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Full Length Research Article

A TWO-PARAMETER ESTIMATOR FOR CORRELATED REGRESSORS IN GAMMA REGRESSION MODEL

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ABSTRACT
Gamma Modified Two Parameter (GMTP) is a novel biased two-parameter estimator proposed to address the effects of multicollinearity in Generalized Linear Models (GLMs). An expansion of the linear regression model's Modified Two Parameters (MTP) is the newly suggested estimator. The performance of the GMTP estimator over the maximum likelihood estimator (MLE), gamma ridge estimator (GRE), gamma Liu estimator (GLE), and gamma Liu-type estimator (GLTE) reviewed in this article are theoretically compared, and the estimator's properties is discussed. A simulation study that examine the effects of the dispersion parameter, sample size, explanatory variables, and degree of correlation are used to examine the superiority of the GMTP with four different biasing parameters over the MLE, GRE, GLE, and GLTE with regard to the estimated MSE criterion. The GMTP estimator with biasing parameters k_2 and k_4 outperforms the MLE, GRE, GLE, and GLTE, according to simulation research. More research can be done to see how well the GMTP estimator performs in comparison to other estimators that were not examined in this study.

Keywords: Multicollinearity, Generalized Linear Models, Two-Parameter estimator, Liu estimator, Mean squared error matrix, Simulation.

INTRODUCTION
In regression models, the distribution of the response variable is frequently an issue to be addressed. In real-world analysis, the response variable does not always follow a normal distribution. For example, datasets derived from economic and social sciences datasets typically contain positive values, resulting in positively skewed datasets. Similarly, epidemiological data is frequently positively skewed, violating the response variable's normality. In cases where the dependent variable no longer follows a normal distribution but is positively skewed, the Gamma regression model is an appropriate model to use (Malehi *et al*., 2015; Asar and Genc, 2015; Hattab, 2016; Asar and Genc, 2018; Algamal, 2018a; Amin *et al*., 2019).
The Maximum Likelihood (ML) estimator is the most commonly employed estimator to determine the Gamma Regression Model's parameters under certain assumptions, similar to how the Ordinary Least Squares (OLS) estimator is used to calculate the Linear Regression Model's parameters.
When using the Gamma regression model, non-correlated explanatory variables must be assumed. However, it is often obvious in research that there is a strong or nearly strong linear relationship between the explanatory variables, which brings up the multicollinearity issue. The estimated coefficients by the maximum likelihood (ML) method become unstable and have a high variance in the presence of multicollinearity, resulting in low statistical significance (Kurtoglu and Ozkale, 2016; Amin *et al*., 2017; Perez-Melo and Kibria, 2020). Furthermore, the regression coefficients' sampling variance can influence both inference and prediction (Hattab, 2016; Algamal 2018a; Amin *et al*., 2019).
Several techniques have been put forth in the literature to address the issue of multicollinearity in linear regression, including the Stein (1956) estimator, principal component estimator (Massy, 1965), ridge regression estimator (Hoerl and Kennard, 1970), modified two-parameter estimator (Dorugade, 2014), Liu estimator (Liu, 1993), Liu-type estimator (Liu, 2003), and a modified Jackknife Ridge estimator proposed by Batah *et al*., (2008). Liu estimation was developed by Kurtoglu and Ozkale (2016). Similarly, Amin *et al*., (2017), Algamal (2018b) and Asar and Genc (2015) presented several methods for estimating the Gamma regression model's ridge parameter, k. Adewale *et al*., (2021) extended new ridge-type estimator to the gamma regression model, and Algboory and Algamal (2022) worked on Liu-Type estimator in gamma regression model on the basis of (r-(k-d)) class estimator.

The goal of this paper is to generalize a modified two-parameter (MTP) estimator proposed by Dorugade (2014) into gamma regression models.
The paper is structured as follows: in Section 2, the Gamma Modified Two-Parameter estimator is proposed and its mean squared error (MSE) properties are derived. Section 3 includes a simulation study to assess the performance of the gamma modified two-parameter estimator and some existing estimators in the presence of multicollinearity. Section 4 concludes with some concluding remarks.

MODEL AND ESTIMATORS
When the predictor variable is positively skewed and the mean is proportional to the dispersion parameter, the Gamma Regression Model is used (Qasim *et al*., 2018; Amin *et al*., 2017)). Let us consider a Gamma distribution with a nonnegative shape α and a nonnegative scale τ parameter, as well as a probability density function:

$$f(y_i)=\frac{1}{\Gamma(\alpha)\tau^{\alpha}}y_i^{\alpha-1}\ell^{-\frac{y_i}{\tau}}, \qquad y_i \geq 0 \qquad (1)$$

where α is the non-negative shape parameter and τ is the scale parameter $E(y_i)=\mu_i=\alpha\tau=\theta_i$ known as the

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canonical parameter, and $Var(y_i) = \alpha\tau^2 = \frac{\theta_i^2}{\alpha}$, $\theta_i = \exp(x_i'\beta)$ where $x_i = (x_{i1}, x_{i2}, ..., x_{ip})'$, $i = 1,2,...,n$ and $j = 1,2,..., p$ where n is the sample size and p is the number of regressors variables such that (n > p). The log-likelihood function of equation (1) is given as follows:

$$l(\beta) = \sum_{i=1}^{n}\left[(\alpha-1)\ln(y_i) - \frac{y_i}{\tau} - \alpha\ln(\tau) - \ln(\Gamma(\alpha))\right] \tag{2}$$

Because equation (2) is nonlinear, it is solved iteratively using the Fisher scoring method as follows:

$$\beta^{(r+1)} = \beta^r + I^{-1}(\beta^r)S(\beta^r) \tag{3}$$

where r is the degree of iteration,

$I^{-1}(\beta) = \left[-E\left(\frac{\partial^2 l(\beta)}{\partial(\beta)\partial(\beta')}\right)\right]^{-1}$ and $S(\beta) = \partial l(\beta)/\partial\beta$. The iterative process continues until the convergence of estimates. Hence, the ML estimator is obtained as:

$$\hat{\beta}_{MLE} = (X'\hat{W}X)^{-1}X'\hat{W}\hat{z} \tag{4}$$

where $\hat{W} = diag(\hat{\theta}_i^2)$ and $\hat{z}$ is the vector in ith element computed by, $\hat{z} = \hat{\theta}_i + \frac{y_i - \hat{\theta}_i}{\hat{\theta}_i^2}$ where $\hat{W}$ and $\hat{z}$ are obtained at the final iteration by the procedure of Fisher scoring (Hardin and Hilbe, 2012). The following is the matrix for the mean squared error (MMSE) and mean square error (MSE) of the ML estimator, respectively:

$$MMSE(\hat{\beta}_{MLE}) = Cov(\hat{\beta}_{MLE}) = \phi(X'\hat{W}X)^{-1} \tag{5}$$

$$MSE(\hat{\beta}_{MLE}) = tr(MMSE(\hat{\beta}_{MLE})) = \phi\sum_{j=1}^{p}\frac{1}{\lambda_j} \tag{6}$$

where $\phi = (n-p)^{-1}\sum_{i=1}^{n}\frac{(y_i-\theta_i)^2}{\theta_i^2}$.

Where λ_j is regarded as jth of the matrix's eigenvalues $A = X'\hat{W}X$. The following is the analysis of the matrix A's eigenvalue decomposition: $A = q\Lambda q'$ such that q is the orthogonal matrix made up of the eigenvectors that correspond to the eigenvalues of A such that $A = diag(\lambda_1, \lambda_2, ..., \lambda_p)$.

Some of the eigenvalues will be small when the predictor variables have a linear association, indicating that the information matrix A is ill-conditioned. As a result, the ML estimator's MSE will be overestimated, and some regression coefficients will be severely impacted.

Estimator of Gamma Ridge

It was Hoerl and Kennard (1970) who initially introduced the ridge estimator, an estimator that is frequently used to reduce multicollinearity in linear regression models. In the context of extended linear models, this was suggested by Segerstedt (1992). The definition of the gamma ridge estimator (GRE) is:

$$\hat{\beta}_{GRE} = (A+kI)^{-1}A\hat{\beta}_{MLE} \tag{7}$$

where $A_k^{-1} = (X'\hat{W}X + kI)^{-1}$ and k > 0 is the biasing parameter. The MMSE and MSE of GRE are defined respectively as:

$$\begin{aligned} MMSE(\hat{\beta}_{GRE}) &= Cov(\hat{\beta}_{GRE}) + Bias(\hat{\beta}_{GRE})Bias(\hat{\beta}_{GRE})' \\ &= \phi A_k^{-1}AA_k^{-1} + k^2A_k^{-1}\beta\beta'A_k^{-1} \end{aligned} \tag{8}$$

$$\begin{aligned} MSE(\hat{\beta}_{GRE}) &= tr(MMSE(\hat{\beta}_{GRE})) \\ &= \phi\sum_{j=1}^{p}\frac{\lambda_j}{(\lambda_j+k)^2} + k^2\sum_{j=1}^{p}\frac{\alpha_j^2}{(\lambda_j+k)^2} \end{aligned} \tag{9}$$

where $\alpha = q'\beta$ in such a way that q is A's eigenvector matrix.

Estimator of Gamma Liu

Kurtoglu and Ozkale (2016) employed the Liu estimator, which was first published by Liu (1993) in the linear regression model and is known as the Gamma Liu estimator given as:

$$\hat{\beta}_{GLE} = F_D\hat{\beta}_{MLE} \tag{10}$$

where $F_D = (A+I)^{-1}(A+dI)$, 0 < d < 1 is the Liu biasing parameter. The MMSE and MSE of GLE are defined respectively as:

$$\begin{aligned} MMSE(\hat{\beta}_{GLE}) &= Cov(\hat{\beta}_{GLE}) + Bias(\hat{\beta}_{GLE})Bias(\hat{\beta}_{GLE})' \\ &= \phi F_D A^{-1}F_D' + (1-d)^2(A+I)^{-1}\beta\beta'(A+I)^{-1} \end{aligned} \tag{11}$$

$$\begin{aligned} MSE(\hat{\beta}_{GLE}) &= tr(MMSE(\hat{\beta}_{GLE})) \\ MSE(\hat{\beta}_{GLE}) &= \phi\sum_{j=1}^{p}\frac{(\lambda_j+d)^2}{\lambda_j(\lambda_j+1)^2} + (1-d)^2\sum_{j=1}^{p}\frac{\alpha_j^2}{(\lambda_j+1)^2} \end{aligned} \tag{12}$$

Estimator of Gamma Liu-type

The Liu-type estimator, first introduced by Liu (2003) in the linear regression model, has been applied to the generalized linear models by Algamal and Asar (2018) and is known as the Gamma

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Liu-type estimator defined as:

$$\hat{\beta}_{GLTE} = A_k^{-1} A_D \hat{\beta}_{MLE} = F_{kD}\hat{\beta}_{MLE} \quad (13)$$

where $A_D = (A - dI)$, $F_{kD} = A_k^{-1} A_D$ and $-\infty < d < \infty$, k > 0. The MMSE and MSE of GLTE are defined respectively as:

$$MMSE(\hat{\beta}_{GLTE}) = Cov(\hat{\beta}_{GLTE}) + Bias(\hat{\beta}_{GLTE})Bias(\hat{\beta}_{GLTE})'$$
$$= \phi F_{kD} A^{-1} F_{kD}' + (d+k)^2 A_k^{-1}\beta\beta' A_k^{-1} \quad (14)$$
$$MSE(\hat{\beta}_{GLTE}) = tr(MMSE(\hat{\beta}_{GLTE}))$$

$$MSE(\hat{\beta}_{GLTE}) = \phi\sum_{j=1}^{p}\frac{(\lambda_j - d)^2}{\lambda_j(\lambda_j + k)^2} + (d+k)^2\sum_{j=1}^{p}\frac{\alpha_j^2}{(\lambda_j + k)^2} \quad (15)$$

A Proposed Estimator of Gamma Ridge-Type

Dorugade (2014) proposed Modified Two-Parameter (MTP) as:

Let $\hat{\beta}^* = (S + kdI)^{-1} Z' y$ and $S = X'X$, then

$$\hat{\beta}_{MTP} = \hat{\beta}^* + k(1-d)(S + kdI)^{-1}\hat{\beta}^* \quad (16)$$

or

$$\hat{\beta}_{MTP} = \left[I + k(1-d)(S+kdI)^{-1}\right]\left[I - kd(S+kdI)^{-1}\right]\hat{\beta}_{MLE} \quad (17)$$

We offer the following Gamma Modified Two-Parameter (GMTP) estimator as the result of our work, which suggests generalizing the modified two-parameter (MTP) estimate in the gamma regression model:

Let $\hat{\beta}^* = (A + kdI)^{-1} A\hat{\beta}_{MLE}$, then (18)

$$\hat{\beta}_{GMTP} = \hat{\beta}^* + k(1-d)(A + kdI)^{-1}\hat{\beta}^* \quad (19)$$

or

$$\hat{\beta}_{GMTP} = \left[I + k(1-d)(A+kdI)^{-1}\right]\left[I - kd(A+kdI)^{-1}\right]\hat{\beta}_{MLE}$$
$$\hat{\beta}_{GMTP} = H\hat{\beta}_{MLE} \quad (20)$$

where

$$H = \left[I + k(1-d)(A+kdI)^{-1}\right]\left[I - kd(A+kdI)^{-1}\right]$$

. The Gamma Modified Two-Parameter Estimator's properties are as follows:

$$Bias(\hat{\beta}_{GMTP}) = E(\hat{\beta}_{GMTP}) - \beta$$
$$Bias(\hat{\beta}_{GMTP}) = (H - I)\beta$$
$$= \sum_{j=1}^{p}\left\{\frac{k\left[(1-2d)\lambda_j - kd^2\right]}{(\lambda_j + kd)^2}\right\}\beta_j \quad (21)$$

$$Cov(\hat{\beta}_{GMTP}) = \phi H A^{-1} H'$$
$$= \phi\sum_{j=1}^{p}\left[\frac{\lambda_j(\lambda_j + k)^2}{(\lambda_j + kd)^4}\right] \quad (22)$$

The Matrix Mean Square Error and Mean Square Error of GMTP are given respectively as:

$$MMSE(\hat{\beta}_{GMTP}) = Cov(\hat{\beta}_{GMTP}) + Bias(\hat{\beta}_{GMTP})Bias(\hat{\beta}_{GMTP})'$$
$$MMSE(\hat{\beta}_{GMTP}) = \phi H A^{-1} H' + H\beta\beta' H' \quad (23)$$

$$MSE(\hat{\beta}_{GMTP}) = tr\left[MMSE(\hat{\beta}_{GMTP})\right]$$

$$= \phi\sum_{j=1}^{p}\left[\frac{\lambda_j(\lambda_j + k)^2}{(\lambda_j + kd)^4}\right] + \sum_{j=1}^{p}\left[\frac{k^2\left((1-2d)\lambda_j - kd^2\right)^2}{(\lambda_j + kd)^4}\right]\alpha_j^2 \quad (24)$$

Setting k = 0 in equation (24), we obtain

$$MSE\hat{\beta}_{MLE} = \phi\sum_{j=1}^{p}\frac{1}{\lambda_j} \quad (25)$$

Also, setting d = 1 in equation (24), we obtain

$$MSE(\hat{\beta}_{GRE}) = \phi\sum_{j=1}^{p}\frac{\lambda_j}{(\lambda_j + k)^2} + k^2\sum_{j=1}^{p}\frac{\alpha_j^2}{(\lambda_j + k)^2} \quad (26)$$

The Effectiveness of Gamma Modified Two-Parameter Estimators

In this part, we employed two important lemmas to provide a comparison between $\hat{\beta}_{GMTP}$ and $\hat{\beta}_{MLE}$, $\hat{\beta}_{GRE}$, $\hat{\beta}_{GLE}$, $\hat{\beta}_{GLTE}$ utilizing lesser MSE criterion.

Lemma 1: Let R be a n × n positive definite matrix and α be a vector. Then, $R - \alpha\alpha'$ is positive definite if and only if $\alpha' R^{-1}\alpha < 1$ (Farebrother, 1976).

We thus arrive at the following conclusion:

Lemma 2: Suppose that $\alpha_i = M_i y, i = 1,2$ be the two competing estimators of α. Assume that $I = Cov(\hat{\alpha}_1) - Cov(\hat{\alpha}_2) > 0$, then,

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$MMSE(\hat{\alpha}_1) - MMSE(\hat{\alpha}_2) > 0$ if and only if $v_2'(I + v_1 v_1') \leq 1$, where v_i denotes the bias of $\hat{\alpha}_i$ (Trenkler and Toutenburg, 1990).
In the context of the proof of $MMSE(\hat{\alpha}_1) - MMSE(\hat{\alpha}_2) > 0$ leads to the inference that $MMSE(\hat{\alpha}_2)$ is better than $MMSE(\hat{\alpha}_1)$. To compare the proposed estimator with certain existing ones, the same criterion is applied; (see Akdeniz and Erol, 2003).

Comparison of GMTP and MLE

Theorem 1: $\hat{\beta}_{GMTP}$ is superior to $\hat{\beta}_{MLE}$ if

$$\beta' H' \left[\phi\left(A^{-1} - HA^{-1}H'\right)\right]^{-1} H\beta < 1 \tag{27}$$

Proof: The difference of the MSE is

$$V_1 = MSE\left(\hat{\beta}_{MLE}\right) - MSE\left(\hat{\beta}_{GMTP}\right) = \phi \sum_{j=1}^{p} \left[\frac{1}{\lambda_j} - \frac{\lambda_j(\lambda_j + k)^2}{(\lambda_j + kd)^4}\right] - k^2 \sum_{j=1}^{p} \left[\frac{\left((1-2d)\lambda_j - kd^2\right)^2}{(\lambda_j + kd)^4}\right] \alpha_j^2 \tag{28}$$

The equation (28) shows that V_1 is positive definite if

$$\phi\left[(\lambda_j + kd)^4 - \lambda_j^2(\lambda_j + k)^2\right] \geq \lambda_j \alpha_j^2 k^2 \left[(1-2d)\lambda_j - kd^2\right]^2$$

. By Lemma 1, the proof is completed.

Comparison of GMTP and GRE

Theorem 2: $\hat{\alpha}_{GMTP}$ is superior to $\hat{\alpha}_{GRE}$ if:

$$\beta' H' \left[\phi\left(A_k^{-1} A A_k^{-1} - HA^{-1}H'\right) + k^2 A_k^{-1} \beta\beta' A_k^{-1}\right] H\beta < 1 \tag{29}$$

Proof: we investigate the difference between equation (24) and (9)

$$V_2 = MSE\left(\hat{\beta}_{GRE}\right) - MSE\left(\hat{\beta}_{GMTP}\right) = \phi \sum_{j=1}^{p} \left[\frac{\lambda_j}{(\lambda_j + k)^2} - \frac{\lambda_j(\lambda_j + k)^2}{(\lambda_j + kd)^4}\right] + k^2 \sum_{j=1}^{p} \left[\frac{1}{(\lambda_j + k)^2} - \frac{\left[(1-2d)\lambda_j - kd^2\right]^2}{(\lambda_j + kd)^4}\right] \alpha_j^2 \tag{30}$$

It can be seen from the above equation that V_2 is positive definite if

$$\phi\lambda_j\left[(\lambda_j + kd)^4 - (\lambda_j + k)^4\right] \geq \alpha_j^2 k^2 \left[(\lambda_j + kd)^4 - (\lambda_j + kd)^2\left((1-2d)\lambda_j - kd^2\right)^2\right]$$

Hence, by lemma 1, the proof is completed.

Comparison of $\hat{\alpha}_{GMTP}$ and $\hat{\alpha}_{GLE}$

Theorem 3: $\hat{\alpha}_{GMTP}$ is better than $\hat{\alpha}_{GLE}$ if

$$\beta' H' \left[\phi\left(F_D A^{-1} F_D' - HA^{-1}H'\right) + (1-d)^2 (A+I)^{-1} \beta\beta' (A+I)^{-1}\right] H\beta < 1 \tag{31}$$

Proof: we investigate the difference between equation (24) and (12)

$$V_3 = MSE\left(\hat{\beta}_{GLE}\right) - MSE\left(\hat{\beta}_{GMTP}\right) = \phi \sum_{j=1}^{p} \left[\frac{(\lambda_j + d)^2}{\lambda_j(\lambda_j + 1)^2} - \frac{\lambda_j(\lambda_j + k)^2}{(\lambda_j + kd)^4}\right] + \sum_{j=1}^{p} \left[\frac{(1-d)^2}{(\lambda_j + 1)^2} - \frac{k^2\left[(1-2d)\lambda_j - kd^2\right]^2}{(\lambda_j + kd)^4}\right] \alpha_j^2 \tag{32}$$

It can be seen from the above equation that V_3 is positive definite if

$$\phi\left[(\lambda_j + d)^2(\lambda_j + kd)^4 - \lambda_j^2(\lambda_j + k)^2(\lambda_j + 1)^2\right] \geq \alpha_j^2\left[(1-d)^2(\lambda_j + kd)^4 - (\lambda_j + 1)^2\left[k^2\left((1-2d)\lambda_j - kd^2\right)^2\right]\right]$$

Hence, by lemma 1, the proof is completed.

Comparison of $\hat{\alpha}_{GMTP}$ and $\hat{\alpha}_{GLTE}$

Theorem 4: $\hat{\alpha}_{GMTP}$ is better than $\hat{\alpha}_{GLTE}$ if

$$\beta' H' \left[\phi\left(F_{kD} A^{-1} F_{kD}' - HA^{-1}H'\right) + (d+k)^2 A_k^{-1} \beta\beta' A_k^{-1}\right] H\beta < 1 \tag{33}$$

Proof: we investigate the difference between equation (24) and (15)

$$V_4 = MSE\left(\hat{\beta}_{GLTE}\right) - MSE\left(\hat{\beta}_{GMTP}\right) = \phi \sum_{j=1}^{p} \left[\frac{(\lambda_j - d)^2}{(\lambda_j + k)^2} - \frac{\lambda_j(\lambda_j + k)^2}{(\lambda_j + kd)^4}\right] + \sum_{j=1}^{p} \left[\frac{(d+k)^2}{(\lambda_j + k)^2} - \frac{k^2\left[(1-2d)\lambda_j - kd^2\right]^2}{(\lambda_j + kd)^4}\right] \alpha_j^2 \tag{34}$$

It can be seen from the above equation that V_4 is positive definite if

$$\phi\left[(\lambda_j - d)^2(\lambda_j + kd)^4 - \lambda_j(\lambda_j + k)^4\right] \geq \alpha_j^2\left[(d+k)^2(\lambda_j + kd)^4 - (\lambda_j + k)^2\left[k^2\left((1-2d)\lambda_j - kd^2\right)^2\right]\right]$$

Hence, by lemma 1, the proof is completed.

Choosing the Gamma Modified Two-Parameter Estimator's parameters, *k* and *d*
The biasing parameter recommended by Hoerl *et al*. (1975) is utilized in accordance with Dorugade (2014) so that

$$k = \frac{p\phi}{\sum_{i=1}^{p} \alpha_j^2} \tag{35}$$

But k is dependent on both ϕ and α_j. They will be practically replaced by their unbiased estimator $\hat{\phi}$ and $\hat{\alpha}_j$. This is thus obtained as:

$$\hat{k} = \frac{p\hat{\phi}}{\sum_{i=1}^{p} \hat{\alpha}_j^2} \tag{36}$$

The value of k was proposed by Hoerl *et al*. (1975) by taking the harmonic mean of the k values (for j = 1, 2,..., p). Thus, for the suggested estimator, the Gamma Modified Two-Parameter Estimator, four shrinkage parameters are investigated. This is how they are defined:

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$$\hat{k}_{AM} = \frac{1}{p}\sum_{j=1}^{p}\frac{\hat{\phi}}{\hat{\alpha}_j^2} = k_1 \tag{37}$$

$$\hat{k}_{HM} = \frac{p\hat{\phi}}{\sum_{j=1}^{p}\hat{\alpha}_j^2} = k_2 \tag{38}$$

$$\hat{k}_{MX} = \max\left(\frac{\hat{\phi}}{\hat{\alpha}_j^2}\right) = k_3 \tag{39}$$

$$\hat{k}_{MN} = \min\left(\frac{\hat{\phi}}{\hat{\alpha}_j^2}\right) = k_4 \tag{40}$$

Therefore, the *d* that minimizes $MSE(\hat{\beta}_{GMTP})$ can be thought of as the best value of *d*.

Let

$$g(k,d) = MSE(\hat{\beta}_{GMTP}) = \phi\sum_{j=1}^{p}\left[\frac{\lambda_j(\lambda_j+k)^2}{(\lambda_j+kd)^4}\right] + \sum_{i=1}^{p}\left[\frac{k^2((1-2d)\lambda_j-kd^2)^2}{(\lambda_j+kd)^4}\right]\alpha_j^2$$

Then, we obtain by differentiating g (k, d) w.r.t. *d* and equating to 0, we have

$$d = \sum_{j=1}^{p}\left[\frac{(\lambda_j+k)(\phi+\lambda_j\alpha_j^2)-\lambda_j}{k\alpha_j^2}\right] \tag{41}$$

On the other hand, *d* is dependent upon ϕ and α_j. They will be practically replaced by their unbiased estimator $\hat{\phi}$ and $\hat{\alpha}_j$. Thus, this will be acquired.

$$\hat{d} = \sum_{j=1}^{p}\left[\frac{(\lambda_j+k)(\hat{\phi}+\lambda_j\hat{\alpha}_j^2)-\lambda_j}{\hat{k}\hat{\alpha}_j^2}\right] \tag{42}$$

MONTE CARLO SIMULATION

This section looks at the performance of GMTP with various multicollinearity levels using a Monte Carlo simulation experiment.

Design of Simulation

In accordance with Amin *et al.* (2017) and Amin *et al.* (2019) the response variable of n observations from the Gamma Regression model is constructed as, $y_i \sim Gamma(\mu_i, \phi)$ where dispersion parameter, $\phi \in \{0.5, 1, 1.5\}$ and $\mu = \theta_i = \exp(x_i'\beta)$, $\beta = (\beta_1, \beta_2, ..., \beta_p)$ with parameter vector, β chosen such that $\sum_{j=1}^{p}\beta_j^2 = 1$. The explanatory factors are obtained by following Gibbons (1981); Kibria (2003); Lukman and Ayinde (2017) and Idowu *et al.* (2023) as follows:

$$x_{ij} = (1-\rho^2)^{1/2} w_{ij} + \rho w_{ip+1}, \qquad i = 1,2,...,n, \qquad j = 1,2,...,p \tag{43}$$

where ρ depict the correlation between the explanatory variables, w_{ij} are independent standard normal pseudorandom numbers. The effectiveness of the suggested estimators is assessed in relation to many parameters, including the degree of correlation $(\rho = 0.8, 0.9, 0.95, 0.99)$, the number of explanatory variables (p = 4, 8), and the sample size (n = 30, 50, 100, 150, 200, 250). The generated data is repeated 1,000 times in the R programming language in order to incorporate these various values of n, ϕ, ρ and p. The average MSE is then determined by:

$$MSE(\hat{\beta}) = \frac{1}{1000}\sum_{i=1}^{1000}\sum_{j=1}^{p}(\hat{\beta}_{ij} - \beta_i)^2 \tag{44}$$

SIMULATION RESULT

The estimated Mean Square Errors (MSEs) of the suggested estimators with four different biasing parameters under various circumstances, including the degree of correlation, sample size, dispersion parameter, and explanatory variables, are provided in Tables 1 and 2. From the simulation results, it can be seen that the estimated MSE increases as the dispersion parameter values increase; the estimated MSE values of all the estimators decrease with increasing n; and that the estimated MSE increases as the degree of multicollinearity increases. The GMTP estimated MSE values with k_2 and k_4 are consistently less than those of the compared estimators. Out of all these estimators, MLE performs the worst. Therefore, GMTP with k_2 and k_4 are regarded as a robust alternative among others in nearly all scenarios when all other parameters are held constant and the dispersion parameters are 1.5 and 0.5, respectively.

Table 1: Estimated MSE of the estimators when n = 30, 50 and 100

phi			0.5								1.5							
n	p	rho	MLE	GRE	GLE	GLTE	GMTP 1	GMTP 2	GMTP 3	GMTP 4	MLE	GRE	GLE	GLTE	GMTP 1	GMTP 2	GMTP 3	GMTP 4
30	4	0.8	0.015	0.0065	0.012	0.009	0.0581	0.0074	0.5211	**0.006**	4.48	2.007	3.553	2.755	1.8786	**0.7045**	27.939	0.778
		0.9	0.025	0.0084	0.018	0.014	0.1096	0.0084	1.1221	**0.005**	8.358	3.465	6.0475	5.276	4.9609	**0.8619**	66.786	0.903
		0.95	0.047	0.0123	0.03	0.025	0.057	0.0115	0.3383	**0.007**	16.39	6.529	10.947	11.13	14.092	1.3971	185.35	**1.349**
		0.99	0.233	0.0453	0.127	0.132	0.0775	0.0179	0.478	**0.012**	83.66	32.69	50.802	77.12	147.82	12.985	1767.2	**12.27**
	8	0.8	0.016	0.0045	0.009	0.007	0.1023	0.0019	4.668	**0.002**	1.185	0.429	0.8826	0.76	0.2067	**0.094**	13.671	0.1021

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		0.9	0.03	0.007	0.016	0.012	0.2077	0.0024	10.018	**0.002**	2.19	0.743	1.4904	1.514	0.5359	**0.095**	32.98	0.1023
		0.95	0.061	0.0125	0.031	0.024	0.3787	0.0031	18.861	**0.002**	4.229	1.384	2.672	3.236	1.5516	**0.104**	83.689	0.111
		0.99	0.332	0.0638	0.163	0.133	0.9048	0.007	46.421	**0.006**	20.8	6.624	12.058	20.17	15.956	0.3659	846.31	**0.362**
50	4	0.8	0.629	0.3492	0.515	0.428	1.2409	0.1928	9.6533	**0.158**	2.617	1.262	2.2312	1.65	1.8242	**0.7062**	28.616	0.786
		0.9	1.117	0.5599	0.845	0.74	2.6166	0.258	21.862	**0.217**	4.691	1.961	3.6281	2.911	5.6269	**0.877**	78.285	0.972
		0.95	2.138	1.005	1.521	1.447	5.6787	0.4873	39.775	**0.431**	9.041	3.462	6.338	5.936	16.954	**1.6058**	231.4	1.668
		0.99	10.8	**4.829**	7.224	8.909	52.282	7.6322	616.39	7.4967	46.16	16.64	28.352	42.16	160.86	19.987	1841.8	**19.68**
	8	0.8	0.011	0.0048	0.009	0.006	0.0031	0.0021	0.0071	**0.002**	1.085	0.451	0.8975	0.705	0.2168	**0.149**	8.3507	0.165
		0.9	0.019	0.008	0.014	0.01	0.0036	0.0026	0.0083	**0.002**	1.987	0.757	1.4933	1.336	0.6856	**0.153**	32.828	0.1699
		0.95	0.036	0.0147	0.025	0.019	0.0052	0.0038	0.0135	**0.003**	3.84	1.391	2.6156	2.754	2.0383	**0.175**	94.931	0.1917
		0.99	0.178	0.0709	0.113	0.089	0.0355	0.0296	0.0756	**0.027**	19.16	6.722	11.326	17.02	21.597	0.7395	854.41	**0.739**
100	4	0.8	2.731	1.7503	2.438	2.113	7.0096	1.013	52.738	**0.85**	1.764	1.045	1.6493	1.327	1.1669	**0.6864**	16.892	0.783
		0.9	4.757	2.7389	3.934	3.673	14.023	1.2604	134.52	**1.063**	3.097	1.597	2.7184	2.332	3.4123	**0.86**	45.251	0.955
		0.95	9.116	4.9134	7.005	7.449	30.998	2.6941	341.92	**2.557**	5.96	2.823	4.815	4.831	10.532	**1.491**	153.48	1.571
		0.99	47.38	**24.56**	33.36	52.67	211.38	38.486	2556.1	37.806	31.19	**14.1**	21.391	37.08	103.92	18.317	1062.2	**18.079**
	8	0.8	0.126	0.063	0.107	0.1	0.0853	0.0218	1.3704	**0.019**	2.918	1.46	2.6839	2.153	**0.662**	0.7661	20.903	0.8329
		0.9	0.237	0.1112	0.186	0.216	0.1199	0.0226	2.5152	**0.02**	5.238	2.319	4.5079	4.083	1.8933	**0.749**	74.464	0.8231
		0.95	0.471	0.2157	0.344	0.52	0.171	0.0247	4.1829	**0.021**	10.11	4.17	7.9811	8.838	7.7216	**0.768**	337.64	0.8463
		0.99	2.478	1.1422	1.667	4.238	0.451	0.0962	9.5293	**0.087**	51.82	20.56	34.203	65.78	84.332	**1.672**	2744.6	1.7251

Table 2: Estimated MSE of the estimators when n = 150, 200 and 250

phi			0.5								1.5							
n	p	ρ	MLE	GRE	GLE	GLTE	GMTP 1	GMTP 2	GMTP 3	GMTP 4	MLE	GRE	GLE	GLTE	GMTP 1	GMTP 2	GMTP 3	GMTP 4
150	4	0.8	4.738	3.248	4.37	3.892	11.46	2.012	78.495	**1.69**	1.472	0.944	1.408	1.15	0.972	**0.681**	11.46	0.771
		0.9	8.011	4.899	6.911	6.591	23.62	2.241	208.42	**1.85**	2.518	1.391	2.295	1.94	2.531	**0.827**	31.84	0.917
		0.95	15.11	8.562	12.13	13.21	48.44	3.235	500.98	**2.771**	4.7943	2.403	4.075	3.929	8.421	**1.355**	103.6	1.441
		0.99	78.05	**42.04**	56.79	94.39	359.4	50.44	3796.9	49.24	25.179	**11.92**	18.23	29.25	103.4	15.2	1102	**15.09**
	8	0.8	1.515	0.877	1.366	1.2	2.076	0.321	39.917	**0.285**	2.3473	1.293	2.229	1.777	**0.603**	0.757	14.79	0.818
		0.9	2.808	1.516	2.384	2.354	4.105	0.344	112.62	**0.301**	4.1931	2.048	3.804	3.281	1.511	**0.733**	54.48	0.799
		0.95	5.566	2.915	4.45	5.127	9.089	0.367	318.69	**0.318**	8.1316	3.716	6.918	6.964	4.717	**0.729**	207.3	0.8
		0.99	29.57	15.51	21.87	34.89	75.82	0.626	2441.7	**0.545**	42.456	18.93	30.89	51.9	56.72	**1.247**	2141	1.294
200	4	0.8	4.387	3.209	4.158	3.641	8.457	1.914	58.135	**1.656**	1.3357	0.918	1.299	1.069	0.747	**0.688**	8.54	0.763
		0.9	7.385	4.888	6.66	6.009	13.81	2.067	117.64	**1.767**	2.2848	1.381	2.152	1.759	1.886	**0.838**	23.18	0.938
		0.95	13.95	8.662	11.87	11.73	28.94	2.861	302.86	**2.503**	4.3736	2.444	3.923	3.431	5.614	**1.366**	70.31	1.425
		0.99	72.81	**43.7**	56.61	79.01	201	45.03	2191.5	43.73	23.336	12.61	18.33	22.95	67.28	14.87	731.3	**14.67**
	8	0.8	2.22	1.345	2.05	1.772	3.178	0.61	46.01	**0.537**	1.9143	1.135	1.848	1.513	**0.618**	0.779	9.949	0.828
		0.9	4.06	2.253	3.548	3.354	6.313	0.639	128.08	**0.56**	3.3594	1.736	3.132	2.71	1.218	**0.757**	35.45	0.812
		0.95	8.039	4.276	6.615	7.165	14.54	0.687	446.49	**0.596**	6.489	3.092	5.738	5.63	3.97	**0.757**	136.2	0.817
		0.99	43.25	22.86	32.59	49.4	96.05	1.84	3173.3	**1.68**	34.306	15.77	26.23	40.29	48.63	**1.311**	2200	1.361
250	4	0.8	4.123	3.137	3.97	3.443	7.074	1.85	42.253	**1.633**	1.277	0.919	1.253	1.046	0.701	**0.725**	6.054	0.785
		0.9	6.878	4.759	6.369	5.528	12.02	1.934	105.95	**1.701**	2.1959	1.411	2.106	1.749	1.594	**0.933**	16.85	0.987
		0.95	12.96	8.442	11.42	10.45	23.28	2.932	243.16	**2.628**	4.2336	2.551	3.918	3.461	4.522	**1.64**	47.51	1.676
		0.99	68.02	**42.99**	55.07	63.36	165.1	57.03	1520.5	55.57	22.905	**13.53**	19.02	23.27	52	19.69	430	19.37
	8	0.8	0.86	0.53	0.802	0.697	1.273	0.254	17.472	**0.227**	1.8074	1.111	1.756	1.454	**0.655**	0.782	8.25	0.827
		0.9	1.561	0.871	1.384	1.324	2.646	0.261	47.811	**0.232**	3.1492	1.685	2.97	2.6	1.228	**0.767**	32.75	0.815
		0.95	3.088	1.637	2.58	2.879	5.668	0.269	183.9	**0.238**	6.0694	2.986	5.465	5.438	3.766	**0.782**	116.7	0.835
		0.99	16.69	8.739	12.72	22.67	34.38	0.537	1606.5	**0.493**	32.2	15.23	25.33	40.11	53.87	**1.704**	1565	1.744

Conclusion

In order to address the consequences of multicollinearity in GLMs, this study suggested a new biased two parameter estimator called Gamma Modified Two Parameter (GMTP). The new proposed estimator is an extension of the Modified Two Parameters (MTP) in the linear regression model. The properties of the GMTP and the existing estimators are compared theoretically. The superiority of the GMTP with four different biasing parameters over the MLE, GRE, GLE, and GLTE with respect to the estimated MSE criterion is investigated through a simulation study that looks at the impact of the dispersion parameter, sample size, explanatory variables, and degree of correlation. Simulation research shows that the

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GMTP estimator with biasing parameters k_2 and k_4 performs better than the MLE, GRE, GLE, and GLTE. More research can be done to see how well the GMTP estimator performs in comparison to other estimators that were not examined in this study.

REFERENCES

Adewale F. Lukman; Issam Dawoud; B. M. Golam Kibria; Zakariya Y. Algamal and Benedicta Aladeitan (2021). A New Ridge-Type Estimator for the Gamma Regression Model. Scientifica Hindawi, 2021(5545356), pp. 1-8.

Akdeniz, F. and Erol, H. (2003). Mean squared error matrix comparisons of some biased estimators in linear regression. Communications in Statistics - Theory and Methods, 32(12), p. 2389 – 413.

Algamal, Z. Y. and Asar, Y. (2018). Liu-type estimator for the gamma regression model. Communications in Statistics—Simulation and Computation.

Algamal, Z. Y. (2018a). Developing a ridge estimator for the gamma regression model. Journal of Chemometrics, 32(10), p. e3054.

Algamal, Z. Y. (2018b). Shrinkage estimators for gamma regression model. Electronic Journal of Applied Statistical Analysis, 11(1), p. 253–268.

Algboory, A. M. and Algamal, Z. Y. (2022). Liu-Type estimator in gamma regression model based on (r-(k-d)) class estimator. Int. J. Nonlinear Anal. Appl., 13(1), p. 2455–2465.

Amin, M., Qasim, M., Amanullah, M. and Afzal, S. (2017). Performance of some ridge estimators for the gamma regression model. Statistical Papers.

Amin, M., Qasim, M., and Amanullah, M. (2019). Performance of Asar and Genç and Huang and Yang's two-parameter estimation methods for the gamma regression model. Iranian Journal of Science and Technology, Transactions A: Science, 43(6), p. 2951-2963.

Asar, Y. and Genc, A. (2015). New shrinkage parameters for the liu-type logistic estimators. Communications in Statistics - Simulation and Computation, Volume 45(3), p. 1094–1103.

Asar, Y. and Genc, A. (2018). A new two-parameter estimator for the poisson regression model. Iran Journal of Science Technology Trans A Sci, 42(2), p. 793–803.

Batah, F. S., Ramnathan, T. and Gore, S. D. (2008). The efficiency of modified jackknife and ridge type regression estimators: a comparison. Surveys in Mathematics and its Applications, 24(2), p. 157–174.

Dorugade, A. (2014). A Modified Two-Parameter Estimator in Linear Regression. Statistics in Transition, Volume 15(1), p. 23–36. Farebrother, R. (1976). Further results on the mean square error of ridge regression. J R Stat Soc., Volume B38, p. 248-250.

Gibbons, D. G. (1981). A simulation study of some ridge estimators. Journal of the American Statistical Association, 76(373), p. 131–139.

Hardin, J. W. and Hilbe, J. M. (2012). Generalized linear models and extensions. Stata Press Publication, College Station.

Hattab, M. W. (2016). A derivation of prediction intervals for gamma regression. Journal of Statistical Computation and Simulation, 86(17), p. 3512–3526.

Hoerl, A. E. and Kennard, R. W. (1970). Ridge regression: biased estimation for nonorthogonal problems. Technometrics, 12(1), p. 55–67.

Hoerl, A. E., Kennard, R. W. and Baldwin, K. F. (1975). Ridge regression: Some simulations. Communications in Statistics, 4(2), p. 105–23.

Idowu, J. I.; Oladapo, O. J.; Owolabi, A.T.; Ayinde, K. and Akinmoju, O. (2023). Combating multicollinearity: A new two-parameter approach. Nicel Bilimler Dergisi, 5(1), pp. 90-116.

Kibria, B. M. G. (2003). Performance of some new ridge regression estimators. Communications in Statistics—Simulation and Computation, Volume 32 (2), p. 419–35.

Kurtoglu, F. and Ozkale, M. R. (2016). Liu estimation in generalized linear models: application on gamma distributed response variable. Statistical Papers, Volume 57(4), p. 911–928.

Liu, K. (1993). A new class of blased estimate in linear regression. 22 ((2)), p. 393–402.

Liu, K. (2003). Using Liu-Type estimator to combat collinearity. Commun. Statist. Theor. Meth., Volume 32, p. 1009–1020.

Lukman A. F. and Ayinde, K. (2017). Review and classifications of the ridge parameter estimation techniques. Hacettepe Math Stat, Volume 46(5), p. 953-967.

Malehi, A. S., Pourmotahari, F. and Angali, K. A. (2015). Statistical models for the analysis of skewed healthcare cost data: a simulation study. Health Econ. Rev., 5(1), pp. 1 - 16.

Massy, W. F. (1965). Principal components regression in exploratory statistical research. Journal of the American Statistical Association, Volume 60, no. 309, p. 234–256.

Perez-Melo, S. and Kibria, B. M. G. (2020). On some test statistics for testing the regression coefficients in presence of multicollinearity: a simulation study. Stat, 3(1), pp. 40 - 55.

Qasim,M., Amin, M. and Amanullah, M. (2018). On the performance of some new Liu parameters for the gamma regression model. Journal of Statistical Computation and Simulation.

Segerstedt, B. (1992). On ordinary Ridge regression in generalized linear models. Communications in Statistics-Theory and Methods, Volume 21(8), p. 2227–46.

Stein, C. M. (1956). Inadmissibility of the usual estimator for mean of a multivariate normal Distribution. Proceedings of the Third Berkley Symposium on Mathematical Statistics and Probability, 1954 -1955, I, 197 – 206. University California Press, Berkeley.

Trenkler, G. and Toutenburg, H. (1990). Mean squared error matrix comparisons between biased estimators-an overview of recent results. Stat Pap., 31(1), p. 165-179.