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ROBUST M KIBRIA-LUKMAN ESTIMATOR FOR LINEAR REGRESSION MODEL WITH OUTLIERS IN THE X-DIRECTION: SIMULATIONS AND APPLICATIONS

*[1]Adejumo Taiwo Joel, [2]Ayinde Kayode, [1]Okegbade Ayobami Ibukun, [3]Akomolafe A.A. , [1]Oshuporu Opeoluwa Abosede and [3]Koleoso Sunday Olawale

[1]Department of Statistics, Ladoke Akintola University of Technology, Ogbomoso, Nigeria
[2]Department of Mathematics and Statistics, Northwest Missouri State University, Maryville, Missouri, USA
[3]Department of Statistics, Federal University of Technology, Akure, Nigeria

*Corresponding Author Email Address: tjadejumo@lautech.edu.ng

ABSTRACT
The Ordinary Least Square (OLS) estimator remains Best Linear Unbiased Estimator (BLUE) when all the assumptions surrounding it stay intact, but at an iota of violation of the assumptions, it becomes inefficient and unstable. Some causes of the violation are the multicollinearity and the presence of extreme values (outliers). Recently robust Kibria –Lukman based on M estimator was proposed by Majid *et al.* (2022) but when there are outlying cases in the y-direction. Since, outliers in the x-direction may be inevitable in the data set, therefore it becomes imperative to examine the performance of the robust-M Kibria–Lukman (KL-M) estimator as alternative to already proposed robust estimators that can handle these problems when there are outliers in the x-direction. Through the Monte Carlo experiment, theoretical results under some conditions and factors, including application to real-life data, the new estimator outperformed other estimators considered in this study in the presence of multicollinearity and extreme values in the x-direction. As the error variances (σ^2), level of multicollinearity (rho) and percentage (px), and magnitude (mx) of outliers increase, the Mean Square Error (MSE) of the estimators' considered increase. Meanwhile the MSEs of the estimators decrease as the sample size (n) increases. When rho>0, mx>0, the (px) increases, and sample size (n) increases KL-M along sides, ordinary Kibria-Lukman (KL) estimator outperformed other estimators as the two anomalies occur simultaneously. The KL-M performed better, especially when the sample size was n=100. Conclusively, at the different biasing parameters of the estimators, KL-M performed better than other estimators considered in the study. In the same vein, real-life data was adopted to affirm the claim.

Keywords: BLUE, Multicollinearity, outliers, estimators, Monte Carlo

INTRODUCTION
Considering the matrix form of the general linear regression model as given in equation (1),

$$y = X\beta + e_i \quad (1)$$

where the response variable y is an $(n \times 1)$ vector, the exogenous variable X is an $(n \times p)$ design matrix, the unknown parameter β is a $(p \times 1)$ vector, and the random error e_i is an $(n \times 1)$ vector with $E(e_i)=0$ and $V(e_i)=\sigma^2 I_n$ such that σ^2 and I_n are unknown parameter and identity matrix of order n respectively. Meanwhile, the Ordinary Least Square (OLS) estimator of (1) can be expressed as;

$$\hat{\beta}_{OLS} = (X^T X)^{-1} X^T y \quad (2)$$

OLS is considered Best Linear Unbiased Estimator (BLUE) if some fundamental assumptions surrounding it are intact, such as unbiasedness and minimum variance. Hence, if these assumptions are violated, OLS becomes inappropriate. Some of the causes for the violation are extreme values and multicollinearity in the data set, which have posed a great threat to the OLS estimator in linear regression analysis. Several erudite scholars have proposed several estimators to combat the problem of outliers in the literature, among the robust regression estimators are, M-estimator by Hubber, (1981) and MM-estimator by Yohai, (1987). Least Trimmed estimator (LTS) by Rousseeuw and Yohai (1998), Least Median Squares (LMS) by Rousseeuw and Yohai, (1998, S-estimator by Rousseeuw and Yohai (1984), Least Absolute Deviation (LAD) which was first introduced by Boscovich in 1757 Birkes and Dodge (1993). likewise Least Quartile of Squares (LQS) proposed by Rousseeuw and Yohai, (1984). Also, multicollinearity is another challenge of OLS as the parameter estimated is usually affected; hence Hoerl and Kennard (1970). proposed a Ridge Estimator (RE) to circumvent this problem. Others are the Liu estimator by Liu, (1993), Principal Component Regression (PCR) by Massey, (1965). Several researchers have combined two or more estimators to deal with the problem of multicollinearity in a regression model. These include: the two-parameter estimator Liu-Ridge estimator by Ozkale and Kaciranlar (2007), the *k–d* class estimator by Sakallioglu and Kaciranlar (2008), the new two-parameter estimator by Yang and Chang (2010), the new Liu-type estimator by Yang and Chang (2012) in the same year Chang and Yang combined the principal component regression estimator with the two-parameter estimator proposed by Ozkale and Kaciranlar (2007). Lukman *et al*. (2020) also combined the PCR estimator with the modified ridge–type estimator. A modified two-parameter estimator was proposed by Dorugade (2014), a Modified ridge-type estimator Lukman *et al*. (2020). When exogenous variables are correlated in a multiple regression model, Ahmad and Aslam, (2020) proposed a new two-parameter estimator called the Modified New Two-type Parameter Estimator (MNTPE), Stein, (1956) proposed James-Stain estimator. Dawoud and Kibria (2020) proposed a DK estimator. More recently is a new ridge-type estimator by Owolabi, *et al*. (2022) to mitigate the problem of multicollinearity in the linear regression model.

However, the presence of twin anomalies may be inevitable, especially in economic data. The two problems (outliers and multicollinearity) can jointly appear in the data. Researchers have proposed several robust estimators that can co-handle the problems; Modified Ridge M-estimator by Hassan, (2017) and Lukman, *et al.* (2019) proposed a robust estimator that can handle three joint problems in

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regression analysis which include; multicollinearity, outliers, and autocorrelation. Dawoud and Abonazel (2021) proposed Robust Dawoud-Kibria and so on. Several estimators have been proposed in the literature to handle situations when outliers are present in the y-direction. Therefore this study proposed a new robust estimator that can handle outliers and multicollinearity simultaneously, especially when the anomaly is in the x-direction.

Already existing one-parameter estimators are Ridge Estimator (RE) by Hoerl and Kennard (1970), Liu Estimator (LE) by Liu, (1993) and Kibria-Lukman (KL) estimator by Kibria and Lukman (2020). The above one–parameter estimators are expressed as follows:

Ridge Estimator

The Ridge regression estimator is one of the most commonly used one-parameter estimators to deal with multicollinearity in regression analysis. The estimator is defined in (3):

$$\hat{\beta}_K = \left(X^T X + kI_p\right)^{-1} X^T y \tag{3}$$

where;

k is a biasing parameter suggested by [13] and is denoted by its Harmonic mean version:

$$K_{HM} = k = \frac{p\sigma^2}{\sum_{i=1}^{p} \hat{\alpha}_i^2} \tag{4}$$

Such that *0<k<1*

$\sigma^2 = \dfrac{e^T e}{n-p}$ which is the estimated MSE from OLS in (1) and α_i is the ith coefficient of $\alpha_i = Q^T \beta$ and p is the number of parameters estimated.

Liu Estimator

Liu estimator is another one-parameter estimator aside from RE to tackle multicollinearity in the linear regression model. It was proposed by Liu, (1993). The estimator is denoted by $\hat{\beta}_d$ and defined as:

$$\hat{\beta}_d = \left(X^T X + I_p\right)^{-1} (X^T y + d\hat{\beta}) \tag{5}$$

The biasing parameter for the Liu estimator is given as:

$$\hat{d}_{opt} = d = 1 - \sigma^2 \left[\frac{\sum_{i=1}^{p} \frac{1}{\lambda_i(\lambda_i + 1)}}{\sum_{i=1}^{p} \frac{\alpha_i^2}{(\lambda_i + 1)^2}} \right]$$

such that λ_i is the ith Eigen value $X^T X$ and $\hat{\alpha}_i = Q^T \hat{\beta}$

Such that *0<d<1*.

Because of the sensitivity of the Ridge regression estimator and Liu estimator to outliers in the y-direction Silvapulle, (1991) and Arslan and Billor (2000) proposed robust Ridge-M estimator and Robust Liu-M estimator, respectively, which are defined as:

$$\hat{\beta}_M(k) = \left(X^T X + kI_p\right)^{-1} \left(X^T X\right) \hat{\beta}_M \tag{6}$$

and

$$\hat{\beta}_M(d) = \left(X^T X + I_p\right)^{-1} (X^T X + dI_p) \hat{\beta}_M \tag{7}$$

where d is the biasing parameter for Liu estimator and $\hat{\beta}_M$ is the M-estimator obtained as follows:

$$\hat{\beta}_M = \min_{\beta} \sum_{i=1}^{n} \theta\left(\frac{e_i}{k}\right) \tag{8}$$

where $\theta(.)$ is some suitably chosen function and *k* represents a scale parameter estimate.

THEORETICAL METHODOLOGY

Robust One-parameter Ridge-Type Estimator

As an alternative method of dealing with highly correlated regressors in linear regression analysis, Kibria and Lukman (2020) proposed a new estimator known as Kibria-Lukman (KL) estimator, expressed as:

$$\hat{\beta}_{KL} = (X^T X + kI_p)^{-1} (X^T X - kI_p) \hat{\beta}_{OLS} \tag{9}$$

where $k = \dfrac{\hat{\sigma}^2}{2\hat{\alpha}_i^2 + (\hat{\sigma}^2 / \lambda_i)}$ and $\hat{\beta}_{OLS}$ is the OLS estimator.

The presence of extreme observation in the response direction (y-direction) has been noted to affect the KL-estimator; this is because KLE was given by shrinking the Ordinary Least Squares Estimator (OLSE) using the matrix $(X^T X + kI_p)^{-1} (X^T X - kI_p)$.

Therefore, to address this problem, Robust Kibria-Lukman (RKL) was proposed based on M-estimator by Majid *et al*. (2022) and denoted by $\hat{\beta}_M^{KL}$. This is achieved by introducing a robust estimator $\hat{\beta}_M$ instead of OLSE $\hat{\beta}_{OLS}$.

Consequently, the estimator is defined as:

$$\hat{\beta}_M^{KL} = (X^T X + kI_p)^{-1} (X^T X - kI_p) \hat{\beta}_M \tag{10}$$

2.2 *Properties of One-parameter Ridge-Type Estimator*

Given the general linear regression model as in equation (1) as follows:

$y = X\beta + e_i$.

Then, the canonical form of the above model indicated in equation (i) is given as:

$$y = W\theta + e_i \tag{11}$$

Such that $W = XT$ where $\theta = T^T \beta$ and T is the orthogonal matrix whose columns constitute the eigen vectors of $X^T X$.

Therefore,

$W^T W = T^T X^T XT = \gamma = diag(\lambda_1, \lambda_2, ..., \lambda_p)$ such that $\lambda_1, \lambda_2, ..., \lambda_p$ >0 and are the ordered eigen values of $X^T X$.

Also, let $\hat{\alpha}_M$ be an M-estimator proposed by Hubber, (1981) and given by $\sum_{i=1}^{n} \theta\left(\frac{e_i}{k}\right) = 0$ and $\sum_{i=1}^{n} \theta\left(\frac{e_i}{k}\right) w_i = 0$ where $e_i = y_i - w^T \hat{\alpha}_M$ is an estimator of scale for the error and $\theta(.)$ is some suitably chosen function. This is according to Hampel *et al*. (1986).

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Hence, the estimator can then be written in a canonical form as follows:

$$\hat{\alpha} = \gamma^{-1} W^T y \qquad (12)$$

where, $\gamma = diag(\lambda_1, \lambda_2, ..., \lambda_p)$ and $W^T = T^T X^T$

$$\hat{\alpha}_M = \min_{\alpha} \sum_{i=1}^{n} \theta\left(\frac{y_i - w_i^T \alpha}{k}\right) \qquad (13)$$

$$MSE(\hat{\alpha}) = \sigma^2 \gamma^{-1} \qquad (14)$$

$$\hat{\alpha}_K = (I_p + k\gamma^{-1})^{-1} \hat{\alpha} \qquad (15)$$

$$\hat{\alpha}(d) = (\gamma + I_p)^{-1}(W^T y + d\,\hat{\alpha}) \qquad (16)$$

$$\hat{\alpha}(d) = (\gamma + I_p)^{-1}(\gamma + dI_p)\hat{\alpha} \qquad (17)$$

$$\hat{\alpha}_m(k) = (I_p + k\gamma^{-1})^{-1} \hat{\alpha}_M \qquad (18)$$

$$\hat{\alpha}_m(d) = (\gamma + I_p)^{-1}(\gamma + dI_p)\hat{\alpha}_M \qquad (19)$$

Therefore, the proposed robust KL estimator of α can be expressed as:

$$\hat{\alpha}_m(KL) = (\gamma + kI_p)^{-1}(\gamma - kI_p)\hat{\alpha}_M \qquad (20)$$

Performance of Robust One-parameter Ridge-Type Estimator under Mean Square Error (MSE)

The Mean Square Error (MSE) of the OLS estimator $\hat{\alpha}$ is given as:

$$MSE(\hat{\alpha}) = E\left[(\hat{\alpha} - \alpha)^T (\hat{\alpha} - \alpha)\right] \qquad (21)$$

$$= tr\left[\operatorname{cov}(\hat{\alpha}) + bias(\hat{\alpha}^T)bias(\hat{\alpha})\right]$$

where $bias(\hat{\alpha}) = E(\hat{\alpha}) - \alpha$

Therefore, the $MSE(\hat{\alpha}) = \sum_{i=1}^{p} \frac{\sigma^2}{\lambda_i}$. (22)

The MSE of the M-estimator is defined as $MSE(\hat{\alpha}_M) = \sum_{i=1}^{n} \Omega ii$

where Ωii is the diagonal element for $Cov(\hat{\alpha}_M) = \Omega$, which is finite.

Hoerl and Kennard (1970) provided the MSE for Ridge estimator $\hat{\alpha}(K)$ as follows:

$$MSE(\hat{\alpha}(k)) = \sigma^2 \sum_{i=1}^{p} \frac{\lambda_i}{(\lambda_i + k)^2} + \sum_{i=1}^{p} \frac{k^2 \alpha_i^2}{(\lambda_i + k)^2} \qquad (23)$$

Liu (1993) , estimated the MSE for the Liu estimator $\hat{\alpha}_d$ as follows:

$$MSE(\hat{\alpha}_d) = \sigma^2 \sum_{i=1}^{p} \frac{(\lambda_i + d)^2}{(\lambda_i + 1)^2 \lambda_i} + (d-1)^2 \sum_{i=1}^{p} \frac{\alpha_i^2}{(\lambda_i + 1)^2}$$

(24)

MSE for Robust Ridge –M estimator $\hat{\alpha}_M(k)$ is defined as:

$$MSE(\hat{\alpha}_M(k)) = \sum_{i=1}^{p} \frac{\lambda_i^2 \Omega_{ii} + k^2 \alpha_i^2}{(\lambda_i + k)^2} \qquad (25)$$

MSE for robust Liu M-estimator $\hat{\alpha}_M(d)$ denoted as $MSE(\hat{\alpha}_M(d))$ is given as:

$$MSE(\hat{\alpha}_M(d)) = \sum_{i=1}^{p} \frac{(\lambda_i + d)^2 \Omega_{ii} + (1-d)^2 \alpha_i^2}{(\lambda_i + 1)^2} \qquad (26)$$

In the case of the KL estimator, Kibria and Lukman (2020). defined the canonical form of their proposed MSE as follows:

$$MSE(\hat{\alpha}_M(KL))$$

$$= \sum_{i=1}^{p} \frac{(\lambda_i - k)^2 \Omega_{ii}}{\lambda_i (\lambda_i + k)^2} + \sum_{i=1}^{p} \frac{4k^2 \alpha_i^2}{(\lambda_i + k)^2} \qquad (27)$$

To propose the main theorems, the following conditions are hereby imposed:

(i) ψ is non-decreasing and skew-symmetric

(ii) The residuals are symmetric; they must have mean zero and finite variance.

(iii) Ω is finite.

Theorem I

If $\sum^{p} \Omega_{ii} < \sum^{p} \sigma^2$ $MSE(\hat{\alpha}_M(KL)) < MSE(\hat{\alpha}(KL))$

such, that Ω is the diagonal elements of Ω_{ii} .

Proof:

The difference between the $MSE(\hat{\alpha}_M(KL))$ and $MSE(\hat{\alpha}(KL))$ is given by $MSE(\hat{\alpha}_M(KL))$-$MSE(\hat{\alpha}(KL))$. $MSE(\hat{\alpha}_M(KL)) - MSE(\hat{\alpha}(KL))$

$$= \sum_{i=1}^{p}\left[\frac{(\lambda_i - k)^2 \Omega_{ii} + 4k^2 \alpha_i^2 \lambda_i}{\lambda_i (\lambda_i + k)^2} - \frac{(\lambda_i - k)^2 \sigma^2 + 4k^2 \alpha_i^2 \lambda_i}{\lambda_i (\lambda_i + k)^2}\right]$$

(28)

$$= \sum_{i=1}^{p}\left[\frac{(\lambda_i - k)^2 \Omega_{ii} - (\lambda_i - k)^2 \sigma^2}{\lambda_i (\lambda_i + k)^2}\right]$$

$$= \sum_{i=1}^{p}\left[\frac{(\lambda_i - k)^2 (\Omega_{ii} - \sigma^2)}{\lambda_i (\lambda_i + k)^2}\right]$$

(29)

If $(\Omega_{ii} - \sigma^2) < 0$ this implies that $\sum_{i=1}^{p} \Omega_{ii} < \sigma^2$, $MSE(\hat{\alpha}_M(KL)) < MSE(\hat{\alpha}(KL))$. In this case, the *RKL* estimator is better than the *KLE*

Theorem II

There exists a positive constant $k > k_{i1} > 0$ such that

$MSE(\hat{\alpha}_M(KL)) < MSE(\hat{\alpha}_M(K))$.

Therefore, $MSE(\hat{\alpha}_M(KL)) - MSE(\hat{\alpha}_M(K))$

$$=\sum_{i=1}^{p}\left[\frac{(\lambda_i-k)^2\Omega_{ii}+4k^2\alpha_i^2\lambda_i}{\lambda_i(\lambda_i+k)^2}-\frac{\lambda_i^2\Omega_{ii}+k^2\alpha_i^2}{(\lambda_i+k)^2}\right] \quad (30)$$

$$=\sum_{i=1}^{p}\left[\frac{(\lambda_i-k)^2\Omega_{ii}+4k^2\alpha_i^2\lambda_i-\lambda_i^3\Omega_{ii}-k^2\alpha_i^2\lambda_i}{\lambda_i(\lambda_i+k)^2}\right]$$

$$=\sum_{i=1}^{p}\left[\frac{(\lambda_i-k)^2\Omega_{ii}-\lambda_i^3\Omega_{ii}+4k^2\alpha_i^2\lambda_i-k^2\alpha_i^2\lambda_i}{\lambda_i(\lambda_i+k)^2}\right]$$

$$=\sum_{i=1}^{p}\left[\frac{((\lambda_i-k)^2-\lambda_i^3)\Omega_{ii}+3k^2\alpha_i^2\lambda_i}{\lambda_i(\lambda_i+k)^2}\right] \quad (31)$$

The difference is strictly less than zero if and only if $((\lambda_i-k)^2-\lambda_i^3)\Omega_{ii} < 3k^2\alpha_i^2\lambda_i$.

Next is to solve the inequality $((\lambda_i-k)^2-\lambda_i^3)\Omega_{ii} < 3k^2\alpha_i^2\lambda_i$ for *k* as follows:

$$(\lambda_i^2-2\lambda_i k+k^2-\lambda_i^3)\Omega_{ii} < 3k^2\alpha_i^2\lambda_i$$

$$\Omega_{ii}k^2-3\alpha_i^2k^2\lambda_i-2\lambda_i\Omega_{ii}k+(1-\lambda_i)\lambda_i^2\Omega_{ii} < 0$$

$$(\Omega_{ii}-3\alpha_i^2\lambda_i)k^2-2\lambda_i\Omega_{ii}k+(1-\lambda_i)\lambda_i^2\Omega_{ii} < 0$$

$$k=\frac{2\lambda_i\Omega_{ii}+\sqrt{4\lambda_i^2\Omega_{ii}^2k-4(\Omega_{ii}-3\alpha_i^2\lambda_i)(1-\lambda_i)\lambda_i^2\Omega_{ii}}}{2(\Omega_{ii}-3\alpha_i^2\lambda_i)} \quad (32)$$

$$k=\frac{2\lambda_i\Omega_{ii}+2\lambda_i\sqrt{\Omega_{ii}^2-(\Omega_{ii}-3\alpha_i^2\lambda_i)(1-\lambda_i)\Omega_{ii}}}{2(\Omega_{ii}-3\alpha_i^2\lambda_i)}$$

$$k=\frac{2\lambda_i\left[\sqrt{[\Omega_{ii}^2-(\Omega_{ii}-3\alpha_i^2\lambda_i)(1-\lambda_i)]\Omega_{ii}}+\Omega_{ii}\right]}{2(\Omega_{ii}-3\alpha_i^2\lambda_i)}$$

Therefore,

$$k_{1i}=\frac{\lambda_i\left[\sqrt{[\Omega_{ii}^2-(\Omega_{ii}-3\alpha_i^2\lambda_i)(1-\lambda_i)]\Omega_{ii}}+\Omega_{ii}\right]}{(\Omega_{ii}-3\alpha_i^2\lambda_i)} \quad (33)$$

Note that $\Omega_{ii} < \sqrt{[\Omega_{ii}^2-(\Omega_{ii}-3\alpha_i^2\lambda_i)(1-\lambda_i)]\Omega_{ii}}$

So that $k_{1i} > 0$ and there is, therefore positive constant $k > k_{1i} > 0$

Theorem III

$MSE(\hat{\alpha}_M(KL)) < MSE(\hat{\alpha}_M)$ hence, to achieve this;

$MSE(\hat{\alpha}_M(KL)) - MSE(\hat{\alpha}_M) < 0$

Proof:

$MSE(\hat{\alpha}_M(KL))$-

$$MSE(\hat{\alpha}_M)=\sum_{i=1}^{p}\frac{(\lambda_i-k)^2\Omega_{ii}+4k^2\alpha_i^2\lambda_i}{\lambda_i(\lambda_i+k)^2}-\Omega_{ii} \quad (34)$$

$$=\sum_{i=1}^{p}\frac{(\lambda_i-k)^2\Omega_{ii}+4k^2\alpha_i^2\lambda_i-(\lambda_i-k)^2\lambda_i\Omega_{ii}}{\lambda_i(\lambda_i+k)^2}$$

$$=\sum_{i=1}^{p}\frac{(\lambda_i-k)^2(1-\lambda_i)\Omega_{ii}+4k^2\alpha_i^2\lambda_i}{\lambda_i(\lambda_i+k)^2} \quad (35)$$

If $(\lambda_i-k)^2(1-\lambda_i)\Omega_{ii} < 4k^2\alpha_i^2\lambda_i$, implies that $\sum_{i=1}^{p}\Omega_{ii} < \sum_{i=1}^{p}\frac{4k^2\alpha_i^2\lambda_i}{(\lambda_i+k)^2(1-\lambda_i)}$. Hence, *RKL* is better than M-estimator.

Choice of Robust biasing parameter

The biasing parameter for robust K-L estimator can be obtained by taking the partial derivative of $MSE(\hat{\alpha}_M(KL))$ with respect to *k*.

$$\frac{\partial}{\partial k}MSE(\hat{\alpha}_M(KL))=\frac{\partial}{\partial k}\left[\sum_{i=1}^{p}\frac{(\lambda_i-k)^2\Omega_{ii}}{\lambda_i(\lambda_i+k)^2}+\sum_{i=1}^{p}\frac{4k^2\alpha_i^2}{(\lambda_i+k)^2}\right] \quad (36)$$

$$\frac{(\lambda_i-k)8k\alpha_i^2}{(\lambda_i+k)^3}=\frac{4\lambda_i\Omega_{ii}(\lambda_i-k)}{\lambda_i(\lambda_i+k)^3}$$

$$2k\alpha_i^2=\Omega_{ii} \quad (37)$$

Therefore, $k_{RKLi}=\frac{\Omega_{ii}}{2\alpha_i^2}$ where *i = 1, 2, 3,…, p*

Ω_{ii} and α_i^2 are substituted with their unbiased estimates in $k_i=\frac{\Omega_{ii}}{2\alpha_i^2}$ where $\hat{\alpha}_M$ are assumed to be normally distributed with mean α and covariance matrix $G^2\wedge^{-1}$. The assumption holds if $\sqrt{n}(\alpha_M^2-\alpha)\rightarrow N(0,G^2\wedge^{-1})$ such that;

$G^2=\frac{v_0^2E(\psi^2(e/v_0))}{E(\psi^T(e/v_0))^2}$ with the scale estimates v_0. Therefore, the estimate of α_i^2 is α_{Mi}^2 and the unbiased estimator of Ω_{ii} is asymptotically $\frac{\hat{G}^2}{\lambda_i}$ where *G* is written as:

$$\hat{G}^2 = \frac{v^2(n-p)^{-1}\sum_{i=1}^{p}\left(\psi(e_i/v)\right)^2}{\sum_{i=1}^{p}\frac{1}{n}\left(\psi^T(e_i/v)\right)^2}$$ according to Hubber (1981)

PARAMETERS FOR SIMULATION

The following parameters were used to conduct the simulation study with the aid of R-statistical programming codes.

(i) Sample sizes (n) = (10, 30, 50 and100)

(ii) Equation to generate exogenous variables

$x_{ij} = (1-\rho^2)^{\frac{1}{2}} z_{ij} + \rho z_{ip+1}$, *i = 1, 2, …, n, j = 1, 2,…, p.*

(iii) Magnitude of outliers (mx) = (0, 5, and 10)

(iv) percentage of outliers (px) = (10% and 20%)

(v) Point of outliers; $X(i)_{outlier}$ = Mo*Max (X_i) + X_i

(vi) $\beta_i = (0,\ 1)$

(vii) $e_i \sim iidN(0,\sigma^2)$

(viii) Replication (RR) = 1000

(ix) Levels of correlation (0, 0.8, 0.9, 0.95, and 0.99)

(x) $MSE(\beta^*) = \frac{1}{1000}\sum_{i=1}^{1000}(\beta^* - \beta)^2$

Simulation procedures

The Monte Carlo experiment of this research was conducted using R-statistical programming language. The performances of some estimators were compared and contrasted, including OLS, Ridge, Liu, KL, DK, M-estimator, Ridge-M, Liu-M, DK-M, and (KL-M) Estimator. All the exogenous variables were generated using the equation given in (38) used by Lukman *et al*. (2020) among other researchers.

$$x_{ij} = (1-\rho^2)^{\frac{1}{2}} z_{ij} + \rho z_{ip+1}, \quad i = 1, 2, \dots, n,\ j = 1, 2, \dots, p. \tag{38}$$

where z_{ij} are independent standard normal pseudo-random numbers and ρ^2 means the correlation between any two exogenous variables. To exhibit the degrees of correlations between the explanatory variable, five (5) levels of different correlations were considered: 0, 0.8, 0.9, 0.95, and 0.99. Meanwhile, the number of exogenous variables is *p* = 3 and expressed in a standardized form. 10% and 20% of x_2 was inflated with the following degrees of outliers (0, 5, and 10). Likewise, the response variable was generated using the following equations:

$$y = \beta_0 + \beta_1 x_1 + \dots + \beta_p x_p + e_i,\ i = 1,\ \dots,\ p. \tag{39}$$

where $e_i \sim iidN(0,\sigma^2)$ and without any loss of generality, zero intercept was assumed for the model in (39). Following the research of Kaciranlar and Sakallioglu (2001), Yang and Chang (2010), Lukman *et al*. (2020), and Ayinde *et al*. (2020), the values of β was chosen to satisfy the constraints $\beta^T\beta = 1$ suggested by Lukman *et al*. (2020). The simulation study was replicated 1000 for the sample sizes n= 10, 30, 50, and 100, respectively, with error variance: 1, 5, and 10. Furthermore, the estimated MSE for each of the estimators was obtained for each replicate as in equation (40).

$$MSE(\beta^*) = \frac{1}{1000}\sum_{i=1}^{1000}(\beta^* - \beta)^2 \tag{40}$$

In the same vein, individual Ridge parameters of the estimators considered in the study were used.

SIMULATION RESULTS AND DISCUSSION

Following the simulation results as presented in Tables 1–4, likewise from figures 1 – 6. We hereby have the following comments:

(i) As expected, the OLS estimator performed woefully as the multicollinearity and outliers increased simultaneously in the x-direction.

(ii) As the error variances (σ^2), level of multicollinearity (rho) and percentage (px), and magnitude (mx) of outliers increase, and the MSE of the estimators' considered increase.

(iii) The MSEs of the estimators decreases as the sample size (n) increases.

(iv) When rho>0, mx>0, the percentage of outliers (px) increases, and sample size (n) increases KL-M along sides, KL outperformed other estimators as the two anomalies occur simultaneously in the x-direction.

(v) The KL-M performed better, especially when the sample size was n=100. Meanwhile, ordinary KL competes favourably with KL-M at different values of rho, error variances, the magnitude of outliers (mx), percentage of an outlier (px), and sample sizes (n).

(vi) Conclusively, at the different biasing parameters of the estimators, KL-M performed better than other estimators considered in the study. Also, in some cases, ordinary KL did well when there was a multicollinearity problem and outliers in the x-direction.

TABLE 1: Estimated MSEs of the estimators when there are no outliers

		px=0.1, mx=0									
		OLS	RE	LE	KL	DK	M	RidgM	Liu-M	DK-M	KL-M
, σ=1	rho=0	0.8378	0.4537	0.5184	0.4227	0.5478	0.9109	0.4488	0.5156	0.5422	**0.4169**
	rho=0.8	2.1436	0.7975	0.8397	0.7001	1.2173	2.3502	0.785	0.8303	1.2038	**0.6877**
	rho=0.9	4.0071	1.3067	1.111	1.12	2.0672	4.3998	1.2717	1.1137	2.0527	**1.0897**
	rho=0.95	7.7475	2.2736	2.0323	1.9212	3.7668	8.5133	2.241	1.9929	3.7189	**1.8931**
	rho=0.99	37.699	10.101	5.2451	8.4144	16.35	41.452	9.9637	**5.1168**	16.15	8.316

	σ=5	rho=0	20.944	7.3591	12.571	6.9533	10.986	22.772	7.2697	12.521	10.933	**6.859**
n=10		rho=0.8	53.589	16.412	19.868	14.675	25.732	58.756	16.225	19.542	25.445	**14.498**
		rho=0.9	100.18	28.895	26.296	25.089	45.757	109.99	28.547	26.241	45.263	**24.801**
		rho=0.95	193.69	53.591	48.77	45.58	89.109	212.83	52.917	48.031	88.17	**45.073**
		rho=0.99	942.47	249.46	127.24	208.08	399.31	1036.3	246.18	**123.69**	395.15	205.85
	σ=10	rho=0	83.777	28.508	50.228	27.12	43.5	91.089	28.16	50.026	43.287	**26.749**
		rho=0.8	214.36	65.007	79.257	58.31	102.18	235.02	64.294	77.909	100.99	**57.639**
		rho=0.9	400.71	115.04	105.21	100.03	181.84	439.98	113.68	104.79	179.89	**98.914**
		rho=0.95	774.75	213.89	194.78	182.02	355.95	212.83	52.917	191.98	88.17	**45.073**
		rho=0.99	3769.9	997.46	508.06	832.02	1595	4145.2	984.36	**493.86**	1578.9	823.16
	, σ=1	rho=0	0.1651	0.1456	0.1518	**0.1444**	0.1531	0.176	0.1458	0.1518	0.1531	0.1445
		rho=0.8	0.4626	0.2441	0.3342	**0.2291**	0.3448	0.4957	0.2454	0.3347	0.3448	0.2306
		rho=0.9	0.8686	0.3578	0.4714	**0.3188**	0.5702	0.9312	0.3615	0.4726	0.5712	0.3231
		rho=0.95	1.6827	0.5655	0.6085	**0.4796**	0.9785	1.8043	0.5745	0.6131	0.982	0.4905
		rho=0.99	8.199	2.1703	1.8392	**1.7535**	3.8961	8.7937	2.2248	1.8982	3.9379	1.8245
	σ=5	rho=0	4.1271	1.7972	3.7734	**1.7523**	2.5294	4.3989	1.8203	3.7734	2.5489	1.7762
n=30		rho=0.8	11.565	3.661	8.1876	**3.2672**	5.9723	12.392	3.7413	8.1872	6.0214	3.3616
		rho=0.9	21.716	6.2323	11.479	5.3554	10.255	12.823	3.1926	11.508	5.0427	**2.7343**
		rho=0.95	42.067	11.311	14.399	**9.4375**	18.836	45.108	11.603	14.453	19.165	9.8154
		rho=0.99	204.98	51.446	43.537	**41.625**	90.002	219.84	52.886	45.074	91.341	43.552
	σ=10	rho=0	16.508	6.0734	15.091	**5.9745**	9.4275	17.596	6.1915	15.091	9.5335	6.0949
		rho=0.8	46.262	14.022	32.734	**12.635**	23.019	49.566	14.355	32.726	23.223	13.024
		rho=0.9	86.865	24.419	45.878	**21.083**	40.082	93.12	25.028	46	40.572	21.844
		rho=0.95	168.27	44.805	57.547	**37.459**	74.352	180.43	45.979	57.87	75.652	38.984
		rho=0.99	819.9	205.42	173.6	**166.24**	358.78	879.37	211.18	179.92	364.02	173.97
	, σ=1	rho=0	0.0976	0.09	0.0933	**0.0896**	0.0923	0.1028	0.09	0.0933	0.0923	**0.0896**
		rho=0.8	0.2598	0.1648	0.2241	**0.1598**	0.2109	0.2729	0.1647	0.2241	0.2107	**0.1598**
		rho=0.9	0.485	0.2396	0.3523	0.2229	0.3547	0.5091	0.2397	0.3515	0.3539	**0.223**
		rho=0.95	0.9368	0.3626	0.5139	**0.3183**	0.6088	0.983	0.363	0.5134	0.6072	0.3189
		rho=0.99	4.5536	1.2498	**0.7956**	1.0019	2.2695	4.7767	1.2545	0.8027	2.2677	1.01
	σ=5	rho=0	2.4406	1.1623	2.3292	**1.1245**	1.5778	2.5691	1.1677	2.3292	1.5812	1.1303
		rho=0.8	6.4948	2.1566	5.5896	**1.9156**	3.4787	6.8215	2.1709	5.5896	3.4886	1.9323
n=50		rho=0.9	12.126	3.5601	8.5632	**3.0335**	5.9508	12.728	3.5809	8.5679	5.9656	3.0622
		rho=0.95	23.42	6.3454	12.268	**5.2324**	10.886	24.574	6.3793	12.241	10.911	5.2887

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		rho=0.99	113.84	28.385	**18.222**	22.583	49.058	119.42	28.535	18.419	48.984	22.878
	σ=10	rho=0	9.7624	3.638	9.3171	3.5605	5.6295	10.305	3.6307	9.3171	5.6579	**3.5426**
		rho=0.8	25.979	7.9666	22.357	**7.1606**	13.048	29.069	8.2078	22.357	13.478	7.2999
		rho=0.9	48.503	13.687	34.243	**11.738**	22.817	54.726	14.237	34.264	24.195	12.124
		rho=0.95	93.679	24.902	48.974	**20.591**	42.54	106.17	26.021	48.85	45.002	21.436
		rho=0.99	455.36	113.15	**72.944**	90.036	194.95	517.96	119.14	73.759	206.02	94.907
	, σ=1	rho=0	0.0457	0.0458	**0.0408**	0.0458	0.0448	0.045	0.0434	1.0494	0.0424	0.0434
		rho=0.8	0.1454	0.098	0.1603	0.0955	0.1217	0.1382	0.0935	0.1177	0.1131	**0.0918**
		rho=0.9	0.2773	0.147	0.2864	0.1376	0.2091	0.2624	0.1416	0.1982	0.1954	**0.1346**
		rho=0.95	0.5414	0.2199	0.4774	0.193	0.3618	0.5111	0.2115	0.3134	0.3396	**0.1899**
		rho=0.99	2.6554	0.6828	0.8641	0.5312	1.3225	2.5017	0.6436	0.6947	1.2449	**0.5071**
	σ=5	rho=0	1.1414	0.6774	1.0384	0.6791	0.8213	1.125	**0.6693**	1.0494	0.8007	0.676
		rho=0.8	3.6353	1.0899	4.0045	0.9277	1.8694	3.4554	1.0414	2.9404	1.7658	**0.8965**
		rho=0.9	6.9319	1.7993	7.1627	1.4722	3.1716	6.5589	1.7004	4.953	2.95	**1.4041**
		rho=0.95	13.535	3.2125	11.931	2.555	5.7089	12.777	3.016	7.765	5.3312	**2.4142**
n=100		rho=0.99	66.384	14.437	21.599	11.16	26.716	62.544	13.452	16.328	24.763	**10.447**
	σ=10	rho=0	4.5655	1.8173	4.1642	1.8054	2.6897	4.4999	**1.7867**	4.1984	2.6067	1.7889
		rho=0.8	14.541	3.9054	16.016	3.3625	6.7264	13.822	3.7091	11.762	6.3105	**3.2188**
		rho=0.9	27.728	6.7887	28.652	5.5999	11.867	26.236	6.3925	19.812	10.897	**5.3107**
		rho=0.95	54.141	12.475	47.721	9.9666	21.922	51.108	11.684	31.047	20.317	**9.3918**
		rho=0.99	265.54	57.411	86.393	44.414	106.01	250.17	53.467	65.248	97.982	**41.556**

Table 2: Estimated MSEs of the estimators when there are 10% outliers and the magnitude of outliers is 5

			Px=0.1, MX=5									
			OLS	RE	LE	KL	DK	M	RidgM	Liu-M	DK-M	KL-M
	, σ=1	rho=0	1.0015	0.4343	0.5108	0.38996	0.5517	1.0739	0.4295	0.512	0.5487	**0.3856**
		rho=0.8	1.7256	0.6097	0.7232	0.5209	0.8604	1.8794	0.6019	0.724	0.8468	**0.5196**
		rho=0.9	2.8917	0.8706	0.7761	**0.71949**	1.2386	3.1942	0.8619	0.778	1.2223	0.7285
		rho=0.95	2.4952	0.7221	0.691	**0.5869**	1.034	2.6965	0.7118	0.685	1.0161	0.5903
		rho=0.99	18.103	4.0717	4.2383	**3.226**	6.7327	20.088	4.027	4.212	6.6201	3.3588
	σ=5	rho=0	25.038	8.222	12.673	**7.49739**	12.271	26.848	8.1721	12.71	12.223	7.5181
		rho=0.8	43.139	12.61	17.744	**11.3752**	20.547	46.984	12.526	17.74	20.235	11.591
n=10		rho=0.9	72.292	19.322	19.253	**16.9245**	29.773	79.856	19.231	19.4	29.425	17.444
		rho=0.95	62.38	15.621	17.276	**13.5535**	24.749	67.412	15.452	17.17	24.323	13.903
		rho=0.99	452.58	99.803	105.45	**80.5831**	167.76	502.2	98.788	104.7	164.96	84.29
	σ=10	rho=0	100.15	32.406	50.678	**29.6918**	49.088	107.39	32.222	50.81	48.894	29.801
		rho=0.8	172.56	49.952	70.849	**45.3135**	82.075	187.94	49.634	70.84	80.859	46.211
		rho=0.9	289.17	76.875	77.021	**67.6008**	118.94	319.42	76.53	77.63	117.61	69.712

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		rho=0.95	249.52	62.053	69.134	54.0993	98.827	67.412	15.452	68.74	24.323	**13.903**
		rho=0.99	1810.3	398.92	421.6	**322.336**	670.84	2008.8	394.87	418.5	659.63	337.21
	, σ=1	rho=0	0.1194	0.0953	0.106	0.09493	0.1107	0.126	0.0952	0.106	0.1106	**0.0948**
		rho=0.8	0.2775	0.1539	0.1922	**0.14309**	0.2032	0.2924	0.154	0.192	0.2033	0.1432
		rho=0.9	0.4874	0.2015	0.2472	**0.17015**	0.2713	0.5129	0.2018	0.247	0.2715	0.1706
		rho=0.95	1.1693	0.3434	0.3838	**0.25628**	0.469	1.246	0.3453	0.384	0.4724	0.2601
		rho=0.99	4.7509	1.0214	0.8556	**0.72704**	1.5909	4.9928	1.032	0.854	1.5993	0.7648
	σ=5	rho=0	2.984	1.1077	2.6488	**1.03419**	1.6516	3.1512	1.1168	2.649	1.6596	1.0445
		rho=0.8	6.9372	2.0949	4.7997	**1.82202**	3.1941	7.3105	2.1107	4.8	3.2081	1.8558
n=30		rho=0.9	12.185	3.1663	6.1707	**2.65478**	5.0246	12.823	3.1926	6.165	5.0427	2.7343
		rho=0.95	29.233	6.7521	9.5966	**5.52036**	11.334	31.15	6.8326	9.598	11.38	5.7225
		rho=0.99	118.77	23.833	21.352	**18.1724**	39.629	124.82	24.132	24.11	39.827	19.371
	σ=10	rho=0	11.936	3.8783	10.595	**3.59132**	6.3491	12.605	3.9275	10.6	6.3724	3.6474
		rho=0.8	27.749	7.8519	19.199	**6.9733**	12.712	29.242	7.9225	19.2	12.767	7.1235
		rho=0.9	48.74	12.228	24.682	**10.4585**	20.074	51.29	12.338	24.66	20.155	10.799
		rho=0.95	116.93	26.672	38.392	**22.0611**	45.339	124.6	27.002	38.4	45.52	22.89
		rho=0.99	475.09	95.066	85.413	**72.7301**	158.47	499.28	96.258	85.23	159.25	77.561
	, σ=1	rho=0	0.0919	0.0752	0.0773	**0.07489**	0.0861	0.096	0.0752	0.077	0.0861	**0.0749**
		rho=0.8	0.1664	0.1057	0.164	0.10126	0.1375	0.1739	0.1054	0.164	0.1373	**0.101**
		rho=0.9	0.2899	0.1411	0.2216	0.12562	0.1945	0.3025	0.1404	0.222	0.1942	**0.1249**
		rho=0.95	0.4696	0.175	0.2768	0.14018	0.2443	0.488	0.1739	0.277	0.2434	**0.1391**
		rho=0.99	2.3277	0.5085	0.6667	0.33332	0.7875	2.426	0.5021	0.67	0.7774	**0.3326**
	σ=5	rho=0	2.2978	0.8766	1.9325	**0.80814**	1.2637	2.3996	0.8787	1.932	1.265	0.8112
		rho=0.8	4.1608	1.2788	4.0942	1.09895	1.9915	4.3484	1.2773	4.096	1.9843	**1.1003**
n=50		rho=0.9	7.2478	1.9231	5.5305	**1.58708**	2.9898	7.5637	1.914	5.529	2.972	1.5902
		rho=0.95	11.739	2.6771	6.8967	**2.11404**	4.4626	12.2	2.6641	6.898	4.4235	2.1565
		rho=0.99	58.193	11.035	16.594	**7.99938**	19.447	60.651	10.898	16.72	19.166	8.137
	σ=10	rho=0	9.1912	2.9962	7.73	2.74405	4.7823	8.5124	2.5939	7.73	4.3331	**2.381**
		rho=0.8	16.643	4.6801	16.375	**4.10533**	7.92	22.223	5.5901	16.38	9.774	4.9273
		rho=0.9	28.991	7.2955	22.119	**6.18214**	11.907	36.926	8.2267	22.11	13.87	6.9236
		rho=0.95	46.957	10.36	27.575	**8.38187**	17.798	59.536	12.448	27.58	20.295	10.27
		rho=0.99	232.77	43.872	66.349	**32.0317**	77.825	308.89	56.666	66.87	99.997	44.66
	, σ=1	rho=0	0.0292	**0.0272**	0.0288	**0.02718**	0.0287	0.0308	0.0273	0.721	0.0288	0.0273
		rho=0.8	0.0584	0.0472	0.1455	**0.04674**	0.0545	0.0629	0.0481	0.056	0.0563	0.0476
		rho=0.9	0.2773	0.147	0.1667	0.13757	0.2091	0.1154	0.071	0.095	0.0939	**0.0678**
		rho=0.95	0.3279	0.1211	0.5258	0.0957	0.1798	0.2406	0.1039	0.159	0.152	**0.0895**
		rho=0.99	0.9535	0.2335	0.8844	**0.15131**	0.3163	1.0547	0.2469	0.308	0.3318	0.163
	σ=5	rho=0	0.7312	0.3462	0.7202	**0.3321**	0.4904	0.77	0.3512	0.721	0.4954	0.3376
		rho=0.8	1.4597	0.5036	3.6347	**0.43644**	0.7001	1.5733	0.5121	1.412	0.7069	0.4392
n=100		rho=0.9	2.4987	0.6959	4.1664	**0.56524**	1.0364	2.8862	0.7237	2.376	1.0835	0.5787
		rho=0.95	8.1973	1.5891	13.143	1.16086	2.7079	6.014	1.2929	3.969	2.072	**1.0187**

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		rho=0.99	23.837	4.3726	22.108	**3.19124**	7.3989	26.367	4.7526	7.731	7.973	3.7053
	σ=10	rho=0	2.9246	1.008	2.8804	**0.93066**	1.5419	3.08	1.0361	2.884	1.5891	0.9612
		rho=0.8	5.8389	1.6535	14.538	**1.44357**	2.6096	6.2932	1.706	5.647	2.6586	1.4896
		rho=0.9	9.9947	2.4556	16.665	**2.07699**	4.0953	11.545	2.5801	9.503	4.3082	2.1714
		rho=0.95	32.789	6.0894	52.573	4.64158	10.807	24.056	4.902	15.87	8.2803	**4.0608**
		rho=0.99	95.348	17.259	88.433	**12.8134**	29.599	105.47	18.796	30.94	31.894	14.912

Table 3: Estimated MSEs of the estimators when there are 20% outliers and the magnitude of outliers is 5

			Px=0.2, MX=5									
			OLS	RE	LE	KL	DK	M	RidgM	Liu-M	DK-M	KL-M
	, σ=1	rho=0	1.47383	0.5394	0.6915	0.4684	0.7962	1.592	0.5324	0.693	0.7849	**0.4638**
		rho=0.8	3.85975	1.0984	0.7555	0.9204	1.6852	4.0085	1.0617	**0.7351**	1.641	0.8997
		rho=0.9	6.51617	1.6526	1.9184	1.3669	2.798	3.4833	1.0061	1.8716	1.4471	**0.8669**
		rho=0.95	3.34841	1.0177	0.9679	0.8721	1.4619	3.4833	1.0061	0.9668	1.4471	**0.8669**
		rho=0.99	34.4908	7.4584	6.842	**5.9823**	11.794	35.565	7.2303	6.6257	11.423	5.9834
	σ=5	rho=0	36.8457	10.623	17.051	**9.3745**	19.193	39.8	10.545	17.097	18.895	9.4535
n=10		rho=0.8	96.4937	24.798	18.561	21.785	41.966	100.21	23.975	**18.037**	40.883	21.511
		rho=0.9	162.904	38.916	47.759	**33.609**	69.923	173.4	37.853	46.609	68.306	33.645
		rho=0.95	83.7103	22.766	23.999	20.597	36.439	87.082	22.549	24.005	36.069	**20.572**
		rho=0.99	862.269	184.54	170.94	**149.7**	294.81	889.12	178.92	165.5	285.54	149.97
	σ=10	rho=0	129.395	36.691	68.173	**32.637**	65.834	159.2	41.767	68.36	75.575	37.563
		rho=0.8	337.709	90.271	74.186	80.159	150.34	400.85	95.521	**72.081**	163.53	85.971
		rho=0.9	518.203	130.45	191.03	**111.57**	241.54	693.62	151.07	186.43	273.23	134.58
		rho=0.95	334.841	90.636	95.981	82.256	145.76	348.33	89.778	96.014	144.28	**82.174**
		rho=0.99	3054.76	651.55	683.76	**524.26**	1053.9	3556.5	715.42	662	1142.2	599.94
	, σ=1	rho=0	0.14228	0.1047	0.1265	0.1035	0.1278	0.1494	0.1045	0.1265	0.1277	**0.1033**
		rho=0.8	0.24151	0.1448	0.1931	0.1384	0.191	0.2526	0.1444	0.1931	0.1906	**0.1379**
		rho=0.9	0.40951	0.1894	0.2377	0.1681	0.2585	0.4253	0.1881	0.2373	0.2581	**0.1666**
		rho=0.95	0.6787	0.25	0.3586	0.2046	0.3505	0.7097	0.2492	0.3563	0.3498	**0.2039**
		rho=0.99	3.41887	0.7582	0.7306	**0.5302**	1.1947	3.5079	0.745	0.7297	1.1696	0.5351
	σ=5	rho=0	3.55701	1.1903	3.1602	**1.0817**	1.8977	3.7352	1.1918	3.1602	1.8975	1.0835
		rho=0.8	6.03769	1.8338	4.826	1.613	2.962	6.3158	1.829	4.8261	2.9512	**1.6112**
		rho=0.9	10.2377	2.7488	5.9395	2.3344	4.5941	10.633	2.7251	5.9301	4.5378	**2.3318**
n=30		rho=0.95	16.9674	4.1538	8.7882	**3.4546**	7.145	17.744	4.1636	8.7573	7.0888	3.5349
		rho=0.99	85.4717	17.108	18.457	**12.991**	29.147	87.696	16.809	18.455	28.589	13.407
	σ=10	rho=0	14.228	4.2858	12.64	**3.874**	7.4517	14.941	4.2998	12.64	7.4499	3.89
		rho=0.8	24.1508	6.8755	19.304	**6.0933**	11.78	25.263	6.8615	19.304	11.736	6.0975
		rho=0.9	40.9509	10.568	23.758	**9.0963**	18.326	42.533	10.478	23.72	18.108	9.1067
		rho=0.95	67.8696	16.197	35.097	**13.665**	28.466	70.974	16.246	34.982	28.255	14.015
		rho=0.99	341.887	68.142	73.964	**51.983**	116.49	350.79	66.949	74.681	114.28	53.696
	, σ=1	rho=0	0.08368	0.0694	0.0788	**0.0692**	0.0791	0.0868	0.0694	0.0788	0.0791	**0.0692**
		rho=0.8	0.1789	0.1081	0.1462	0.1026	0.1448	0.1885	0.108	0.1463	0.1446	**0.1025**
		rho=0.9	0.30941	0.1405	0.2302	**0.1222**	0.1996	0.3261	0.1403	0.2299	0.1993	0.1302

		rho=0.95	0.42457	0.1718	0.2552	0.1428	0.2385	0.4476	0.1715	0.2557	0.238	**0.1425**
		rho=0.99	2.36945	0.5055	0.5374	**0.328**	0.7812	2.5066	0.5062	0.5355	0.7806	0.3338
	σ=5	rho=0	2.092	0.7719	1.9705	**0.7192**	1.1661	2.1711	0.7726	1.9705	1.163	0.7198
		rho=0.8	4.47249	1.2388	3.656	**1.0451**	2.0018	4.7114	1.248	3.6565	2.0093	1.0556
		rho=0.9	7.73523	1.8427	5.7547	**1.4836**	2.9524	8.1526	1.8523	5.746	2.9573	1.5011
n=50		rho=0.95	10.6142	2.5646	6.3595	**2.0661**	4.0668	11.191	2.5745	6.3824	4.0613	2.1083
		rho=0.99	59.2364	10.943	13.451	**7.9252**	19.525	62.664	10.994	13.414	19.509	8.2114
	σ=10	rho=0	8.368	2.6292	7.8822	**2.4063**	4.4413	8.7951	2.6473	7.8822	4.3991	2.4247
		rho=0.8	17.89	4.5584	14.624	**3.9181**	8.0037	18.857	4.702	14.626	8.0785	4.0829
		rho=0.9	30.9409	7.0178	23.018	**5.7988**	11.811	40.374	8.7716	22.984	14.629	7.2901
		rho=0.95	42.4568	9.9255	25.433	**8.1733**	16.27	49.022	10.409	25.526	16.994	8.5986
		rho=0.99	236.945	43.522	53.823	**31.745**	78.099	243.19	43.636	53.673	76.874	33.432
	, σ=1	rho=0	0.03115	0.0288	0.0332	0.0288	0.0305	0.0325	**0.0286**	0.0303	0.0303	**0.0286**
		rho=0.8	0.0622	0.0495	0.1561	0.049	0.0578	0.0643	0.0489	0.0577	0.0575	**0.0484**
		rho=0.9	0.10661	0.0696	0.2225	0.0669	0.0908	0.1176	0.0698	0.0946	0.0937	**0.0664**
		rho=0.95	0.29382	0.1181	0.4395	0.0972	0.1743	0.2267	0.1011	0.154	0.1484	**0.0879**
		rho=0.99	0.9928	0.2449	0.8031	0.1613	0.3261	1.0041	0.2343	0.2947	0.3146	**0.1527**
	σ=5	rho=0	0.77865	0.359	0.8231	0.3439	0.5145	0.8127	0.3569	0.7579	0.512	**0.342**
		rho=0.8	1.55501	0.5212	3.9007	0.4489	0.7239	1.6063	0.5096	1.4415	0.7077	**0.4383**
		rho=0.9	2.66533	0.7309	5.5628	0.5944	1.0744	2.9406	0.707	2.3639	1.0496	**0.5693**
		rho=0.95	7.34545	1.5349	10.988	1.1568	2.5464	5.6666	1.2255	3.8477	1.971	**0.9703**
		rho=0.99	24.82	4.6654	20.103	3.5157	7.8399	25.103	4.4334	7.3767	7.5383	**3.4392**
	σ=10	rho=0	3.1146	1.0549	3.3219	**0.9735**	1.6461	3.2509	1.0547	3.0315	1.636	0.9749
n=100		rho=0.8	6.22004	1.7324	15.603	1.5072	2.7326	6.4252	1.6948	5.7662	2.6772	**1.4837**
		rho=0.9	10.6613	2.6028	22.251	2.2079	4.2749	11.762	2.5224	9.4554	4.1921	**2.1517**
		rho=0.95	29.3818	5.8662	43.952	4.6088	10.184	22.666	4.6208	15.39	7.8777	**3.8528**
		rho=0.99	99.2801	18.435	80.41	14.124	31.368	100.41	17.511	29.512	30.152	**13.845**

Table 4: Estimated MSEs of the estimators when there are 20% outliers and the magnitude of outliers is 10

						Px=0.2, MX=10						
			OLS	RE	LE	KL	DK	M	RidgM	Liu-M	DK-M	KL-M
	, σ=1	rho=0	1.5017	0.543	0.6957	0.4695	0.8035	1.6227	0.5356	0.6963	0.7925	**0.4644**
		rho=0.8	3.9086	1.106	**0.7395**	0.9252	1.71619	4.0438	1.0637	0.7182	1.6645	0.9
		rho=0.9	6.6372	1.671	1.9684	1.3802	2.87179	7.0272	1.6209	1.9181	2.7987	**1.3672**
		rho=0.95	3.3499	1.007	0.9638	0.8631	1.46355	3.4853	0.9945	0.962	1.4481	**0.8574**
		rho=0.99	34.739	7.486	6.8063	**6.0086**	11.8295	35.819	7.2549	6.5936	11.455	6.0132
n=10	σ=5	rho=0	37.541	10.72	17.177	**9.448**	19.4479	40.568	10.635	17.187	19.16	9.5124
		rho=0.8	97.714	25.04	18.222	21.963	42.8346	101.1	24.084	17.703	41.544	**21.6**
		rho=0.9	165.93	39.43	49.023	33.985	71.7941	175.68	38.269	47.78	69.972	**33.958**
		rho=0.95	83.748	22.56	23.876	20.385	36.5319	87.133	22.328	23.87	36.136	**20.344**
		rho=0.99	868.48	185.3	170.04	**150.37**	295.781	895.47	179.57	164.71	286.37	150.72
	σ=10	rho=0	129.24	36.37	68.68	**32.312**	65.5485	162.27	42.129	68.722	76.638	37.804
		rho=0.8	337.96	89.52	72.841	79.333	150.42	404.38	95.99	**70.766**	166.17	86.329

		rho=0.9	505.15	124.2	196.08	**105.3**	231.412	702.72	152.77	191.11	279.9	135.84
		rho=0.95	528.31	127.8	95.477	110.76	212.232	348.53	88.948	95.464	144.54	**81.267**
		rho=0.99	3043.6	649.6	680.14	**522.49**	1053.36	3581.9	718.03	658.81	1145.5	602.94
	, σ=1	rho=0	0.1401	0.103	0.1247	0.1014	0.12591	0.1473	0.1024	0.1247	0.1258	**0.1012**
		rho=0.8	0.2377	0.142	0.1895	0.1351	0.18858	0.2485	0.141	0.1895	0.1881	**0.1345**
		rho=0.9	0.4031	0.184	0.2373	0.1624	0.25272	0.4185	0.1827	0.2369	0.2526	**0.1609**
		rho=0.95	0.6599	0.237	0.3418	0.1905	0.3216	0.6907	0.2358	0.3407	0.3217	**0.1899**
		rho=0.99	3.4053	0.747	0.7248	**0.5181**	1.1548	3.4942	0.7339	0.7248	1.1321	0.5236
	σ=5	rho=0	3.5015	1.146	3.1173	**1.0382**	1.84346	3.6835	1.149	3.1173	1.8461	1.0415
		rho=0.8	5.9429	1.755	4.7357	1.5368	2.86678	6.213	1.749	4.7359	2.8584	**1.535**
		rho=0.9	10.078	2.633	5.9295	**2.2238**	4.43007	10.462	2.6085	5.9198	4.3762	2.2242
		rho=0.95	16.499	3.894	8.4959	**3.2069**	6.7177	17.267	3.9019	8.5796	6.6698	3.2894
n=30		rho=0.99	85.133	16.86	18.197	**12.726**	28.7837	87.355	16.565	18.212	28.23	13.163
	σ=10	rho=0	14.006	4.124	12.469	**3.7092**	7.27887	14.734	4.1432	12.469	7.2847	3.7307
		rho=0.8	23.772	6.588	18.943	**5.8002**	11.4522	24.852	6.5705	18.943	11.415	5.8055
		rho=0.9	40.312	10.14	23.717	**8.6713**	17.7107	41.85	10.045	23.679	17.494	8.6948
		rho=0.95	65.995	15.22	33.966	**12.706**	26.8504	69.069	15.255	33.883	26.667	13.067
		rho=0.99	340.53	67.16	72.833	**50.93**	115.121	349.42	65.991	72.902	112.92	52.725
	, σ=1	rho=0	0.0832	0.069	0.0786	**0.0687**	0.0787	0.0865	0.0689	0.0786	0.0787	**0.0687**
		rho=0.8	0.1778	0.107	0.145	0.1018	0.14406	0.1872	0.1072	0.145	0.1439	**0.1016**
		rho=0.9	0.3082	0.14	0.2297	0.1213	0.19921	0.3247	0.1394	0.2293	0.1989	**0.121**
		rho=0.95	0.4233	0.171	0.2531	0.1415	0.23816	0.4463	0.1704	0.254	0.2375	**0.1412**
		rho=0.99	2.368	0.505	0.5366	**0.327**	0.78004	2.5052	0.5052	0.535	0.7797	0.3328
	σ=5	rho=0	2.0804	0.758	1.9645	**0.7049**	1.14806	2.1624	0.7594	1.9645	1.1458	0.7063
		rho=0.8	4.4454	1.222	3.625	**1.0283**	1.98024	4.6799	1.2306	3.6256	1.9865	1.0381
		rho=0.9	7.706	1.824	5.7391	**1.4649**	2.92591	8.1179	1.8327	5.73	2.9304	1.4818
		rho=0.95	10.582	2.539	6.3212	**2.0404**	4.03497	11.158	2.5493	6.3445	4.0294	2.0827
		rho=0.99	59.199	10.92	13.426	**7.9022**	19.4981	62.631	10.971	13.384	19.485	8.1896
	σ=10	rho=0	8.3217	2.572	7.8582	**2.3468**	4.38393	8.7673	2.5959	7.8582	4.3528	2.3706
		rho=0.8	17.782	4.492	14.5	**3.8533**	7.91966	18.716	4.6081	14.502	7.959	3.9898
		rho=0.9	30.824	6.945	22.956	**5.7273**	11.7051	40.32	8.7064	22.92	14.574	7.2339
		rho=0.95	42.327	9.827	25.283	**8.0745**	16.1411	48.848	10.294	25.377	16.819	8.4829
		rho=0.99	236.8	43.43	53.713	**31.653**	77.9927	243.01	43.506	53.544	76.737	33.3
	, σ=1	rho=0	0.0308	0.028	0.0324	0.0285	0.03018	0.0323	0.0284	0.0301	0.0301	**0.0283**
		rho=0.8	0.0615	0.049	0.1554	0.0483	0.05723	0.0635	0.0483	0.057	0.0569	**0.0477**
		rho=0.9	0.1059	0.069	0.2242	0.0663	0.09037	0.1169	0.0693	0.094	0.0932	**0.0659**
		rho=0.95	0.2933	0.118	0.4359	0.0966	0.17423	0.2279	0.1024	0.1534	0.148	**0.0892**
		rho=0.99	0.992	0.244	0.8013	0.1606	0.32442	1.0038	0.2337	0.2947	0.3137	**0.1521**
	σ=5	rho=0	0.7708	0.351	0.8088	0.3357	0.50622	0.8072	0.3491	0.752	0.5038	**0.334**
		rho=0.8	1.5376	0.507	3.8861	0.4349	0.70274	1.5882	0.4945	1.425	0.6863	**0.4227**
		rho=0.9	2.6485	0.718	5.6051	0.5813	1.05527	2.9236	0.6969	2.3496	1.0378	**0.5593**
		rho=0.95	7.3328	1.526	10.897	1.1478	2.5349	5.6545	1.2136	3.8355	1.9537	**0.958**
n=100		rho=0.99	24.799	4.651	20.032	3.5026	7.81912	25.096	4.4213	7.3688	7.5284	**3.4269**

σ=10	rho=0	3.0832	1.021	3.2351	**0.9388**	1.6068	3.2289	1.0215	3.0082	1.6006	0.9409
	rho=0.8	6.1504	1.679	15.544	1.4559	2.67222	6.3526	1.6364	5.7001	2.6162	**1.4263**
	rho=0.9	10.594	2.553	22.421	2.1607	4.2149	11.694	2.4838	9.3982	4.15	**2.1156**
	rho=0.95	29.331	5.83	43.589	4.5749	10.1397	22.618	4.5755	15.342	7.8139	**3.8078**
	rho=0.99	99.195	18.38	80.127	14.072	31.2805	100.38	17.465	29.476	30.114	**13.798**

Source: Simulation results

Note: The values in bold form indicate the minimum MSE of the estimator.

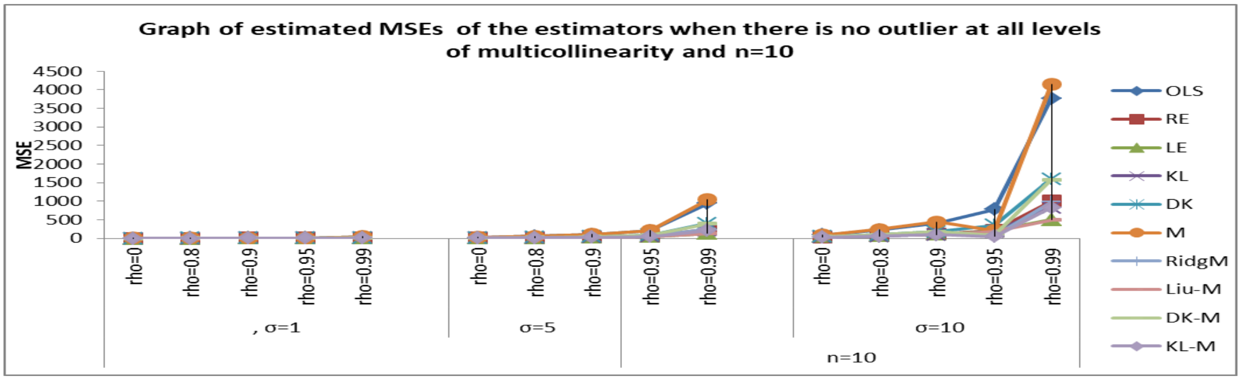

Figure 1: Graph of estimated MSEs of the estimators when there is no outlier at all levels of multicollinearity and n=10

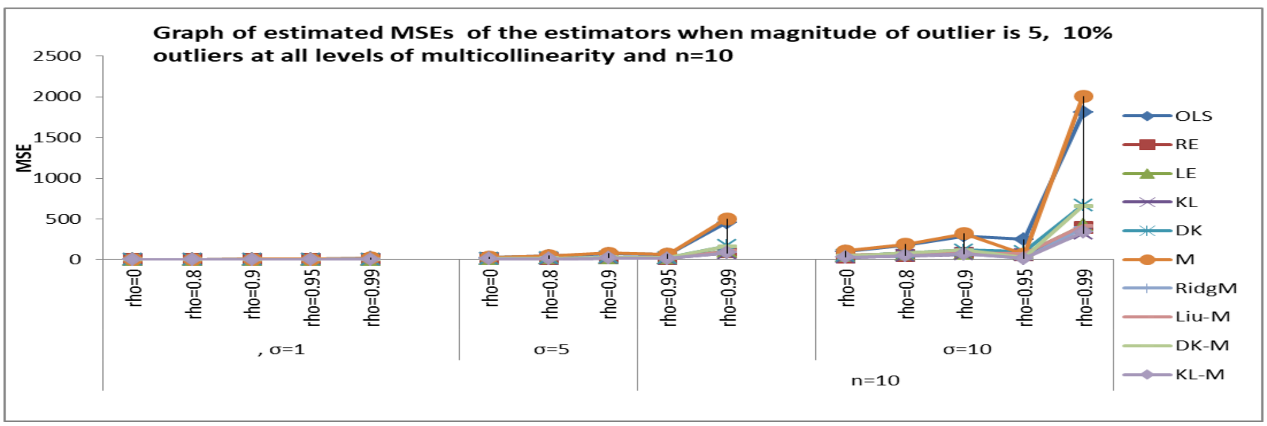

Figure 2: Graph of estimated MSEs of the estimators when the magnitude of outlier is 5, 10% outliers at all levels of multicollinearity and n=10

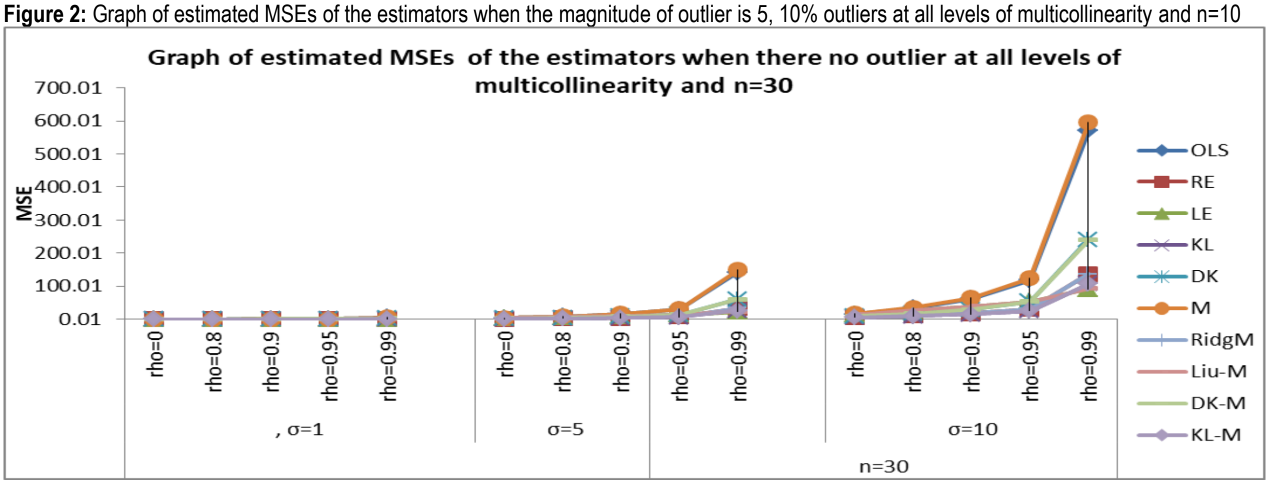

Figure 3: Graph of estimated MSEs of the estimators when there is no outlier at all levels of multicollinearity and n=30

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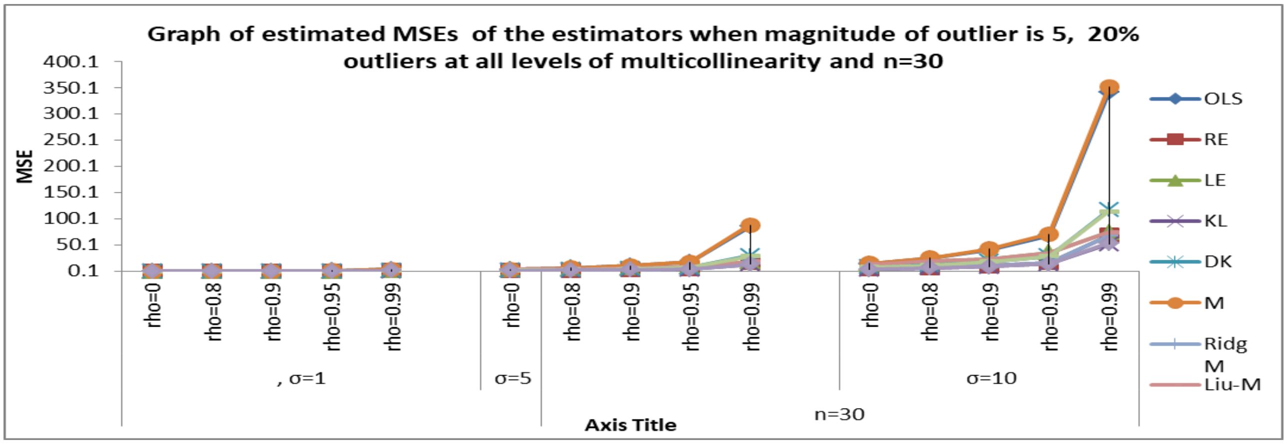

Figure 4: Graph of estimated MSEs of the estimators when the magnitude of outlier is 5, 20% outliers at all levels of multicollinearity and n=30

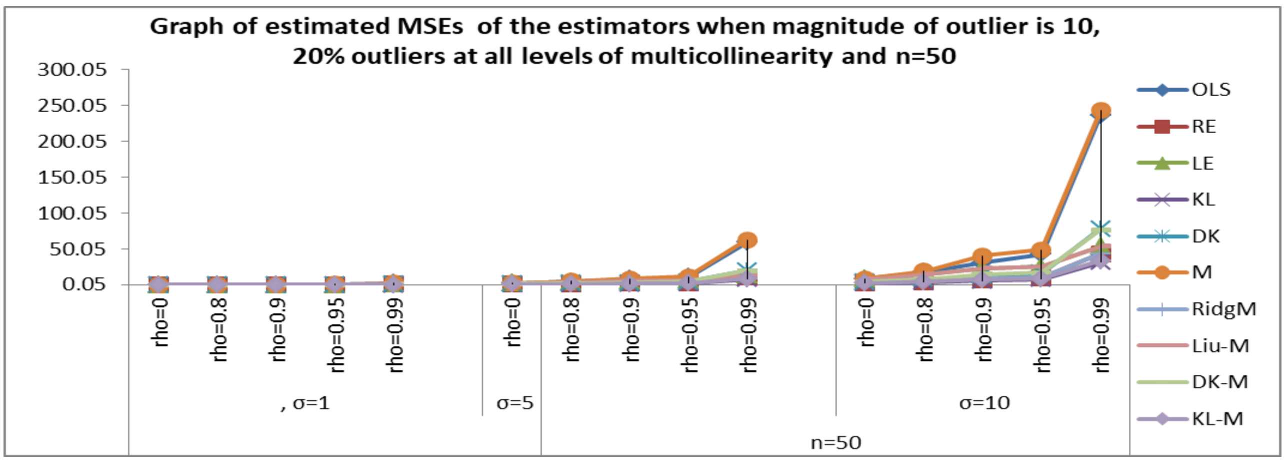

Figure 5: Graph of estimated MSEs of the estimators when the magnitude of outlier is 10, 20% outliers at all levels of multicollinearity and n=50

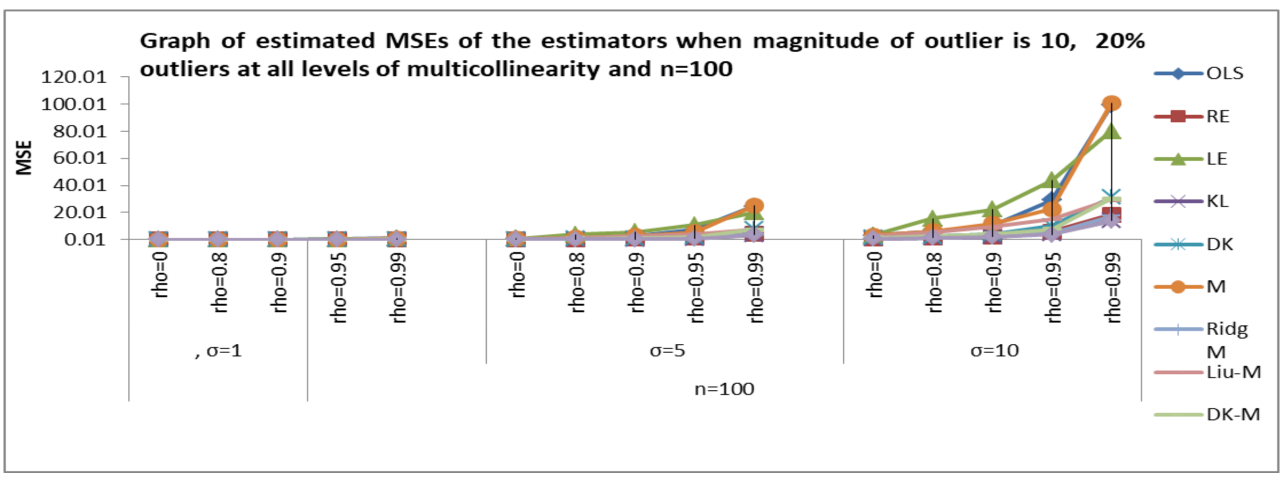

Figure 6: Graph of estimated MSEs of the estimators when the magnitude of outlier is 10, 20% outliers at all levels of multicollinearity and n=100

Application to real-life data
Longley data was adopted for real-life applications. Longley data was adopted by Longley (1967). Whereby the regression equation is defined as:

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + .\beta_4 x_4 + \beta_5 x_5 + \beta_6 x_6 \quad (41)$$

Where *y* is the total derived employment, x_1 is the gross national product implicit price deflator, x_2 is the gross national product, x_3 is unemployment, x_4 is the size of armed forces, x_5 is the non-institutional population 14 years of age and over and x_6 is the time. Meanwhile, Walke and Birch (1988) affirmed that the scaled condition number of the data is 43.275. This means that it suffers from the problem of multicollinearity and outliers. In the same vein, many researchers have used this data to identify influential points, such as; (Walker and Birch (1988); Cook (1977); Jahufer and Jianbao (2009); Jahufer, (2013); Yasin and Murat (2016); and Ullah *et al.* (2013)). As it can be seen from Table 5 that both KL-M and ordinary KL estimators have the least MSEs.

Table 5: Regression coefficients and MSEs of estimators using Longley data

Coef	OLS	RE	LE	KL	Dk	M	RIDGE-M	LIU-M	DK-M	KL-M
β_0	-3482.3	-2388.9	1144.16	-2218.9	-3261.8	-3639	-2404.38	1142	-3266.2	-2239.5
β_1	0.01506	-0.0063	110896	-0.0096	0.0108	-0.0074	-0.006	110810	0.01084	-0.0092
β_2	-0.0358	-0.0023	396318	0.00296	-0.0291	-0.0362	-0.00273	396611	-0.0292	0.00233
β_3	-0.0202	-0.0152	337545	-0.0144	-0.0192	-0.0201	-0.01526	337517	-0.0192	-0.0145
β_4	-0.0103	-0.0089	275440	-0.0087	-0.01	-0.0106	-0.00891	275412	-0.01	-0.0087
β_5	-0.0511	-0.1651	189537	-0.1828	-0.0741	-0.0719	-0.1635	188144	-0.0736	-0.1807
β_6	1.82915	1.27004	2033091	1.1831	1.7164	1.91185	1.27795	2E+06	1.71865	1.19365
MSE	792849	373133	4.1E+14	1.11153	695631	848074	404314	4E+14	746084	**1.18298**

Conclusion
The presence of multicollinearity and outliers in regression analysis is prone to a great threat to the OLS estimator; therefore, it becomes imperative to propose a robust estimator that can handle the two problems. Hence, this study examines the performance of the new robust one-parameter ridge-type estimator to handle the co-existence of the two problems in linear regression analysis when there are outlying cases in x-direction. In order to showcase the new estimator's superiority, theoretical expression under some conditions were established. A Monte Carlo experiment was performed alongside some factors to prove that this new robust estimator is better than the other estimators. In the same vein, real-life data was adopted to establish this fact, as it can be seen from Table 5 that the MSE of robust-M Kibria-Lukman (KL-M) has the least MSE.

Conflict of Interest:
The Authors hereby declare that there is no conflict of interest.

Data Availability
The data generated and analyzed during the current study are available from the corresponding author on reasonable request.

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