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EFFECT OF ELECTRIC FIELD FLOW ON NANOFLUID OVER STRETCHABLE SURFACE

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ABSTRACT
This work explores numerically the influence of the electric field on nanofluid over a stretching sheet surface. The ordinary differential equation (ODE's) are obtained from the partial differential equations (PDE's) employing the transformation technique. Hence the transformed model are computed with the respective conditions using Keller Box method. The functions of the different parameter values on the flow field profiles are graphically presented and analysed. The current result is in good agreement with previous research studied.

Keywords: Electric field, Nanofluid, Stretching sheet, flow.

INTRODUCTION
Features of magnetic and electric field impacts is felt significantly in areas of engineering, medicine and physics due to various functionalities (Kandasamy et al., 2018; Kasaeian et al., 2017; Daniel et al., 2017a and Sheikholeslami & Shehzad, 2017). A lot of industrial equipment for such as bearings, MHD generators, pumps, and boundary layer are governing by the interaction existing with electrically conducting fluid and a magnetic intensity (Daniel et al., 2017b; Sheikholeslami & Rokni, 2017 and Daniel et al., 2019a). Additionally, it is noticed based on experiment that magnetohydrodynamic flow becomes more significant with the heat transfer processes. Therefore, good attempts have been made to examine the influence of the thermal characteristics and magnetic field (Armaghani et al., 2018; Daniel et al., 2019b; Daniel 2015; Waqas et al., 2017, Daniel and Daniel, 2015; and Hayat et al., 2018a).
Furthermore, the physiognomies of the flow firmly subjected on the orientation and intensity of the applied magnetic field (Daniel et al., 2017c). This applied magnetic field tends to controls the suspended nanoparticles and adjusts their concentration in the ordinary fluid which strongly alters the behaviour of heat transfer in the flow (Choi ad Eastman, 1995; Daniel et al. 2017d; Hayat et al. 2018b & Hayat et al., 2019). The magnetic nanomaterial has both the magnetic and fluid properties (Waqas et al., 2019). These nanomaterials have several importance in magneto-optical switches and wavelength filters, besides optical modulators and gratings (Rehman et al., 2019, Daniel, 2016 and Daniel et al., 2018). Additionally, such materials have significance in cancer treatment, tumor analysis, customizing loudspeakers, drug delivery in the body system and sink-float separation. Heat transfer of the magnetic nanoparticles are likewise tunable by giving the vicissitudes in the magnetic field strength (Bhattacharyya et al., 2013, and Hayat et al., 2014).
The present investigation looks at the linear stretched flow of electrical MHD nanofluid with Joule heating using Buongiorno model (Buongiorno 2006). Flow caused is due to a linear stretched sheet and electric field. Numerical solutions are achieved by utilizing implicit finite difference scheme (Cebeci and Bradshaw, 1988). Numerical values are presented to demonstrate the accuracy and efficiency of the solutions (Ibrahim and Shankar, 2013). Moreover graphical results are demonstrated and analyzed.

MATHEMATICAL FORMULATION
Consider a 2D steady electrical magnetohydrodynamic (EMHD) nanofluid over a stretching sheet. The velocity of the stretching sheet is denoted as $U_W = cx$. The boundary layer equations of the fluid flow are composition of the continuity equation, the momentum equation, energy equation and concentration equation, which are formulated based on Maxwell's equation and Ohm's law. The incompressible flow of viscous fluid in the presence of an applied magnetic field $B(x)$ and electric field $E(x)$ are taken into consideration. The flow is due to stretching of sheet from a slot through two equal and opposite force and thermally radiative. The magnetic and electric fields obey the Ohm's law define $\bar{J} = \sigma(\bar{E} + \bar{V} \times \bar{B})$ where $\bar{J}$ is the Joule current, σ is the electrical conductivity and $\bar{V}$ represent the fluid velocity. T, φ is the fluid temperature and concentration, the ambient values of temperature and nanoparticle fraction attained to constant value of T_∞ and φ_∞ . Magnetic field and electric field of strength are applied normal to the flow, such that the Reynolds number is selected small. The induced magnetic field is smaller to the applied magnetic field. Hence the induced magnetic field is absence for small magnetic Reynolds number. We choose the Cartesian coordinate system such that x is chosen along the stretching sheet and y axis denotes the normal to the stretching sheet, u and v are the velocity components of the fluid in the x and y direction. The boundary layer flow equation of an incompressible nanofluid is given as:
Continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

x –direction momentum equation

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho_f}\frac{\partial P}{\partial x} + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) + \frac{\sigma}{\rho_f}(E(x)B(x) - B^2(x)u) \quad (2)$$

y –direction momentum equation

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho_f}\frac{\partial P}{\partial y} + \nu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) + \frac{\sigma}{\rho_f}(E(x)B(x) - B^2(x)v) \quad (3)$$

The energy field for temperature can be expressed in terms of Joule heating terms due to magnetic field as:

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y} = \frac{k}{\rho c_p}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right) + \tau\left\{D_B\left(\frac{\partial \varphi}{\partial x}\frac{\partial T}{\partial x}+\frac{\partial \varphi}{\partial y}\frac{\partial T}{\partial y}\right)+\frac{D_T}{T_\infty}\left[\left(\frac{\partial T}{\partial x}\right)^2+\left(\frac{\partial T}{\partial y}\right)^2\right]\right\} + \frac{\sigma}{\rho c_p}(uB(x) - E(x))^2 \quad (4)$$

Concentration equation

$$u\frac{\partial \varphi}{\partial y}+v\frac{\partial \varphi}{\partial y} = D_B\left(\frac{\partial^2 \varphi}{\partial x^2}+\frac{\partial^2 \varphi}{\partial y^2}\right) + \frac{D_T}{T_\infty}\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) \quad (5)$$

The magnetic field factor $B(x) = B_0, \sigma$ is the electrical conductivity, $E(x) = E_0$ is the electrical field factor, ν, ρ_f are the kinematic viscosity of the fluid and the fluid density. $k/\rho c_p, \mu, \sigma *, \rho\ \rho_f$, and ρ_p is the thermal diffusivity, the kinematic viscosity, the Steffan-Boltzman constant, the density, the fluid density and particles density respectively. We also have B_0, D_B, D_T, $\tau = (\rho c)_p/(\rho c)_f$ which represents magnetic field, the Brownian diffusion coefficient, the thermophoresis diffusion coefficient, the ratio between the effective heat transfer capacity of the ultrafine nanoparticle material and the heat capacity of the fluid Follow with the boundary conditions:

$$u = U_W(x), \quad v = 0, \quad T = T_\infty, \quad \varphi = \varphi_W \quad (6)$$

$$y \to \infty: \quad u \to 0, \quad T \longrightarrow T_\infty, \quad \varphi \longrightarrow \varphi_\infty \quad (7)$$

To obtain similarity solution of equations (1)-(5) the nondimensionalize variables are presented as:

$$\theta(\eta) = (T - T_\infty)/(T_W - T_\infty), \quad \phi(\eta) = (\varphi - \varphi_\infty)/(\varphi_W - \varphi_\infty) \quad (8)$$

Using an order magnitude analysis for the system of equations and boundary layer approximation in (1)-(5). The set of equations are transformed into:

$$f'''(\eta) + f(\eta)f''(\eta) - \left(f'(\eta)\right)^2 + M\left(E_1 - f'(\eta)\right) = 0 \quad (10)$$

$$\theta'' + Pr(f\theta' + Nb\phi'\theta' + Nt\theta'^2 + M(f' - E_1)^2) = 0 \quad (11)$$

$$\phi'' + \frac{Nt}{Nb}\theta'' + Lef\phi' = 0 \quad (12)$$

Boundary conditions

$$\eta = 0: \quad f(\eta) = 0, \quad f'(\eta) = 1, \quad \theta(\eta) = 1, \quad \phi(\eta) = 1$$

$$\eta \to \infty; \quad f'(\eta) = 0, \quad \theta(\eta) = 0, \ \phi(\eta) = 0 \quad (13)$$

where M is the magnetic field parameter, E_1 is the electric parameter. Pr is the Prandtl number, Nb is the Brownian motion parameter, Le is the Lewis number, and Nt is the thermophoresis parameter.

RESULTS AND DISCUSSION

The validation of present numerical scheme, the results are presented and examined with (Ibrahim and Shankar, 2013) in some limiting case when $Le = 20, Nb = Nt = 0.5$, and $Pr = 1.0$. The numerical values are in good agreement as displayed from Tables 1 presents the effects of emerging parameters on the skin friction coefficient.

Influence of electric field parameter on the velocity $f'(\eta)$ is depicted in Fig.1. It is noticed that the velocity profile of the nanofluid significantly enhanced with an increase in the values of E_1. An increase in the parameter result to an increase in dimensionless velocity field and the momentum boundary layer thickness. Why because the electric field introduces accelerating body force which acts to the direction of the applied electric field. This body force, known as the Lorentz force, accelerates the boundary layer flow and thickens the momentum boundary layer. Hence it resulted to decrease in the skin friction at the linear stretching sheet surface.

In Fig. 2, displayed the impact of electric field parameter E_1 on the temperature profile $\theta(\eta)$. It is observed that the temperature profile decreases with an increase in the values of E_1. Variation of electric field parameter E_1, the energy field decreases near the linear stretching sheet. Why because the electric field which behave as accelerating body force, decelerated the amount of energy due to flow of nanofluid distance away from the wall for large η. The thickness of the thermal boundary layer becomes thinner for higher values of the parameter and the profile decreases monotonically. The Nusselt number increase at the surface of the linear stretching sheet for increase in the values of the electric field parameter.

Effect of electric field is displayed in (Fig. 3) on the concentration profile $\phi(\eta)$. It decrease distance away from the wall significantly alone the stretching sheet. The electric field boost the fluid flow which tends to reduce the total amount of nanoparticles concentration and solutal boundary layer after some distance for large η. The rate of mass transfer increased with higher values of electric field.

Table 1: Comparison of Skin friction coefficient $-f''(0)$ for various values of $M$and E_1.

M	E_1	(Ibrahim and Shankar, 2013)	Present results
0	0	1.2808	1.280777
0.5		1.5000	1.500000
1.0		1.6861	1.686141
1.5		1.8508	1.850781
2.0		2.0000	2.000000
1.0		1.4142	1.414214
	0.1	-	1.335083
	0.3	-	1.003660
	0.5	-	0.698797
	0.1	-	1.400699
		-	1.547543
		-	1.774626

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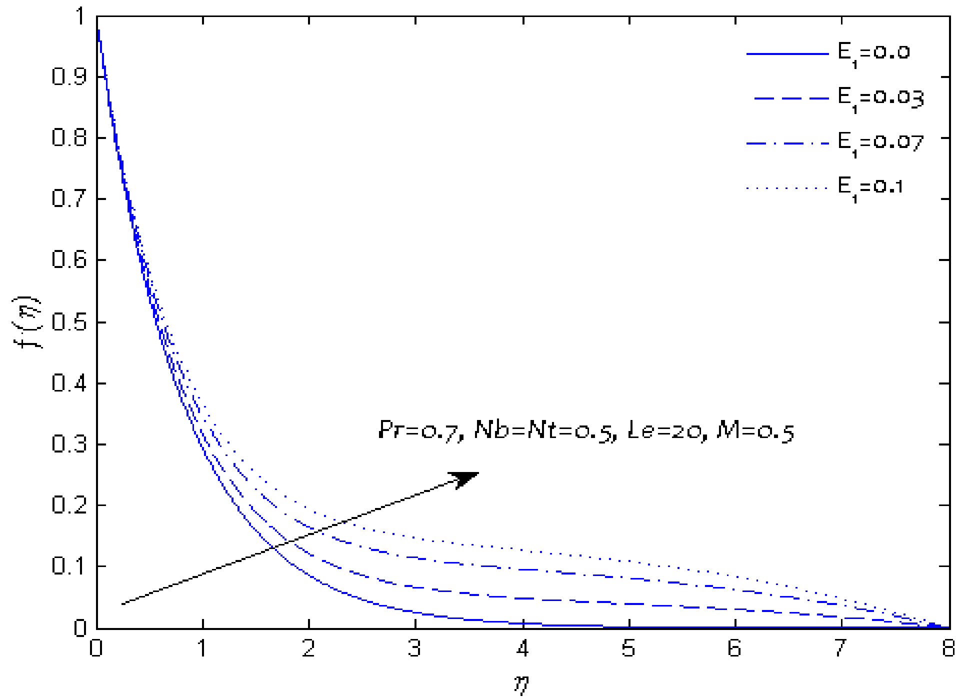

Fig.1 Influence of E_1 on the velocity profile $f'(\eta)$

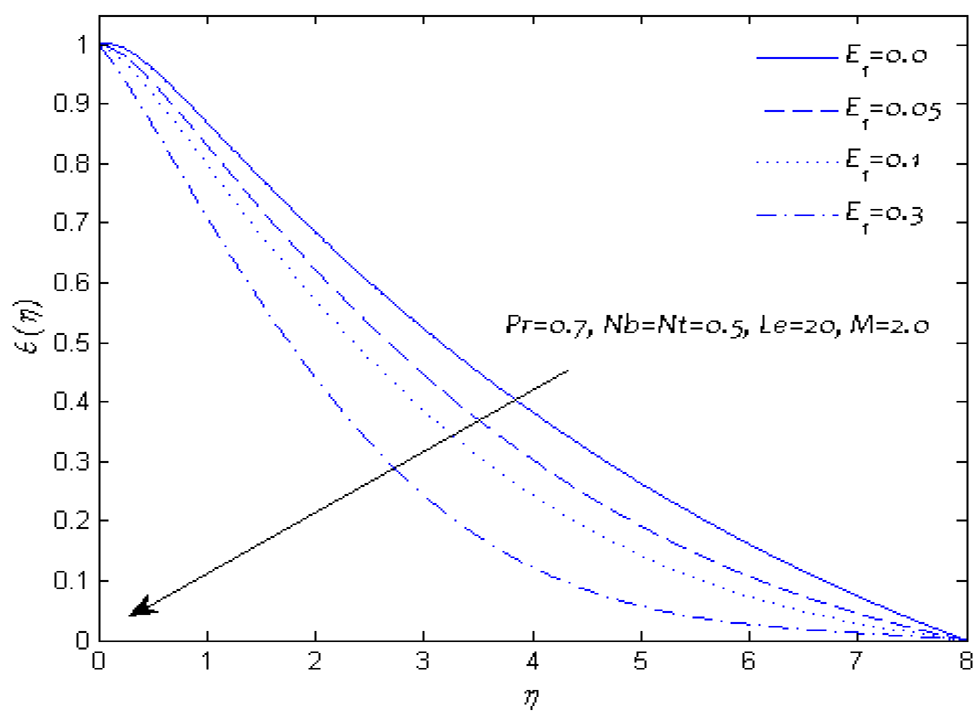

Fig.2 Influence of E_1 on the Temperature profile $\theta(\eta)$

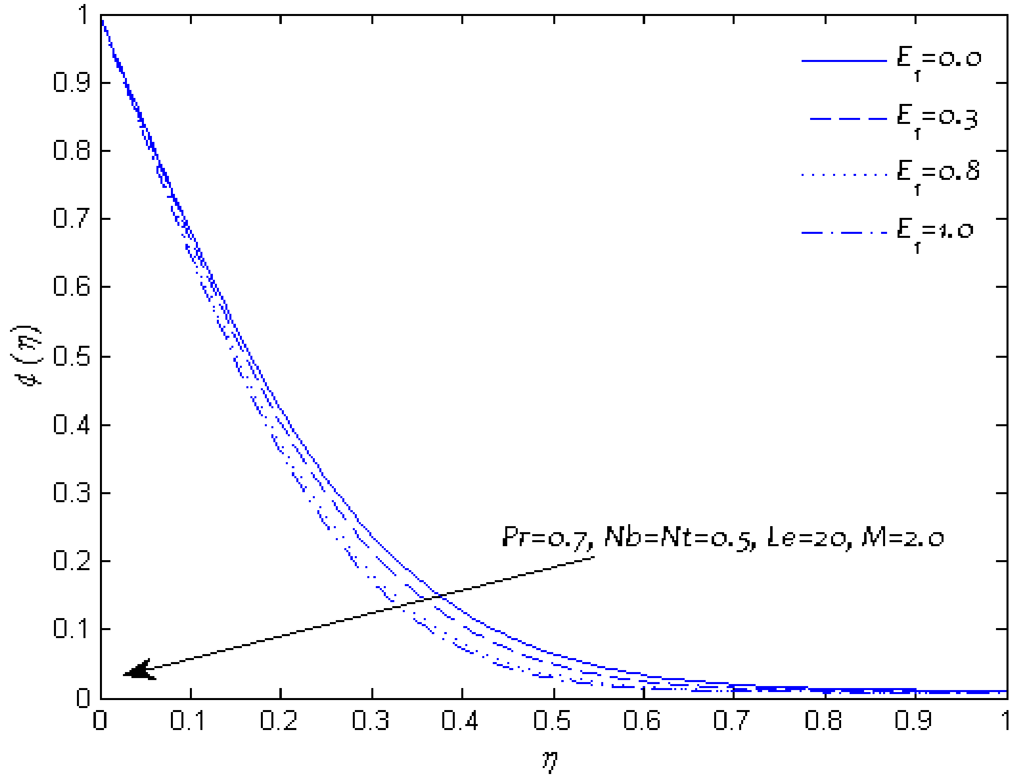

Fig.3 Influence of E_1 on the Concentration profile $\phi(\eta)$

Conclusions

We have studied the influence of electric field on boundary layer flow in MHD nanofluid due to linear stretching sheet. A numerical solution has been employed to study steady state two dimension boundary layer flow for heat and mass transfer due to linear stretching sheet in an electrical conductivity of nanofluid. The effects of various emerging governing parameters on the heat and mass transfer characteristics were examined. The Velocity profile increase distance along η for higher values of electric field parameter. Whereas, temperature and concentration fields decreased by increasing the values of electric field.

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