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INVESTIGATION OF A ONE-STEP HYBRID ALGORITHM TOWARDS THE SOLUTION OF FIRST ORDER LINEAR AND NONLINEAR INITIAL-VALUE PROBLEMS OF ORDINARY DIFFERENTIAL EQUATIONS (FOLNIVP)

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ABSTRACT

The aim of developing any numerical method is to complement the challenges inherent in obtaining the analytical solution of a differential equation, if at all a closed form solution does exist. In this study we present a one-step implicit code of order eight block algorithms for the purpose of utilizing data at points other than a whole step number. The major advantage of hybrid method is that they possess remarkably small error constants which translate to better approximation. These methods constitute a class of methods whose computational potentialities have probably not yet been fully exploited. Therefore, the performance of the derived block hybrid algorithm is investigated using some numerical examples for the purpose of demonstrating its validity and applicability. The results obtained revealed that the algorithm is suitable for solving first order linear and nonlinear initial value problems (IVPs) of ordinary differential equations.

Keywords: First order, Hybrid Algorithm, shifted Legendre polynomials, approximation.

INTRODUCTION

The desire of arriving at accurate approximate solution to any mathematical models in the form of ordinary differential equations arising from science, engineering and even social sciences cannot be ignored. Hence, this study seeks to consider the numerical solution of First Order Initial Value Problems (FOIVP) of the form;

$$\dot{x}(t) = f\big(t, x(t)\big), \qquad x(t_0) = x_0, \qquad a \le t \le b \qquad (1.0)$$

where t_0 is the initial point, $f(t,x)$ is a function which is continuously differentiable within the interval of integration $[a,\ b]$ and $x(t)$ is an unknown function that is being sought for within a set of equally spaced points on the integration interval denoted by

$$a = t_0 < t_1 < \cdots < t_{n-1} < t_n < \cdots < t_{n+k} < t_N = \ b \qquad (2.0)$$

with a specified positive integer step number k greater than zero, h is a constant step-size given by

$$h = \frac{b-a}{N} \ \text{ or } \qquad h = t_{n+1} - t_n,\ \ n = 0,1,2,\dots,N$$

Several scholars have proffered solutions to (1.0) using different techniques, methods and approaches and have established that numerical solution using block method is better than the predictor corrector method in the sense that the predictor-corrector method are very costly as the subroutines involved are very complicated to write because of the special techniques required to supply starting values and also varying the step size leads to longer computer time and more human effort. For example, based on the above mentioned setbacks, the block method which simultaneously generates approximate solution at any given point in the interval of integration and which does not require any starting value is favored by Lambert (1973), Badmus and Yahaya (2009), Jator (2008), Owolabi (2015), Sagir (2014), Fatunla (1991), Yap, *et al.,* (2014), Odejide and Adeniran (2012) and Olabode (2009) be the most admired and preferred methods for the solution of (1.0).

Apart from the advantages of the block method outlined above, several superior arguments have emerged as to why the use of hybrid method as against the conventional step numbers is advanced. To justify the use of hybrid methods, Kamoh, *et al.,* (2017); revealed that the methods performed far much better than the conventional step numbers. One-sixth hybrid block method was derived by Rufai, *et al.,* (2016**)**; comparison of the results showed that the hybrid method performed much better than the conventional step number. A two-step, two-point hybrid method for the direct solution of general second order differential equations was developed by Abdul Majid, *et al.,* (2012); the results showed a better performance over the conventional whole step points methods. A five-step ninth order hybrid linear multistep method with three non-step points for the solution of first order IVPs in block form as simultaneous numerical integrators over non-overlapping intervals was constructed Majid, *et al.,*(2006) and the results showed some remarkable improvement over some existing schemes. Researchers including Lambert, (1973), Badmus and Yahaya (2009), Jator (2008), Kamoh, *et al.,* (2017), Mohammmed *et al.,* (2010), Olabode and Yusuph (2009), Okunuga and Ehigie (2009), Owolabi (2015), Sagir (2014), Fatunla (1991), Yap, *et al.,* (2014), Olabode (2009) and Awoyemi, *et al.,* (2014), Nor Ain *et al.,* (2012) have discussed the advantages of the hybrid block methods and have since suggested the outright replacement of the predictor-corrector methods and the conventional whole step point methods.

MATERIALS AND METHODS

Derivation of the Method

In this section, we desire to construct an algorithm for the solution of (1.0). Assuming that the solution to (1.0) is approximated by the shifted Legendre polynomial of the form:

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$$\bar{x}(t) = \sum_{i=0}^{m+s-1} \delta_i \mathcal{H}_i(t), \quad t_n \le t \le t_{n+1} \qquad (3.0)$$

where $\mathcal{H}_i(t)$ is the shifted Legendre polynomial of degree i, valid in the interval $t_n \le t \le t_{n+1}$ and $\delta_i{}'s$ are real parameters to be determined and $m+s$ is the sum of the collocation and interpolation points respectively.

The well-known shifted Legendre polynomial can be determined in the interval [0, A] using the recurrence relation:

$$\mathcal{H}_i(t) = \sum_{k=0}^{i} (-1)^{(i+k)} \frac{(i+k)!\, t^k}{(i-k)!\,(k!)^2 A^k} \qquad (4.0)$$

for any positive integer A. Our main goal is to construct a continuous formulation of the general linear multistep method $\bar{x}(t)$ of degree $i = m+s-1$.
The first derivative of (3.0) is substituted into (1.0) to obtain a differential system of the form

$$\dot{x}(t) = \sum_{i=0}^{m+s-1} \delta_i \dot{\mathcal{H}}_i(t), t_n \le t \le t_{n+1} \qquad (5.0)$$

Equation (5.0) is evaluated at the collocation points $t_{n+r}, r = 0, \frac{1}{6}, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \frac{5}{6}, k$ and equation (3.0) is evaluated at the interpolation points t_n and $t_{n+\frac{5}{6}}$ respectively to give a system of nonlinear algebraic equations of the form;

$$\sum_{i=0}^{8} \delta_i \mathcal{H}_i(t) = y_{n+t}, t = 0, \frac{5}{6} \quad and \sum_{i=0}^{8} \delta_i \dot{\mathcal{H}}_i(t) = y_{n+t}, t = 0, \frac{1}{6}, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \frac{5}{6}, k \qquad (6.0)$$

Solving for $\delta_i{}'s$, $i = 0(1)8$ in (6.0) and then substituting back into (3.0) gives the continuous scheme

$$x(t) = \sum_{j=0,\ \frac{5}{6}}^{\frac{5}{6}} \alpha_j(t) x_{n+j} + h \sum_{j=0}^{1} \beta_j(t) f_{n+j} \qquad (7.0)$$

where the continuous coefficients $\alpha(t)$ and $\beta(t)$ are given as;

$$\alpha_0(t) = 1 + \frac{559872}{983125h^8} - \frac{62208}{275h^2}t^2 + \frac{3048192}{1375h^3}t^3 - \frac{12628224}{1375h^4}t^4 + \frac{27433728}{1375h^5}t^5 + \frac{564350976}{983125h^8}t^2 - \frac{752467968}{196625h^8}t^3 - \frac{1306368}{55h^6}t^6 + \frac{47029248}{3575h^8}t^4 - \frac{2257403904}{89375h^8}t^5 + \frac{20155392}{1375h^7}t^7 + \frac{188116992}{6875h^8}t^6 - \frac{107495424}{6875h^8}t^7 - \frac{35831808}{983125h^8}t$$

$$\alpha_{\frac{5}{6}}(t) = \frac{62208}{(275h^2}t^2 - \frac{3048192}{1375h^3}t^3 + \frac{12628224}{1375h^4}t^4 - \frac{27433728}{1375h^5}t^5 - \frac{564350976}{983125h^8}t^2 + \frac{752467968}{196625h^8}t^3 + \frac{1306368}{55h^6}t^6 - \frac{47029248}{3575h^8}t^4 + \frac{2257403904}{89375h^8}t^5 - \frac{20155392}{1375h^7}t^7 - \frac{188116992}{6875h^8}t^6 + \frac{107495424}{6875h^8}t^7 - \frac{559872}{983125h^8})/(983125h^8)) + \frac{35831808}{983125h^8}t$$

$$\beta_0(t) = t + \frac{40122}{1376375h^7} - \frac{29151}{1540h}t^2 + \frac{115948}{825h^2}t^3 - \frac{1155531}{2200h^3}t^4 + \frac{298179}{275h^4}t^5 + \frac{5777568}{196625h^7}t^2 - \frac{7703424}{39325h^7}t^3 - \frac{68949}{55h^5}t^6 + \frac{481464}{715h^7}t^4 - \frac{23110272}{17875h^7}t^5 + \frac{1462212}{1925h^6}t^7 + \frac{1925856}{1375h^7}t^6 - \frac{7703424}{9625h^7}t^7 - \frac{2567808}{1376375h^7}t$$

$$\beta_{\frac{1}{6}}(t) = \frac{37584}{275275h^7} - \frac{2405376}{275275h^7}t - \frac{2790}{77h}t^2 + \frac{4698}{11h^2}t^3 - \frac{106749}{55h^3}t^4 + \frac{244674}{55h^4}t^5 + \frac{5412096}{39325h^7}t^2 - \frac{7216128}{7865h^7}t^3 - \frac{60264}{11h^5}t^6 + \frac{451008}{143h^7}t^4 - \frac{21648384}{3575h^7}t^5 + \frac{266328}{77h^6}t^7 + \frac{1804032}{275h^7}t^6 - \frac{7216128}{1925h^7}t^7$$

$$\beta_{\frac{1}{3}}(t) = \frac{2754}{55055h^7} - \frac{176256}{55055h^7}t - \frac{6525}{154h}t^2 + \frac{8145}{22h^2}t^3 - \frac{116631}{88h^3}t^4 + \frac{137079}{55h^4}t^5 + \frac{396576}{7865h^7}t^2 - \frac{528768}{1573h^7}t^3 - \frac{28593}{11h^5}t^6 + \frac{165240}{143h^7}t^4 - \frac{1586304}{715h^7}t^5 + \frac{109836}{77h^6}t^7 + \frac{132192}{55h^7}t^6 - \frac{528768}{385h^7}t^7$$

$$\beta_{\frac{1}{2}}(t) = \frac{6912}{55055h^7} - \frac{442368}{55055h^7}t - \frac{2300}{77h}t^2 + \frac{10540}{33h^2}t^3 - \frac{16134}{11h^3}t^4 + \frac{194004}{55h^4}t^5 + \frac{995328}{7865h^7}t^2 - \frac{1327104}{1573h^7}t^3 - \frac{50472}{11h^5}t^6 + \frac{414720}{143h^7}t^4 - \frac{3981312}{715h^7}t^5 + \frac{234576}{77h^6}t^7 + \frac{331776}{55h^7}t^6 - \frac{1327104}{385h^7}t^7$$

$$\beta_{\frac{2}{3}}(t) = \frac{191484}{77h^6}t^7 + \frac{241056}{55h^7}t^6 - \frac{964224}{385h^7}t^7 + \frac{5022}{55055h^7} - \frac{321408}{55055h^7}t - \frac{14625}{308h}t^2 + \frac{4995}{11h^2}t^3 - \frac{159849}{88h^3}t^4 + \frac{207549}{55h^4}t^5 - \frac{964224}{1573h^7}t^3 + \frac{723168}{7865h^7}t^2 - \frac{46899}{11h^5}t^6 + \frac{301320}{143h^7}t^4 - \frac{2892672}{715h^7}t^5$$

$$\beta_{\frac{5}{6}}(t) = \frac{369954}{275h^4}t^5 + \frac{8771328}{196625h^7}t^2 - \frac{11695104}{39325h^7}t^3 - \frac{92016}{55h^5}t^6 + \frac{730944}{715h^7}t^4 - \frac{35085312}{17875h^7}t^5 + \frac{2085912}{1925h^6}t^7 + \frac{2923776}{1375h^7}t^6 - \frac{11695104}{9625h^7}t^7 + \frac{38466}{275h^2}t^3 - \frac{3898368}{1376375h^7}t - \frac{164097}{275h^3}t^4 - \frac{5382}{385h}t^2 + \frac{60912}{1376375h^7}$$

$$\beta_1(t) = \frac{5}{14h}t^2 - \frac{23}{6h^2})t^3 + \frac{717}{40h^3}t^4 - \frac{45}{h^4}t^5 - \frac{7776}{3575h^7}t^2 + \frac{10368}{715h^7}t^3 + \frac{63}{h^5}t^6 - \frac{648}{13h^7}t^4 + \frac{31104}{325h^7}t^5 - \frac{324}{7h^6}t^7 - \frac{2592}{25h^7}t^6 + \frac{10368}{175h^7}t^7 + \frac{3456}{25025h^7}t - \frac{54}{25025h^7}$$

Substituting for $\alpha_0(t)$, $\alpha_{\frac{5}{6}}(t)$ and $\beta_i(t), i = 0, \frac{1}{6}, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \frac{5}{6}, k$ and evaluating (7.0) at $t = \frac{1}{3}h,\ \frac{1}{2}h,\ \frac{1}{4}h,\ \frac{2}{3}h$ and h with its first derivative evaluated at $t = \frac{1}{4}h$ provides the algorithms needed for solving first order linear and nonlinear initial-value problems of ordinary differential equations of type (1.0) (see appendix I).

Convergence Analysis

Order and error constant

Expanding (8.0) in Taylor's series and collecting like terms in powers of h, the order and error constant are respectively reported in table 1 below;

Table 1: Order and error constants

Algorithm number	Order	Error constant
1	$P = 8$	$C_9 = -9.1004 \times 10^{-11}$
2	$P = 8$	$C_9 = -1.0524 \times 10^{-10}$
3	$P = 8$	$C_9 = -7.7321 \times 10^{-11}$
4	$P = 8$	$C_9 = -1.2537 \times 10^{-10}$
5	$P = 8$	$C_9 = -9.6083 \times 10^{-11}$
6	$P = 8$	$C_9 = -6.1953 \times 10^{-11}$
7	$P = 8$	$C_9 = -6.379 \times 10^{-10}$

Consistency

Following Lambert (1973) and Jator (2008), the block method (8.0) is said to be consistent since $\check{p} = 8 > 1$

Zero stability

The block method (8.0) according to Badmus and Yahaya (2009), Kuboye and Omar (2015) and Olabode and Yusuph (2009), is said to be zero stable if the roots $z_r;\ r = 1, \ldots, n$ of the first characteristic polynomial $p(z)$, defined by

$$p(z) = det|zQ - T|$$

satisfies $|z_r| \leq 1$and every root with $|z_r| = 1$ has multiplicity not exceeding the order of the differential equation in the limit as $h \to 0$. From the block method, we have $z^5(z^2 - 1) = 0$ and $z = (-1,1)$, showing that the method is zero stable.

According to Lambert, (1973) and Jator (2008), the block method (8.0) is convergent since it is consistent and zero stable

RESULTS AND DISCUSSION

In this work, some standard test examples contained in literature are considered in order to test the validity and the applicability of the proposed algorithm; the derived algorithms are implemented more efficiently by combining the individual hybrid algorithm as simultaneous integrators for IVPs without requiring starting value. (See appendix II for results Tables 2-5).

Problem 1:

Consider a first order nonlinear initial value problem of ordinary differential $\dot{x} = -10(x-1)^2$, with initial condition given by $x(0) = 2,\ h = 0.01,\ t \in [0, 0.1]$, the exact solution is $x(t) = 1 + \frac{1}{1+10t}$ which was solved by Rufai, *et al.,* (2016). (See Table 2 below for the performance of the algorithm)

Table 2: Performance of the block algorithm on problem 1

t	Exact value $x(t)$	proposed value $x(t_n)$	Error in prop method $e_n = \lvert x(t) - x(t_n)\rvert$	Error in Rufai, *et al.,* (2016) $e_n = \lvert x(t) - x(t_n)\rvert$
0.0100	1.90909090909091	1.90909090909077	1.4000×10^{-13}	1.558250×10^{-6}
0.0200	1.83333333333333	1.83333333333316	1.7000×10^{-13}	2.399479×10^{-6}
0.0300	1.76923076923077	1.76923076923060	1.7000×10^{-13}	2.830452×10^{-6}
0.0400	1.71428571428571	1.71428571428556	1.5000×10^{-13}	3.020939×10^{-6}
0.0500	1.66666666666667	1.66666666666653	1.4000×10^{-13}	3.069564×10^{-6}
0.0600	1.62500000000000	1.62499999999987	1.3000×10^{-13}	3.034568×10^{-6}
0.0700	1.58823529411765	1.58823529411753	1.2000×10^{-13}	2.951147×10^{-6}
0.0800	1.55555555555556	1.55555555555545	1.1000×10^{-13}	2.840882×10^{-6}
0.0900	1.52631578947368	1.52631578947359	9.0000×10^{-14}	2.717126×10^{-6}
0.1000	1.50000000000000	1.49999999999992	8.0000×10^{-14}	2.588157×10^{-6}

Problem 2:

Consider a first order nonlinear initial value problem of ordinary differential which was solved by Majid, *et al.,*(2006) given as; $\dot{x} = -(x)^2$ with

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initial condition given as $x(0) = 1,\ h = 0.1,\ t \in [0, 1]$, the exact solution is $x(t) = \frac{1}{1+t}$. (Table 3 below shows the performance of the algorithm)

Table 3: Performance of the block algorithm on problem 2

t	Exact value $x(t)$	proposed value $x(t_n)$	Error in prop method $e_n = \|x(t) - x(t_n)\|$	Error in Majid, *et al.*, (2006) $e_n = \|x(t) - x(t_n)\|$
0.100	0.909090909090909	0.909090909090772	1.3700×10^{-13}	2.91799×10^{-11}
0.200	0.833333333333333	0.833333333333163	1.6700×10^{-13}	3.71577×10^{-11}
0.300	0.769230769230769	0.769230769230600	1.6900×10^{-13}	3.93663×10^{-11}
0.400	0.714285714285714	0.714285714285557	1.5700×10^{-13}	3.39963×10^{-11}
0.500	0.666666666666667	0.666666666666524	1.4300×10^{-13}	2.94922×10^{-11}
0.600	0.625000000000000	0.624999999999872	1.2800×10^{-13}	2.61278×10^{-11}
0.700	0.588235294117647	0.588235294117532	1.1500×10^{-13}	2.31487×10^{-11}
0.800	0.555555555555556	0.555555555555452	1.0400×10^{-13}	6.80704×10^{-6}
0.900	0.526315789473684	0.526315789473591	9.3000×10^{-14}	8.31745×10^{-6}
1.000	0.500000000000000	0.499999999999916	8.4000×10^{-14}	7.50649×10^{-6}

Problem 3:
Given a first order nonlinear initial value problem of ordinary differential solved by AbdulAzeez and Aolat (2020). $\dot{x} = tx, x(0) = 1,\ h = 0.1,$ $t \in [0,1]$ with exact solution given by $x(t) = e^{\frac{1}{2}t^2}$, the result is shown in Table 4. (See Table 4 for the performance of the algorithm)

Table 4: Performance of the block algorithm on problem 3

t	Exact value $x(t)$	proposed value $x(t_n)$	Error in prop method $e_n = \|x(t) - x(t_n)\|$ Order $p = 8$	AbdulAzeez and Aolat (2020) $e_n = \|x(t) - x(t_n)\|$ Order $p = 9$
0.100	1.00501252085940106	1.00501252085940109	3.0000×10^{-17}	4.0345×10^{-13}
0.200	1.02020134002675581	1.02020134002675594	1.3000×10^{-16}	9.2974×10^{-13}
0.300	1.04602785990871694	1.04602785990871726	3.2000×10^{-16}	1.6266×10^{-12}
0.400	1.08328706767495855	1.08328706767495913	5.8000×10^{-16}	2.6270×10^{-12}
0.500	1.13314845306682632	1.13314845306682731	9.9000×10^{-16}	4.1049×10^{-12}
0.600	1.19721736312181016	1.19721736312181178	1.6200×10^{-15}	6.3136×10^{-12}
0.700	1.27762131320488661	1.27762131320488915	2.5400×10^{-15}	9.6096×10^{-12}
0.800	1.37712776433595708	1.37712776433596096	3.8800×10^{-15}	1.4527×10^{-12}
0.900	1.49930250005676687	1.49930250005677272	5.8500×10^{-15}	2.1842×10^{-12}
1.000	1.64872127070012815	1.64872127070013693	8.7800×10^{-15}	3.2709×10^{-12}

Problem 4:
An initial value problem of a single nonlinear ODE $\dot{x} = -x - x^2$ with initial condition $x(0) = 1,\ \ h = 0.1,\ t \in [0,1]$ that has exact solution $x(t) = \frac{1}{2e^t - 1}$ (See Table 5 below for the performance of the algorithm)

Table 5: Performance of the block algorithm on problem 4

t	Exact value $x(t)$	proposed value $x(t_n)$	Error in prop method $e_n = \|x(t) - x(t_n)\|$
0.10	0.826212868242120	0.826212868237828	4.292×10^{-12}
0.20	0.225399673560564	0.225399673559270	1.294×10^{-12}
0.30	0.588333021371067	0.588333021367038	4.029×10^{-12}
0.40	0.504121344416091	0.504121344412679	3.412×10^{-12}
0.50	0.435266598393583	0.435266598390716	2.867×10^{-12}
0.60	0.378180841125863	0.378180841123446	2.417×10^{-12}
0.70	0.330304941839221	0.330304941837173	2.048×10^{-12}
0.80	0.289764207700844	0.289764207699098	1.746×10^{-12}
0.90	0.255153707989778	0.255153707988279	1.499×10^{-12}
1.00	0.225399673560564	0.225399673559270	1.294×10^{-12}

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Problem 5:

(SIR Model) The SIR model is an epidemiological model that computes the theoretical number of people infected with a contagious illness in a closed population over time. The name of this class of models derives from the fact that they involve coupled equations relating the number of susceptible people $S(t)$, number of people infected $I(t)$ and the number of people who have recovered $R(t)$. This is a good and simple model for many infectious diseases including measles, mumps and rubella Sunday, *et al.,* (2013). It is given by the following three coupled equations,

$$\frac{dS}{dt} = \mu(1-S) - \beta IS \qquad (i)$$

$$\frac{dI}{dt} = -\mu I - \gamma I + \beta IS \qquad (ii)$$

$$\frac{dR}{dt} = -\mu I + \gamma I \qquad (iii)$$

where μ, γ and β are positive parameters x is defined to be,

$$x(t) = S + I + R \qquad (iv)$$

and adding equations $(i), (ii)$ and (iii), we obtain the following evolution equation for ,

$$\dot{x}(t) = \mu(1-x) \qquad (v)$$

Taking $\mu = 0.5$ and attaching an initial condition $x(0) = 0.5$ (for a particular closed population), we obtain,

$$\dot{x}(t) = \mu(1-x), \qquad x(0) = 0.5 \qquad (v)$$

whose exact solution is,

$$x(t) = 1.0 - 0.5e^{-0.5t} \qquad (vi)$$

Applying the block integrator (8) to Problem 5, we obtain the results in Table 6 at different values of t

Table 6: Performance of the block algorithm on problem 5

t	Exact value $x(t)$	proposed value $x(t_n)$	Error in prop method $e_n = \lvert x(t) - x(t_n)\rvert$ Order $p = 8$	Error in Sunday, *et al.,* (2013) $e_n = \lvert x(t) - x(t_n)\rvert$ Order $p = 6$
0.100	0.52438528774964299546	0.52438528774964299541	5.0000×10^{-20}	5.5744×10^{-12}
0.200	0.54758129098202021342	0.54758129098202021333	9.0000×10^{-20}	3.9462×10^{-13}
0.300	0.56964601178747109638	0.56964601178747109626	1.2000×10^{-19}	8.1832×10^{-12}
0.400	0.59063462346100907066	0.59063462346100907050	1.6000×10^{-19}	3.4361×10^{-11}
0.500	0.61059960846429756588	0.61059960846429756566	2.2000×10^{-19}	1.9300×10^{-10}
0.600	0.62959088965914106696	0.62959088965914106672	2.4000×10^{-19}	1.8790×10^{-10}
0.700	0.64765595514064328282	0.64765595514064328253	2.9000×10^{-19}	1.7768×10^{-10}
0.800	0.66483997698218034963	0.66483997698218034930	3.3000×10^{-19}	1.7247×10^{-10}
0.900	0.68118592418911335343	0.68118592418911335307	3.6000×10^{-19}	1.8475×10^{-10}
1.000	0.69673467014368328820	0.69673467014368328780	4.0000×10^{-19}	3.0058×10^{-10}

Conclusion

In this work, a one-step block hybrid algorithm for solving first order linear and nonlinear initial value problems of ordinary differential equations is derived and presented. The algorithm was found to be consistent, zero stable and convergent. The method was implemented on some existing linear and nonlinear stiff initial value problems of ordinary differential equations and the numerical results were found to be more accurate when compared with some existing numerical methods. The proposed hybrid block algorithm can be a suitable candidate for all forms of linear and nonlinear first order initial value problems of ordinary differential equations.

Conflict of Interests:

No potential conflict of interest was reported by the authors.

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Appendix I

$$\begin{bmatrix} y_{n+\frac{1}{6}} - y_{n+\frac{5}{6}} \\ y_{n+\frac{1}{3}} - \frac{351}{1375} y_n - \frac{1024}{1375} y_{n+\frac{5}{6}} \\ y_{n+\frac{1}{2}} - \frac{32}{275} y_n - \frac{243}{275} y_{n+\frac{5}{6}} \\ y_{n+\frac{2}{3}} - \frac{351}{1375} y_n - \frac{1024}{1375} y_{n+\frac{5}{6}} \\ y_{n+\frac{1}{4}} - \frac{52381}{352000} y_n - \frac{299619}{352000} y_{n+\frac{5}{6}} \\ y_{n+\frac{5}{6}} - y_n \\ y_{n+1} - y_n \\ y_{n+\frac{1}{6}} - y_{n+\frac{5}{6}} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{4}{2835} & -\frac{19}{315} & 0 & -\frac{68}{315} & -\frac{332}{2835} & -\frac{68}{315} & -\frac{19}{315} & \frac{68}{315} \\ \frac{559}{46200} & \frac{27}{385} & 0 & -\frac{197}{3080} & -\frac{122}{1155} & -\frac{477}{3080} & -\frac{89}{1925} & \frac{1}{840} \\ \frac{53}{9240} & \frac{27}{385} & 0 & -\frac{197}{3080} & -\frac{122}{1155} & -\frac{477}{3080} & -\frac{89}{1925} & \frac{1}{840} \\ \frac{202727}{27033600} & \frac{7119}{225280} & 0 & -\frac{286587}{1802240} & -\frac{18193}{168960} & -\frac{326277}{1802240} & -\frac{58527}{1126400} & \frac{623}{491520} \\ \frac{2305}{54432} & \frac{4825}{12096} & -\frac{2200}{5103} & \frac{5875}{12096} & \frac{2375}{54432} & \frac{725}{3024} & \frac{95}{1728} & -\frac{275}{326592} \\ \frac{41}{840} & \frac{9}{35} & 0 & \frac{9}{280} & \frac{34}{105} & \frac{9}{280} & \frac{9}{35} & \frac{41}{840} \\ \frac{4}{2835} & -\frac{19}{315} & 0 & -\frac{68}{315} & -\frac{332}{2835} & -\frac{68}{315} & -\frac{19}{315} & \frac{68}{315} \end{bmatrix} \begin{bmatrix} hf_n \\ hf_{n+\frac{1}{6}} \\ hf_{n+\frac{1}{4}} \\ hf_{n+\frac{1}{3}} \\ hf_{n+\frac{1}{2}} \\ hf_{n+\frac{2}{3}} \\ hf_{n+\frac{5}{6}} \\ hf_{n+1} \end{bmatrix} \quad (8.0)$$