}{\partial y}\right)^2 \frac{\partial^3 u}{\partial y^2 \partial t}\right] - \frac{\sigma B_0^2}{\rho C_p}u + g\beta_T(T - T_\infty) - \frac{\nu}{k}u \quad (10)$$

$$\frac{\partial T}{\partial t} + \mathrm{v}\frac{\partial T}{\partial y} = \frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y^2} + \frac{Q_0}{\rho C_p}(T_w - T_\infty) - \frac{1}{\rho C_p}\frac{\partial q_r}{\partial y} \quad (11)$$

From equation (11), q_r is the radiative heat flux define as

$$\frac{\partial q_r}{\partial y} = 4\alpha^2(T_w - T_\infty) \quad (12i)$$

The initial and boundary conditions are

$$\left.\begin{array}{c} u = U_0 e^{-yh}, T = T_0 + (T_w - T_\infty)e^{-yh} \;\; at\; t = 0 \; for \;\; 0 \le y \ge h \\ u(y,t) = U, T(y,t) = T_w, at\; y = h\; for\; t \ge 0 \\ u(y,t) \to \infty, T(y,t) \to \infty \; as\; y \to \infty \; for\; t > 0 \end{array}\right\} \quad (13)$$

Where u is the fluid velocity, T is the temperature, q_r is the radiative heat flux, ρ is the density of the fluid, C_p is the heat capacity, B_0 is the external magnetic field.

In order to transform equations (10) – (13), we use the following dimensionless parameters

$$u^* = \frac{u}{U_0},\; p^* = \frac{p}{\mu U_0^2}, t^* = \frac{tU_0^2}{\nu}, G_r = \frac{g\beta_T(T_w - T_\infty)\nu}{U_0^3},, Ha^2 = \frac{\sigma B_0^2 \nu}{\rho U_0^2},, y^* = \frac{yU_0}{\nu}, x^* = \frac{x}{h}, h = \frac{U_0}{\nu}\; P_r = \frac{kU_0^2}{\nu^2}, S = \frac{\mathrm{v}_0}{U_0}, \mathrm{v} = \frac{\mathrm{v}}{U_0}, \theta = \frac{T - T_0}{T_w - T_\infty},, \delta = \frac{4\alpha^2 U_0^2}{\rho c_p \nu}, \alpha = \frac{\alpha_1 U_0^2}{\rho \nu^2}, \beta_a = \frac{\beta_1 U_0^4}{\rho \nu^3}, \beta_b = \frac{(\beta_2+\beta_3)U_0^4}{\rho \nu^3}, \gamma_a = \frac{\gamma_1 U_0^6}{\rho \nu^3}, \gamma_b = \frac{2(3\gamma_2+\gamma_3+\gamma_4+\gamma_5+3\gamma_7+\gamma_8)U_0^6}{\rho \nu^4} \quad (14)$$

Substituting equation (14) into equations (9) – (13) and by dropping the asterisks, we have the following:

$$\frac{\partial v}{\partial y} = 0 \Rightarrow \mathrm{v} = -\mathrm{v}_0 \quad (15)$$

$$\frac{\partial u}{\partial t} - S\frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \frac{\partial^2 u}{\partial y^2} + \alpha\frac{\partial^3 u}{\partial y^2 \partial t} + \beta_a \frac{\partial^4 u}{\partial y^2 \partial t^2} + \beta_b \left(\frac{\partial u}{\partial y}\right)^2 \frac{\partial^2 u}{\partial y^2} + \gamma_a \frac{\partial^5 u}{\partial y^2 \partial t^3} + \gamma_b \left[2\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2}\frac{\partial^2 u}{\partial t \partial y} + \left(\frac{\partial u}{\partial y}\right)^2 \frac{\partial^3 u}{\partial y^2 \partial t}\right] - \left(Ha + \frac{1}{Da}\right)u + G_r\theta \quad (16)$$

$$\frac{\partial \theta}{\partial t} - S\frac{\partial \theta}{\partial y} = \frac{1}{P_r}\frac{\partial^2 \theta}{\partial y^2} + (Q_0 + \delta)\theta \quad (17)$$

And the initial and boundary conditions becomes

$$\left.\begin{array}{c} u(y,t) = e^{-y}, \theta(y,t) = e^{-y}\; at\; t = 0\; for\; 0 \le y \ge 1 \\ u(y,t) = 1, \theta(y,t) = 1\; at\; y = 1\; for\; t \ge 0 \\ u(y,t) \to \infty, T(y,t) \to \infty\; as\; y \to \infty\; for\; t > 0 \end{array}\right\} \quad (18)$$

METHOD OF SOLUTION/SOLUTION OF THE PROBLEM

In this section we employed the He – Laplace scheme to solve equations (16) and (17) subjects to the initial and boundary conditions (18).

Since equation (16) is a coupled non – linear partial differential equation, we have to solve equations (17) first.

Now applying Laplace transform on equation (17), we have;

Next, we consider equation (17), which is rearranged to give;

$$\frac{\partial \theta}{\partial t} - S\frac{\partial \theta}{\partial y} = \frac{1}{P_r}\frac{\partial^2 \theta}{\partial y^2} + (l_1)\theta, \text{ where, } l_1 = Q_0 + \delta \quad (18i)$$

Now applying Laplace transform on equation (18i);

$$L\left\{\frac{\partial \theta}{\partial t}\right\} - L\left\{S\frac{\partial \theta}{\partial y}\right\} = \frac{1}{P_r}L\left\{\frac{\partial^2 \theta}{\partial y^2}\right\} + L\{l_1\theta\} \quad (19)$$

Applying the initial condition and dividing through by s and rearranging we obtain;

$$L\{\theta(y,t)\} = \frac{e^{-y}}{s} + \frac{1}{s}\left\{\frac{1}{P_r}L\left\{\frac{\partial^2 \theta}{\partial y^2}\right\} + SL\left\{\frac{\partial \theta}{\partial y}\right\} + L\{l_1\theta\}\right\} \quad (20)$$

Taking the inverse Laplace transform of both sides of equation (20) gives,

$$\theta(y,t) = e^{-y} + L^{-1}\left[\frac{1}{s}\left\{\frac{1}{P_r}L\left\{\frac{\partial^2 \theta}{\partial y^2}\right\} + SL\left\{\frac{\partial \theta}{\partial y}\right\} + L\{l_1\theta\}\right\}\right] \quad (21)$$

Applying the Homotopy perturbation technique on equation (21), yields

$$P^n \theta_n(y,t) = e^{-y} + \sum_{n=0}^{\infty} P\left[L^{-1}\left\{\frac{1}{s}\left\{\frac{1}{P_r}L\left\{\frac{\partial^2 \theta}{\partial y^2}\right\} + L\{\delta\theta\}\right\}\right\}\right] \quad (22)$$

Comparing the coefficients of the like powers of $'P'$in equation (22), the following approximations are obtained;

$$P^0: \theta_0(y,t) = e^{-y} \quad (23)$$

$$P^1: \theta_1(y,t) = L^{-1}\left[\frac{1}{s}\left\{\frac{1}{P_r}L\left\{\frac{\partial^2\theta_0}{\partial y^2}\right\} + SL\left\{\frac{\partial\theta_0}{\partial y}\right\} + L\{l_1\theta_0\}\right\}\right] = L^{-1}\left\{\frac{1}{P_r}\left(\frac{e^{-y}}{s^2}\right) - S\left(\frac{e^{-y}}{s^2}\right) + l_1\left(\frac{e^{-y}}{s^2}\right)\right\} = \left(\frac{e^{-y}}{P_r} - Se^{-y} + l_1e^{-y}\right)t \quad (24)$$

$$P^2: \theta_2(y,t) = L^{-1}\left[\frac{1}{s}\left\{\frac{1}{P_r}L\left\{\frac{\partial^2\theta_1}{\partial y^2}\right\} + SL\left\{\frac{\partial\theta_1}{\partial y}\right\} + L\{l_1\theta_1\}\right\}\right] = L^{-1}\left[\frac{1}{s}\left\{\frac{1}{P_r}L\left\{\left(\frac{e^{-y}}{P_r} - Se^{-y} + l_1e^{-y}\right)t\right\} - SL\left\{\left(Se^{-y} - \frac{e^{-y}}{P_r} - l_1e^{-y}\right)t\right\} + L\left\{l_1\left(\frac{e^{-y}}{P_r} - Se^{-y} + l_1e^{-y}\right)t\right\}\right\}\right] = \left(\frac{e^{-y}}{P_r^2} + \frac{2l_1e^{-y}}{P_r} - S^2e^{-y} + l_1^2e^{-y}\right)\frac{t^2}{2!} \quad (25)$$

$$P^3: \theta_3(y,t) = L^{-1}\left[\frac{1}{s}\left\{\frac{1}{P_r}L\left\{\frac{\partial^2\theta_2}{\partial y^2}\right\} + SL\left\{\frac{\partial\theta_2}{\partial y}\right\} + L\{l_1\theta_2\}\right\}\right] = L^{-1}\left[\frac{1}{s}\left\{\frac{1}{P_r}L\left\{\left(\frac{e^{-y}}{P_r^2} + \frac{2l_1e^{-y}}{P_r} - S^2e^{-y} + l_1^2e^{-y}\right)\frac{t^2}{2!}\right\} - SL\left\{\left(\frac{e^{-y}}{P_r^2} + \frac{2l_1e^{-y}}{P_r} - S^2e^{-y} + l_1^2e^{-y}\right)\frac{t^2}{2!}\right\} + \left(S^2e^{-y} - \frac{e^{-y}}{P_r^2} - \frac{2l_1e^{-y}}{P_r} - l_1^2e^{-y}\right)\frac{t^2}{2!}\right\}\right] = \left(\frac{e^{-y}}{P_r^3} + \frac{3l_1e^{-y}}{P_r^2} - \frac{S^2e^{-y}}{P_r} + \frac{3l_1^2e^{-y}}{P_r} - \frac{Se^{-y}}{P_r} - \frac{2l_1Se^{-y}}{P_r} - l_1^2Se^{-y} - l_1S^2e^{-y} + l_1^3e^{-y} + S^3e^{-y}\right)\frac{t^3}{3!} \quad (26)$$

Therefore, in view of equations (23), (24), (25) and (26), the solution to equation (17) is,

$$\theta(y,t) = \theta_0(y,t) + \theta_1(y,t) + \theta_2(y,t) + \theta_3(y,t)\cdots$$

$$\theta(y,t) = e^{-y} + \left(\frac{e^{-y}}{P_r} - Se^{-y} + l_1e^{-y}\right)t + \left(\frac{e^{-y}}{P_r^2} + \frac{2l_1e^{-y}}{P_r} - S^2e^{-y} + l_1^2e^{-y}\right)\frac{t^2}{2!} + \left(\frac{e^{-y}}{P_r^3} + \frac{3l_1e^{-y}}{P_r^2} - \frac{S^2e^{-y}}{P_r} + \frac{3l_1^2e^{-y}}{P_r} - \frac{Se^{-y}}{P_r} - \frac{2l_1Se^{-y}}{P_r} - l_1^2Se^{-y} - l_1S^2e^{-y} + l_1^3e^{-y} + S^3e^{-y}\right)\frac{t^3}{3!} + \cdots \quad (29)$$

Finally, we now solve equation (16), which is rearranged to give

$$\frac{\partial u}{\partial t} - S\frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \frac{\partial^2 u}{\partial y^2} + \alpha\frac{\partial^3 u}{\partial y^2\partial t} + \beta_a\frac{\partial^4 u}{\partial y^2\partial t^2} + \beta_b\left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^2 u}{\partial y^2} + \gamma_a\frac{\partial^5 u}{\partial y^2\partial t^3} + \gamma_b\left[2\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2}\frac{\partial^2 u}{\partial t\partial y} + \left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^3 u}{\partial y^2\partial t}\right] - l_2u + G_r\theta \text{ where, } Ha + \frac{1}{Da} = l_2 \quad (29i)$$

Applying the Laplace transform on both sides of equation (29i) gives

$$L\left\{\frac{\partial u}{\partial t}\right\} - L\left\{S\frac{\partial u}{\partial y}\right\} = L\left\{-\frac{\partial p}{\partial x} + \frac{\partial^2 u}{\partial y^2} + \alpha\frac{\partial^3 u}{\partial y^2\partial t} + \beta_a\frac{\partial^4 u}{\partial y^2\partial t^2} + \beta_b\left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^2 u}{\partial y^2} + \gamma_a\frac{\partial^5 u}{\partial y^2\partial t^3} + \gamma_b\left[2\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2}\frac{\partial^2 u}{\partial t\partial y} + \left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^3 u}{\partial y^2\partial t}\right] - l_2u + G_r\theta\right\} \quad (30)$$

But, $L\left\{\frac{\partial u}{\partial t}\right\} = sL\{u(y,t)\} - u(y,0)$ (31)

Hence,

$$L\{u(y,t)\} = \frac{u(y,0)}{s} + \frac{1}{s}L\left\{-\frac{\partial p}{\partial x} + S\frac{\partial u}{\partial y} + \frac{\partial^2 u}{\partial y^2} + \alpha\frac{\partial^3 u}{\partial y^2\partial t} + \beta_a\frac{\partial^4 u}{\partial y^2\partial t^2} + \beta_b\left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^2 u}{\partial y^2} + \gamma_a\frac{\partial^5 u}{\partial y^2\partial t^3} + \gamma_b\left[2\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2}\frac{\partial^2 u}{\partial t\partial y} + \left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^3 u}{\partial y^2\partial t}\right] - l_2u + G_r\theta\right\} \quad (32)$$

Taking the inverse Laplace transform of both sides of equation (32), we have;

$$L^{-1}\{L\{u(y,t)\}\} = L^{-1}\left\{\frac{u(y,0)}{s} - \frac{\partial p}{\partial x} + \frac{1}{s}L\left\{+S\frac{\partial u}{\partial y} + \frac{\partial^2 u}{\partial y^2} + \alpha\frac{\partial^3 u}{\partial y^2\partial t} + \beta_a\frac{\partial^4 u}{\partial y^2\partial t^2} + \beta_b\left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^2 u}{\partial y^2} + \gamma_a\frac{\partial^5 u}{\partial y^2\partial t^3} + \gamma_b\left[2\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2}\frac{\partial^2 u}{\partial t\partial y} + \left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^3 u}{\partial y^2\partial t}\right] - l_2u + \frac{G_r}{s}\left(e^{-y} + \left(\frac{e^{-y}}{P_r} - Se^{-y} + l_1e^{-y}\right)t + \left(\frac{e^{-y}}{P_r^2} + \frac{2l_1e^{-y}}{P_r} - S^2e^{-y} + l_1^2e^{-y}\right)\frac{t^2}{2!} + \left(\frac{e^{-y}}{P_r^3} + \frac{3l_1e^{-y}}{P_r^2} - \frac{S^2e^{-y}}{P_r} + \frac{3l_1^2e^{-y}}{P_r} - \frac{Se^{-y}}{P_r} - \frac{2l_1Se^{-y}}{P_r} - l_1^2Se^{-y} - l_1S^2e^{-y} + l_1^3e^{-y} + S^3e^{-y}\right)\frac{t^3}{3!} + \right)\right\}\right\} \quad (33)$$

Or,

$$u(y,t) = \lambda + e^{-y} + (G_re^{-y})t + \left(\frac{e^{-y}}{P_r} - 2Se^{-y} + l_1e^{-y}\right)\frac{t^2}{2!} + \left(\frac{e^{-y}}{P_r^2} + \frac{2l_1e^{-y}}{P_r} + l_1^2e^{-y}\right)\frac{t^3}{3!} + L^{-1}\left\{\frac{1}{s}L\left\{+S\frac{\partial u}{\partial y} + \frac{\partial^2 u}{\partial y^2} + \alpha\frac{\partial^3 u}{\partial y^2\partial t} + \beta_a\frac{\partial^4 u}{\partial y^2\partial t^2} + \beta_b\left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^2 u}{\partial y^2} + \gamma_a\frac{\partial^5 u}{\partial y^2\partial t^3} + \gamma_b\left[2\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2}\frac{\partial^2 u}{\partial t\partial y} + \left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^3 u}{\partial y^2\partial t}\right] - l_2u\right\}\right\} \quad (34)$$

Applying the Homotopy perturbation method to equation (34), gives

$$\sum_{n=0}^{\infty}P^nu_n(y,t) = \lambda + e^{-y} + (G_ce^{-y} + G_re^{-y})t + \left(\frac{e^{-y}}{P_r} - 2Se^{-y} + l_1e^{-y}\right)\frac{t^2}{2!} + \left(\frac{e^{-y}}{P_r^2} + \frac{2l_1e^{-y}}{P_r} + l_1^2e^{-y}\right)\frac{t^3}{3!} + \left(\frac{e^{-y}}{P_r^3} + \frac{3l_1e^{-y}}{P_r^2} - \frac{S^2e^{-y}}{P_r} + \frac{3l_1^2e^{-y}}{P_r} - \frac{Se^{-y}}{P_r} - \frac{2l_1Se^{-y}}{P_r} - l_1^2Se^{-y} - l_1S^2e^{-y} + l_1^3e^{-y}\right)\frac{t^4}{4!} + P\left(L^{-1}\left\{\frac{1}{s}L\left\{S\frac{\partial u}{\partial y} + \frac{\partial^2 u}{\partial y^2} + \alpha\frac{\partial^3 u}{\partial y^2\partial t} + \beta_a\frac{\partial^4 u}{\partial y^2\partial t^2} + \beta_bH_a(u_n) + \gamma_1\frac{\partial^5 u}{\partial y^2\partial t^3} + \gamma_b\left[2H_b(u_n) + H_c(u_n)\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2}\frac{\partial^2 u}{\partial t\partial y} + \left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^3 u}{\partial y^2\partial t}\right] - l_2u\right\}\right\}\right) \quad (35)$$

Where, $H_a(u_n), H_b(u_n)$ and $H_c(u_n)$ are the He's polynomials for $\left(\frac{\partial u}{\partial y}\right)^2$, $\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2}\frac{\partial^2 u}{\partial t\partial y}$ and $\left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^3 u}{\partial y^2\partial t}$ respectively.

The He's polynomials for $\left(\frac{\partial u}{\partial y}\right)^2$ are as follows

$$\begin{cases} H_0(u) = (u_0')^2 \\ H_1(u) = 2u_0'u_1' \\ H_2(u) = 2u_0'u_2' + (u_1')^2 \\ H_3(u) = 2u_1'u_2' \\ \vdots \end{cases} \quad (36)$$

The He's polynomials for $\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2}\frac{\partial^2 u}{\partial t\partial y}$ are as follows

$$\begin{cases} H_0(u) = u_0''' u''_{0t} \\ H_1(u) = u_0''' u''_{1t} + u_1''' u''_{1t} \\ H_2(u) = u_0''' u'_{2t} + u_1''' u'_{1t} + u_2''' u'_{0t} \\ H_3(u) = u_1''' u''_{2t} + u_2''' u''_{1t} \\ \vdots \end{cases} \quad (37)$$

The He's polynomials for $\left(\frac{\partial u}{\partial y}\right)^2 \frac{\partial^3 u}{\partial y^2 \partial t}$ are as follows

$$\begin{cases} H_0(u) = (u_0')^2 (u_0'' u_{0t}') \\ H_1(u) = (u_0')^2 (u_0'' u_{1t}') + (u_0')^2 (u_1'' u_{0t}') + 2u_0' u_1' (u_0'' u_{0t}') \\ H_2(u) = (u_0')^2 (u_0'' u_{2t}') + (u_0')^2 (u_1'' u_{1t}') + (u_0')^2 (u_2'' u_{0t}') + 2u_0' u_1' (u_0'' u_{1t}') + \\ 2u_0' u_1' (u_1'' u_{0t}') + 2u_0' u_2' (u_0'' u_{0t}') \\ \vdots \end{cases} \quad (38)$$

Now, comparing the like powers of "P" in equation (25) and equating their coefficients gives

$$P^0; u_0(y,t) = \lambda + e^{-y} + (G_r e^{-y})t + \left(\frac{e^{-y}}{P_r} - 2Se^{-y} + l_1 e^{-y}\right)\frac{t^2}{2!} + \left(\frac{e^{-y}}{P_r^2} + \frac{2l_1 e^{-y}}{P_r} + l_1^2 e^{-y}\right)\frac{t^3}{3!} + \left(\frac{e^{-y}}{P_r^3} + \frac{3l_1 e^{-y}}{P_r^2} - \frac{S^2 e^{-y}}{P_r} + \frac{3l_1^2 e^{-y}}{P_r} - \frac{Se^{-y}}{P_r} - \frac{2l_1 Se^{-y}}{P_r} - l_1^2 Se^{-y} - l_1 S^2 e^{-y} + l_1^3 e^{-y}\right)\frac{t^4}{4!} \quad (39)$$

$$P^1; u_1(y,t) = L^{-1}\left\{\frac{1}{s} L\left\{S\frac{\partial u_0}{\partial y} + \frac{\partial^2 u_0}{\partial y^2} + \alpha \frac{\partial^3 u_0}{\partial y^2 \partial t} + \beta_a \frac{\partial^4 u_0}{\partial y^2 \partial t^2} + \beta_b (u_0')^2 (u_0'') + \gamma_a \frac{\partial^5 u_0}{\partial y^2 \partial t^3} + \gamma_b [2u_0''' u''_{0t} + (u_0')^2 (u_0'' u_{0t}')] - l_2 u_0\right\}\right\} \quad (40)$$

Or,

$$u_1(y,t) = \left(e^{-y} - Se^{-y} + \alpha G_r e^{-y} + \frac{\beta_a e^{-y}}{P_r} - \beta_b e^{-y} + \frac{\gamma_a e^{-y}}{P_r^2} + \frac{2\gamma_a l_1 e^{-y}}{P_r} + \gamma_a l_1^2 e^{-y} + 2\gamma_a G_r e^{-2y} + \gamma_b G_r e^{-3y} - l_2 e^{-y} - \lambda l_2\right)t + \left(G_r e^{-y} - G_r Se^{-y} - 2\alpha Se^{-y} - \frac{\alpha e^{-y}}{P_r} + \alpha l_1 e^{-y} + \frac{\beta_a e^{-y}}{P_r^2} - \frac{2\beta_a l_1 e^{-y}}{P_r} + \beta_a l_1^2 e^{-y} + 3\beta_b G_r e^{-3y} + \frac{\gamma_a e^{-y}}{P_r^3} + \frac{3\gamma_a e^{-y}}{P_r^2} - \frac{\gamma_a S^2 e^{-y}}{P_r} + \frac{3\gamma_a l_1^2 e^{-y}}{P_r} - \frac{\gamma_a Se^{-y}}{P_r} - \gamma_a l_1^2 Se^{-y} - \gamma_a l_1 S^2 e^{-y} + \gamma_a l_1^3 e^{-y} - 3\gamma_b G_r^2 e^{-3y} - l_2 G_r e^{-y}\right)\frac{t^2}{2!} + \left(\frac{e^{-y}}{P_r} + l_1 e^{-y} - \frac{Se^{-y}}{P_r} + Sl_1 e^{-y} + \frac{\alpha e^{-y}}{P_r^2} + \frac{2\alpha l_1 e^{-y}}{P_r} + \alpha l_1^2 e^{-y} + \frac{\beta_a e^{-y}}{P_r^3} + \frac{3\beta_a l_1 e^{-y}}{P_r^2} - \frac{\beta_a S^2 e^{-y}}{P_r} + \frac{3\beta_a l_1^2 e^{-y}}{P_r} - \frac{\beta_a Se^{-y}}{P_r} - \frac{2\beta_a l_1 Se^{-y}}{P_r} - \beta_a l_1^2 Se^{-y} - \beta_a l_1 S^2 e^{-y} + \beta_a l_1^3 e^{-y} + \beta_a S^3 e^{-y} - 6\beta_b G_r^2 e^{-3y} - \frac{l_2 e^{-y}}{P_r} + 2\gamma_b G_r^3 e^{-3y} - l_1 l_2 e^{-y}\right)\frac{t^3}{3!} + \left(-\frac{Se^{-y}}{P_r^2} - \frac{2l_1 Se^{-y}}{P_r} - l_1^2 Se^{-y} + \frac{e^{-y}}{P_r^2} + \frac{2l_1 e^{-y}}{P_r} + l_1^2 e^{-y} + \frac{\alpha e^{-y}}{P_r^3} + \frac{3\alpha l_1 e^{-y}}{P_r^2} - \frac{\alpha S^2 e^{-y}}{P_r} + \frac{3\alpha l_1^2 e^{-y}}{P_r} - \frac{\alpha Se^{-y}}{P_r} - \frac{\alpha l_1 Se^{-y}}{P_r} - \alpha l_1^2 Se^{-y} - \alpha l_1 S^2 e^{-y} + \alpha l_1^3 e^{-y} + 6\beta_b G_r^3 e^{-3y} - \frac{l_2 e^{-y}}{P_r} - \frac{2l_1 l_2 e^{-y}}{P_r} - l_1^2 l_2 e^{-y}\right)\frac{t^4}{4!} + \left(\frac{e^{-y}}{P_r^3} + \frac{3l_1 e^{-y}}{P_r^2} - \frac{S^2 e^{-y}}{P_r} + \frac{3l_1^2 e^{-y}}{P_r} - \frac{Se^{-y}}{P_r} - \frac{2Sl_1 e^{-y}}{P_r} - l_1^2 Se^{-y} - l_1 S^2 e^{-y} + l_1^3 e^{-y} - \frac{Se^{-y}}{P_r^3} - \frac{3l_1 Se^{-y}}{P_r^2} + \frac{s^3 e^{-y}}{P_r} - \frac{3l_1^2 Se^{-y}}{P_r} + \frac{S^2 e^{-y}}{P_r} + \frac{2l_1 S^2 e^{-y}}{P_r} + l_1^2 S^2 e^{-y} - l_1 S^3 e^{-y} - l_1^3 Se^{-y} - \frac{l_2 e^{-y}}{P_r^3} - \frac{3l_1 l_2 e^{-y}}{P_r^2} + \frac{l_2 S^2 e^{-y}}{P_r} - \frac{l_2 Se^{-y}}{P_r} + \frac{2l_1 l_2 Se^{-y}}{P_r} + l_1^2 l_2 Se^{-y} + l_1 l_2 S^2 e^{-y} - l_1^3 l_2 e^{-y}\right)\frac{t^5}{5!} \quad (41)$$

Therefore, the solution to equation (16) is;

$$u(y,t) = u_0(y,t) + u_1(y,t) + \cdots \quad (42)$$

Where, $u_0(y,t)$ and $u_1(y,t)$ are defined in equations (40) and (41) respectively.

The physical momentum, heat and mass properties such as skin friction C_f and Nusselt number N_u are given as follows

$$\begin{cases} C_f = \left(\frac{\partial u}{\partial y}\right)_{y=0} \\ N_u = \left(\frac{\partial \theta}{\partial y}\right)_{y=0} \end{cases} \quad (43)$$

RESULTS AND DISCUSSION

Theoretical work on unsteady MHD flow of fourth-grade fluid in horizontal parallel plates channel with thermal radiation, chemical reaction and suction effects has been analyzed. The impact of thermal radiation, chemical reaction, suction, third and fourth-grade parameters along with other pertinent flow parameters are plotted graphically on different flow fields. The default values for the pertinent flow parameters are taken as Arifuzzaman (2018), $\lambda = 0.30, \alpha = 0.20, \beta_a = 0.05, \beta_b = 0.05, \gamma_a = 0.05, \gamma_b = 0.05, S_c = 0.50, G_r = 5, G_c = 5, P_r = 0.71, Ha = 0.30, \delta = 0.05, Da = 1.00, K_r = 0.50$.

To validate the present work; when $G_r = 0$ and $S = 0$, then our results would be in agreement with Zaman et al. (2014). The impression of system parameters on skin friction C_f and Nusselt number N_u are also investigated and presented in table 1 below.

Table 1: Computational values of Skin friction C_f and Nusselt number N_u

S	δ	Ha	P_r	G_r	C_f	N_u
0.10	0.05	0.30	0.71	4.00	8.5875	0.9664
0.15					8.5596	0.9657
0.20	0.10				8.5177	0.9649
0.30	0.20				8.5058	0.9538
	0.25	0.60			8.5659	0.9482
		0.80			8.5559	1.0042
		1.0	1.00		8.5557	1.1447
			1.50		8.5551	1.2023
			2.00	1.00	8.5567	1.2115
				2.00	8.5579	1.3001
				3.00	8.5699	1.3111

Table 1 presented the effect of flow parameters on skin friction C_f and Nusselt number N_u. It is seen that the skin friction develops due to the increase in thermal radiation parameter δ, Grashof

Science World Journal Vol. 16(No 4) 2021
www.scienceworldjournal.org
ISSN: 1597-6343 (Online), ISSN: 2756-391X (Print)
Published by Faculty of Science, Kaduna State University

number G_r and Prandtl number P_r but diminish due to the increase in suction parameter S and Hartmann number Ha. The Nusselt number increases with the increase in Prandtl number P_r and radiation parameter δ. But it decreases with the increase in suction parameter S .

Figs. 2 and 3 depict the velocity and temperature fields for increment of thermal radiation parameter δ $(0.05 \leq \delta \geq 0.40)$. Thermal radiation is known as electromagnetic radiation or the conversion of thermal energy which generates the thermal motion of particles in matter. Thermal radiation could be attributed due to thermal excitation. Both velocity and temperature fields are affected significantly with increase in thermal radiation parameter (δ). Thermal radiation for a medium which contains it inevitably has pressure and density gradients, and the treatment requires the use of hydrodynamics.

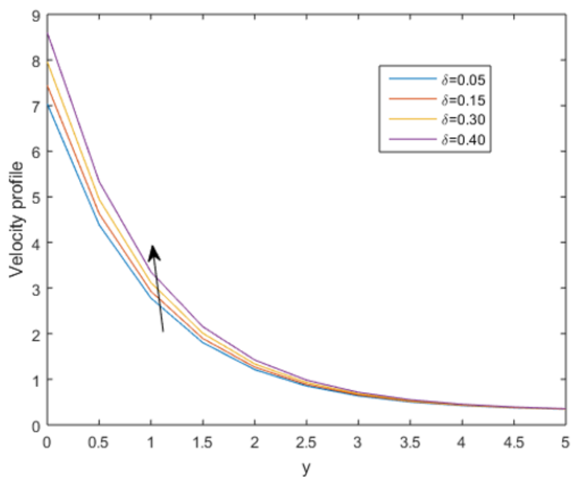

Figure 2: Effect of Thermal radiation parameter δ on Velocity profile u with $Gr = 4, Pr = 0.71, S = 0.10, Ha = 1, \lambda = 0.3, \alpha = 0.20, \beta_a = 0.05, \beta_b = 0.05, \ \gamma_a = 0.05, \gamma_b = 0.05\ Q_0 = 1.0$ and $t = 0.5$

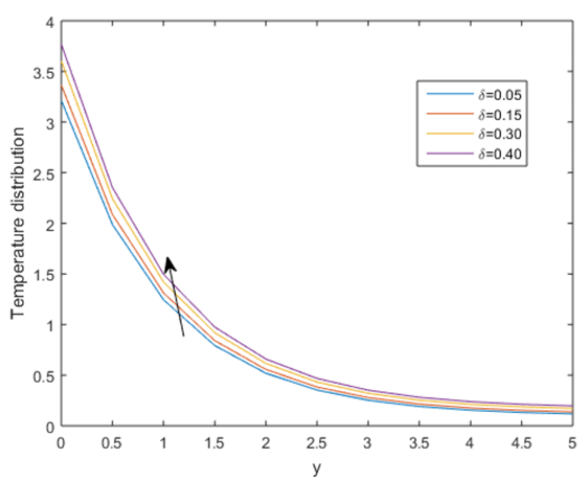

Figure 3: Effect of Thermal radiation parameter δ on Temperature distribution θ with $P_r = 0.71, S = 0.10, Q_0 = 1.0$ and $t = 0.5$

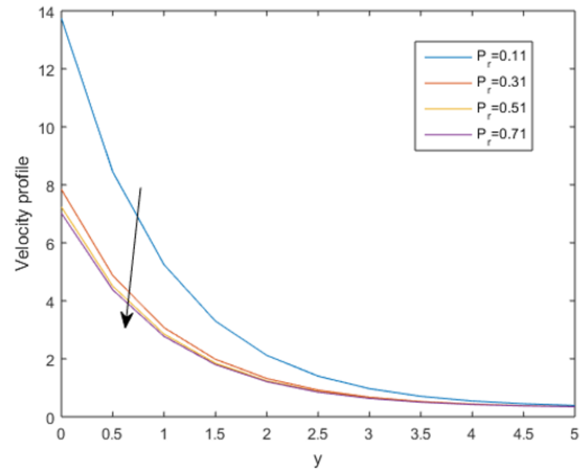

Figure 4: Effect of Prandtl number P_r on Velocity profile u with $G_r = 4, S = 0.10, \delta = 0.05, Ha = 1, \lambda = 0.3, \alpha = 0.20, \beta_a = 0.05, \beta_b = 0.05, \ \gamma_a = 0.05, \gamma_b = 0.05\ Q_0 = 1.0$ and $t = 0.5$

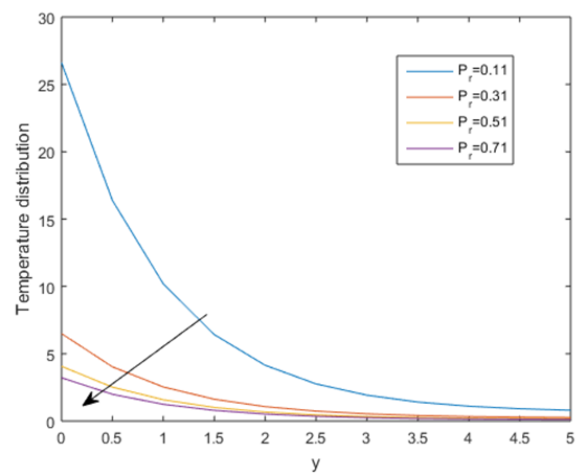

Figure 5: Effect of Prandtl number P_r on Temperature distribution θ with $S = 0.10, \delta = 0.05, Q_0 = 1.0$ and $t = 0.5$

Figure 6 illustrates the drag force effect on fluid flow. The velocity profile decreases with the increment of Hartmann number $(1.0 \leq Ha \geq 4.0)$. The role of Hartmann number which is the magnetic parameter is to suppress turbulence. Physically, when magnetic field is applied to any fluid, the apparent viscosity of the fluid increases to the point of becoming viscous elastic solid. It is of great interest that yield stress of the fluid can be controlled very accurately through variation of the magnetic field intensity. The result is that the ability of the fluid to transmit force can be controlled with help of electromagnet which give rise to many possible control – based applications, including MHD power generation, electromagnetic casting of metals, MHD propulsion etc.

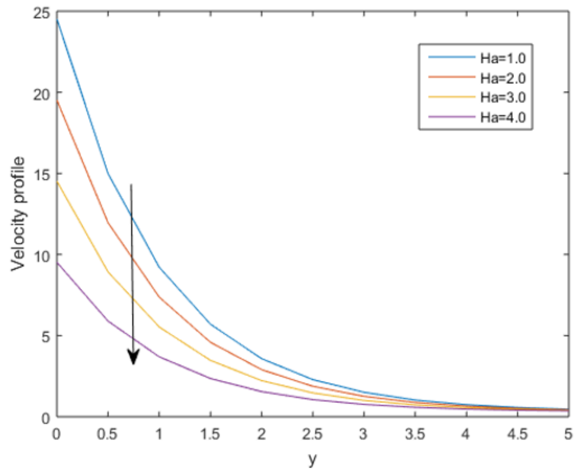

Figure 6: Effect of Hartman number Ha on Velocity profile u with $P_r = 0.71, S = 0.10, \delta = 0.05, G_r = 4, \lambda = 0.3, \alpha = 0.20, \beta_a = 0.05, \beta_b = 0.05,\ \gamma_a = 0.05, \gamma_b = 0.05\ Q_0 = 1.0$ and $t = 0.5$

Grashof number which is the dimensionless quantity with heat transfer that approximates the ratio of buoyancy to viscous force acting on a fluid. The effect of this dimensionless parameter is depicted in figure 7. It is observed that as Grashof number G_r increases $(1.0 \leq G_r \geq 4.0)$, the velocity profile u increases. To this effect, at higher Grashof number G_r the flow at the boundary is turbulent while at lower G_r the flow at the boundary is laminar.

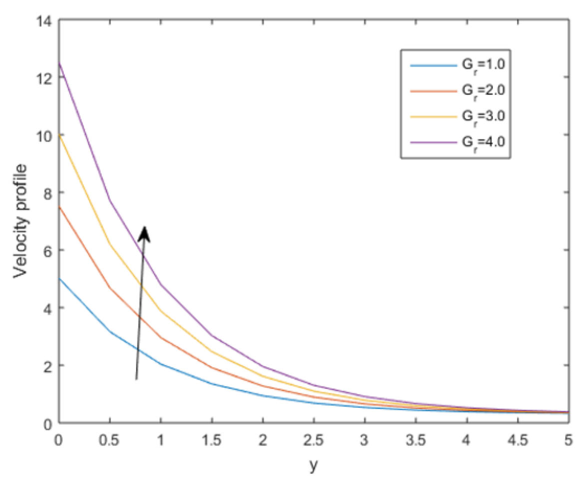

Figure 7: Effect of Grashof number G_r on Velocity profile u with $P_r = 0.71, S = 0.10, \delta = 0.05, Ha = 1, \lambda = 0.3, \alpha = 0.20, \beta_a = 0.05, \beta_b = 0.05,\ \gamma_a = 0.05, \gamma_b = 0.05\ Q_0 = 1.0$ and $t = 0.5$

The impact of suction parameter S on velocity, temperature and concentration profiles are depicted in figures 8 and 9 respectively. It is clearly seen that velocity profile and temperature distribution diminish with the increase $(0.10 \leq S \geq 0.40)$ of S. This is due to the porosity of plates.

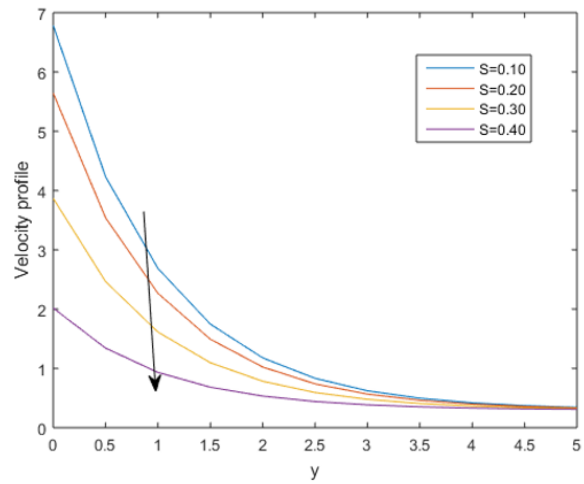

Figure 8: Effect of Suction parameter S on Velocity profile u with $Gr = 4, Pr = 0.71, \delta = 0.05, Ha = 1, \lambda = 0.3, \alpha = 0.20, \beta_a = 0.05, \beta_b = 0.05,\ \gamma_a = 0.05, \gamma_b = 0.05\ Q_0 = 1.0$ and $t = 0.5$

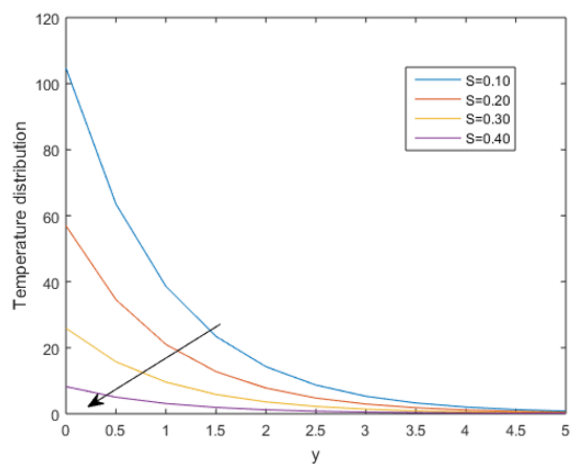

Figure 9: Effect of Suction parameter S on Temperature distribution θ with $P_r = 0.71, \delta = 0.05, Q_0 = 1.0$ and $t = 0.5$

Conclusion

Heat transfer on unsteady MHD flow of fourth-grade in a horizontal infinite parallel plates with suction effects have been investigated. The solution for the nonlinear partial differential equations are obtained by He-Laplace scheme. The effects of flow parameters on velocity profile and temperature distribution depicted in figures and discussed. From the results obtained, the findings are:

(i) Velocity profile and temperature distribution rise due to the increment of thermal radiation parameter.

(ii) Velocity profile and temperature distribution diminish due to the increment of suction parameter.

(iii) Strong values of Hartman number suppresses the turbulence, hence, decrease the velocity of the flow.

(iv) Higher values of Grashof number accelerates the velocity of the flow.

(v) Nusselt number distribution rise due to the enhancement in thermal radiation parameter and drop due to the increase in suction parameter.

(vi) Strong values of thermal radiation parameter, Prandtl number increase the skin friction while higher values of suction parameters diminish the skin friction.

Nomenclature

B_0 – external magnetic field
T – temperature of the fluid
q_r – radiative heat flux
u – fluid velocity
C_f – skin friction
S – suction parameter
N_u – Nusselt number
Ha – Hartmann number
P_r – Prandtl number
G_r – Grashof number due to heat transfer
T_w – temperature at the surface
T_∞ – ambient temperature as $y \to \infty$
x, y – cartesian coordinates

Greek Symbols

μ – coefficient of shear viscosity
α – second grade parameter
β_a, β_b – third grade parameters
γ_a, γ_b – fourth grade parameters
β – thermal expansion coefficient
δ – thermal radiation parameter
σ – Stefan – Boltzmann constant
ρ – density of the fluid
ν – kinematic viscosity

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