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Full Length Research Article

MATHEMATICAL MODELLING AND SIMULATION OF PRESSURE, TEMPERATURE, AND VELOCITY DISTRIBUTION OF TWO-PHASE FLOW IN A PRODUCING GAS WELL

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ABSTRACT
Mathematical modelling of pressure, temperature, and velocity distribution of two-phase flow in a producing gas well is an important phenomena in the management, design, and dynamic analysis of the wells. Many studies in producing gas wells focuses mostly on single phase gas flow during production but pressure traverses in two-phase flow differs from single phase due to the differential flow rates of the different phases and using single phase model may result to insufficient prediction leading to premature closure of wells. This work considered a one-dimensional time-dependent homogeneous model which represents a system of partial differential equations to describe mathematically the transient two-phase gas-liquid mixture flow in a producing well. The numerical solution of the mathematical model, which consists conservation of mass, momentum, and energy equations based on finite difference technique in the implicit scheme has been applied. PVT correlation is used in estimating the thermodynamic and transport properties of the fluids. From the results obtained, it was observed that Joule-Thomson affect sensitively in the prediction of pressure and other flow parameters which agreed with existing work. The curves obtained reflect the gas flowing law which can provide technical reliance and dynamic analysis of multiphase producing gas wells.

Keyword: Two phase flow, homogenous mathematical model, PVT correlation, producing gas well

INTRODUCTION
Two phase flow (liquid and gas) through gas wells and pipes is a complex phenomenon. To model this type flow, it is important to take into account the coexistence of several flow regimes along the well or pipe for a given set of operational conditions. Far back 1972, authors described flow such as bubble, froth, annular mist and slug to be flows that occurs in upward vertical. Generally there are flow regimes that depend on pipe properties such as pipe diameter, thermos-physical properties of fluids, inclination angle and flow rate of each phase. The simultaneous flow gas-liquid two phases in producing well is complex leading to many authors' attempts to develop predictive techniques through empirical or semi empirical methods but these methods were not suitable to transient flow analysis in the two phase flow in the producing gas well due to existence of the liquid holdup variation.

In the early 1980s, mechanistic modelling approach based on the fundamentals of multiphase flow and fluid dynamic has dominated research field in the fluid dynamics. The method started with local instantaneous conservation equation leading to systematic development in averaged relations for the variables of interest (pressure, temperature, velocity and liquid holdup). Research in the two phase flow and using mechanistic model cannot be complete without mentioning (Hemeida, 1987; Taitel et al, 1989; Ouyang and Aziz, 1999, and Ouyang and Aziz 2000). These authors were among the first researchers who proposed the mechanistic models, assuming that flow is under steady state condition. Their models only predict pressure profile neglecting other parameters. Parameters such as temperature profiles are predicted separately by authors such as (Xiao, 1987; Sagar et al., 1991; Alves et al., 1992) assuming steady state conditions. Other authors such as Mbaya and Amin, (2015), apply non isothermal model to predict the unsteady flow and heat transfer in a producing gas well using single phase gas flow. Mbaya and Amin, (2018), modified Mbaya and Amin (2015) by presenting a model which consider the energy equation for single phase gas flow. Farhan et al (2019) apply ANSYS Fluent to simulate the fluid flow and heat transfer in a weak wellbore of crude oil flowing upward in the tubing and gas (air) injection through holes to increase the wellbore production. Results show that crude oil velocity decreases downstream throughout the tubing. Liu et al (2013), presented a model for the determination of wellhead and bottom-hole pressure based on the principles of fluid dynamics considering fluid temperature to be constant neglecting the effect of the earth temperature on the temperature of the flowing fluid. Juiping et al, (2013), developed a couple systems of partial differential equations for the variation of pressure, temperature, velocity and density at different time and depth in high pressure, high temperature well for two phases. Their solution considers splitting techniques with Eulerian Generalized Riemann Problems (GRP) schemes but all these works does not consider two phase flow in the producing gas well.

In this paper a one-dimensional, time-dependent homogeneous mathematical model is presented, which can be used for determining the pressure, temperature, and velocity distributions of two-phase flow with two components (liquid and gas) in gas wells. The numerical solution of the mathematical model, which consists conservation of mass, momentum, and energy equations based on the finite difference technique in the implicit scheme. PVT correlation is used in estimating the thermodynamic and transport properties of the fluids while friction factor and the flow regimes are estimated using Beggs and Brill (1973) correlations. It was chosen because it is multiphase correlation that deals with both friction pressure loss, hydrostatic pressure difference and the liquid holdup calculation.

Developing the Governing Equations
The equation of two phase flow can be analysed using homogeneous model, two fluid model or drift flux models. As

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presented earlier in this work, the components are treated as a pseudo fluid that follows the usual equation of single component flow which allows the application of fluid mechanics. The drift-flux model is essentially a separated flow model in which attention is focused on the relative motion, rather than on the motion of individual phases. This model gives a very useful way for two phase particularly for steady state or quasi-steady state calculations. The two phase model takes into account the different properties and different velocities of the two fluids to some extent that separate equations of continuity, momentum and energy are written for each phase and solved simultaneous. The homogeneous model presented in this work is the type where gas phase and liquid phase (liquid and gas) are treated as a pseudo-single phase (mixture), which its density and velocity followed that of (Yadigaroglu and Lahey, 1976). The model is built upon the following assumptions:

1. The model is homogeneous one- dimensional and time-dependent.
2. The gas well consist only liquid and gas and effect of viscosity is neglected.
3. The gas-liquid mixture flow is transient and treated as a pseudo fluid that follows the usual equation of single component.
4. The flow is in vertical well effect of inclination is neglected

Governing Equations

The pressure, temperature and the velocity of the two fluids are considered simultaneously. The equations governing two phase flow is the continuity, momentum and the energy when considering area of wellbore which is given as.

$$\frac{\partial(\rho A)}{\partial t}=-\frac{\partial(A\rho u)}{\partial x} \qquad 1$$

$$\frac{\partial(\rho u)}{\partial t}+\frac{\partial \rho u^2}{\partial x}+\frac{\partial p}{\partial x}+\rho g\sin\theta=-\frac{\tau_w S}{A} \qquad 2$$

$$\frac{\partial h}{\partial t}+u\frac{\partial h}{\partial x}+u\frac{\partial u}{\partial t}+uu\frac{\partial u}{\partial x}+\frac{q\pi\sin\theta}{\rho A}+ug\sin\theta=\frac{1}{\rho}\frac{\partial p}{\partial t} \qquad 3$$

In equation (3) the enthalpy h, density ρ, and the velocity u of the mixture is obtain as (Yadigaroglu and Lahel, 1976)

$$h=\frac{\varepsilon_I\rho_I h_I+\varepsilon_g\rho_g h_g}{\rho} \qquad 4$$

$$\rho=\rho_I\varepsilon_I+\rho_g\varepsilon_g \qquad 5$$

$$u=\frac{\rho_I\varepsilon_I u_I+\rho_g\varepsilon_g u_g}{\rho}=\frac{\rho_I u_{sI}}{\rho}+\frac{\rho_g u_{sg}}{\rho} \qquad 6$$

In equation (4) to (6) the subscripts I and g represent the liquid and gas phase respectively while ε is volumetric fraction, u is average velocity u_s is superficial velocity and ρ is the density. Considering the two phase mixture, equations (1) to (3) above can be rewritten as in Octavio et al., (2005).

$$\frac{\partial p}{\partial t}+\frac{\rho a^2}{\gamma}\frac{\partial u}{\partial x}+u\frac{\partial p}{\partial x}=0 \qquad 7$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+\frac{\gamma}{\rho}\frac{\partial p}{\partial x}+g\sin\theta=-\frac{\gamma\tau_w S}{\rho A} \qquad 8$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}-D\frac{\partial p}{\partial t}-\frac{\beta}{C_p}\left(\frac{\partial p}{\partial t}+u\frac{\partial p}{\partial x}\right)+\frac{u}{Kg_cC_p}\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}\right)=\frac{ug\sin\theta}{Kg_cC_p} \qquad 9$$

In equation (7) to (9) A is cross sectional area assumed to be constant, other parameters are, β is a parameter that contains gas and liquid Joule-Thomson coefficient, x and t are distance and time respectively while τ_w wall friction shear stress, c_p is the specific heat at constant pressure, g_c is the conversion factor, T is temperature, p pressure, a is the sound wave, g is acceleration due to gravity and θ is the inclination angle, D and K are given as $D=\frac{\gamma}{Kg_c\rho C_\rho}$, $K=778.26$ and $\gamma=144$ respectively.

Numerical Solution Procedure

The flux vector splitting method of Steger Warming used by authors in solving single phase in a producing gas well becomes more complicated when solving the partial differential equations for two phase flow. Other methods such as method of characteristics, finite element, finite volume are used but reliably finite difference method is appreciated in the two phase flow. To apply finite difference method equation (7) to (9) are discretized by applying a first order downstream implicit scheme for spatial derivatives and a first order upstream implicit scheme for time derivatives. In this work it is assumed that the fluid condition in the node is the same as that at the exist point. The discretised form of equations (7) to (9) obtain by LINPACK (Dongara et al., 1990).

$$\frac{\Delta x}{\Delta t}\frac{c^2\rho}{K}(u)_j^{t+\Delta t}+\left[1+\frac{\Delta x}{\Delta t}(u)_0\right](p)_j^{t+\Delta t}=(p)_j^t+\frac{\frac{\Delta x}{\Delta t}c^2\rho}{K}(u)_{j-1}^{t+\Delta t}+\left[1+\frac{\Delta x}{\Delta t}(u)_0\right](p)_{j-1}^{t+\Delta t} \qquad 10$$

$$\left[1+\left(\frac{\Delta x}{\Delta t}u\right)_0\right](u)_j^{t+\Delta t}+\left(\frac{K\frac{\Delta x}{\Delta t}}{\rho}\right)(p)_j^{t+\Delta t}=(u)_j^t+\frac{\Delta x}{\Delta t}(u)_0(u)_{j-1}^{t+\Delta t}+\left(\frac{K}{\rho}\frac{\Delta x}{\Delta t}\right)(p)_{j-1}^{t+\Delta t}-g\Delta t\sin\theta-\left(\frac{K\tau_w S\Delta t}{\rho A}\right) \qquad 11$$

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$$+\frac{\Delta x}{\Delta t}(u)_0(T)_{j-1}^{t+\Delta t}-\left(D+\frac{\beta}{C_p}\right)(p)_j^t-\frac{\beta}{C_p}\frac{\Delta x}{\Delta t}(u)_0(p)_{j-1}^{t+\Delta t}+\frac{(u_0)}{D}(u)_j^t+\frac{(u)_0(u)_0}{D}(u)_{j-1}^{t+\Delta t}$$

$$\left[1+\left(\frac{\Delta x}{\Delta t}u\right)_0\right](T)_j^{t+\Delta t}-\left(D+\frac{\beta}{C_p}\left(1+\frac{\Delta x}{\Delta t}(u)_0\right)\right)(p)_j^{t+\Delta t}+\frac{(u)_0\left[1+\frac{\Delta x}{\Delta t}(u)_0\right]}{D}(u)_{j-1}^{t+\Delta t}=(T)_j^i$$

$$-\frac{(u)_0\, g\Delta t\sin\theta}{D} \qquad 12$$

The discretised equations (9) through (12) can be written in matrix form as follows

$$A_j\left(u_j^0\right)u_j^{t+\Delta t}=B_j\left(u_j^0,u_j^1,u_{j-1}^{t+\Delta t}\right) \qquad 13$$

In equation (13) Δt is the time steps while Δx is the space step. The calculation of odd t is obtain at t and new time is calculated at $t+\Delta t$, the subscript j indicate the cell number where it is calculated. In equation (13) the variable with subscript $j-1$ and the superscript of t are the inlet and are known which served as the initial condition. The dummy variables is represented by the superscript 0 and v represent the column vector of independent variable given by $v=(p,u,T)^{\Gamma}$. Γ is the transpose of the column vector. The matrix A and column vector B obtained from equations (9) through (12) is as follows;

$$A_j=\begin{bmatrix}\left(\frac{\gamma}{\rho}\frac{\Delta x}{\Delta t}\right) & \left[1+(u)_o\frac{\Delta x}{\Delta t}\right] & 0\\ \left[1+(u)_o\right]\frac{\Delta x}{\Delta t} & \frac{a^2\rho}{\gamma}\frac{\Delta x}{\Delta t} & 0\\ -\left[D+\frac{\beta}{C_p}\left(1+(u)_o\frac{\Delta x}{\Delta t}\right)\right] & \frac{(u)_o\left[1+(u)_o\frac{\Delta x}{\Delta t}\right]}{D} & \left[1+(u)_o\frac{\Delta x}{\Delta t}\right]\end{bmatrix}, \qquad 14a,b$$

$$B_j=\begin{bmatrix}u_j^t+\gamma(u)_o(u)_{j-1}^{t+\Delta t}+\left(\frac{\gamma}{\rho}\frac{\Delta x}{\Delta t}\right)p_{j-1}^{t+\Delta t}-\frac{\gamma\tau_w S\Delta t}{\rho A}-g\Delta t\sin\theta\\ p_j^t+\frac{C^2\rho\left(\frac{\Delta x}{\Delta t}\right)}{\gamma}u_{j-1}^{t+}+\gamma u_o p_{j-1}^{t+\Delta t}\\ T_j^t+\gamma u_o T_{j-1}^{t+\Delta t}-\left[D+\frac{\beta}{C_p}\right]p_j^t-\frac{\beta}{C_p}\gamma u_o p_{j-1}^{t+\Delta t}+\\ +\frac{u_o}{KC_p}u_j^t+\frac{u_o u_o\gamma}{K}u_{j-1}^{t+\Delta t}-\frac{u_o g\Delta t\sin\theta}{KC_p}\end{bmatrix}$$

Gaussian elimination method with partial pivoting is used in factorising the matrix of equation. A_j is a matrix of discretised form of equations (7) to (9) which is a function of u and B_j is a column vector matrix which is a function of $T,\ P,\ u$

Initial Condition

The initial condition for all cells is set when $t=0$ for variables u_j^t, p_j^t and T_j^t. To obtain the initial velocity, pressure, temperature profiles, the geothermal gradient of the formation is taken to be $\frac{1}{60}\ \frac{^oF}{ft}$. Equation (8) without the terms involving velocity and equation (7) without the accumulation term were applied.

Boundary Condition

At the system entrance the boundary is either constant or function of time, total gas-oil ratio is $R=751\,ft/bl$, $p_o=9.5\,(Mpa)$ $T_o=180(k)$ $R_g=0.701$, $q_o=3607$, $d_m=2.875(ft)$, well depth $h=3000(m)$ values are given for p_e, T_e and u_e, the subscript e indicate at entrance point while d_m well diameter, h depth of the well and q_o is flow rate.

Procedure for Solution

For accurate solution procedures are outlined in order to understand the basic steps that can be followed. At $t=0$, initial condition is assign to variables with index o and e ie $u_j^o=u_j^t$. The initial condition is used to calculate mixture density, friction factor; two phase Joule-Thomson coefficients and specific heat capacity C_p. It is also applied to each element of matrix A_j and the column vector B_j, after doing these steps convergence is checked, if the condition $\left[A_j u_j^{t+\Delta t}-B_j\right]\le|error|$ is not satisfied, then move to $u_j^o=\left(u_j^t+u_j^{t+\Delta t}\right)/2$ this process is continued until convergence is attain. The simulation continuous in this way until the variables at $t+\Delta t$ are assign to the variables at time $t: u_j^t=u_j^{t+\Delta t}$, the process is continued until total simulation time is completed.

RESULTS AND DISCUSSIONS

We developed a model for two phase flow of Newtonian fluid in a producing gas well and solve numerically. The model has been compared with existing work Bing *et al.,* (2014) when pressure is 10 Mpa and temperature varies from 180k and was observed to be in good agreement. The proposed method enable us to calculate pressure, temperature, density and velocity of the two phase flow simultaneously. We use different values of flow rate on pressure profile and it was observed that pressure of flowing fluid increases figure 2. When we keep the geothermal gradient constant the temperature of the flowing fluid increases figure 3. When liquid and gas density were plotted we observed that both decreases simultaneously figure 4.

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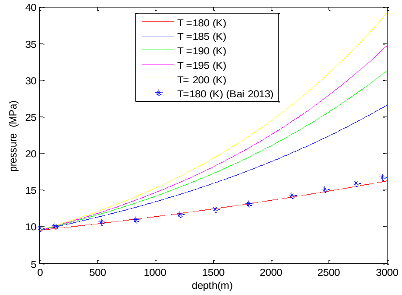

Figure 1: Model compared with Bing et. al., (2014)

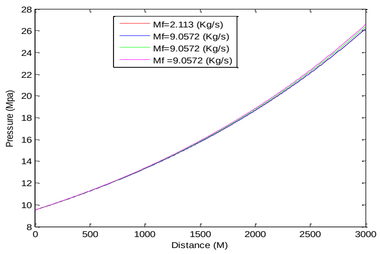

Figure 2: Pressure with different flow rate

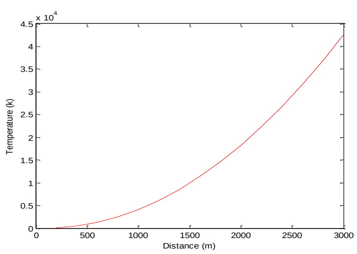

Figure 3: Temperature of flowing fluid

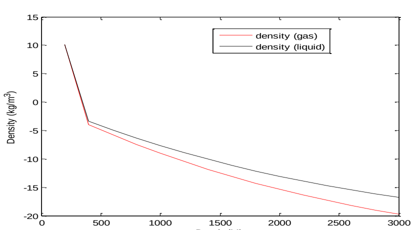

Figure 4: Density of gas and liquid

The velocity of liquid and gas were tested simultaneously and it was observed that both decreases where gas velocity is slower than that of the liquid figure 5 but the case is different if the distance is varied the velocity of the liquid is seen to be stable as compared to the gas velocity. We also tested the pressure of the mixture fluid at different time and was observed to be faster when it is one minute after the start of production as in figure 7.

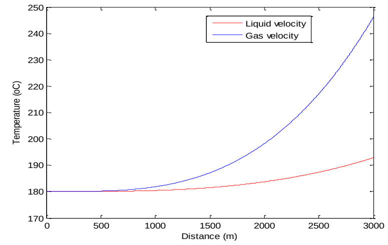

Figure 5: Velocity of gas and liquid

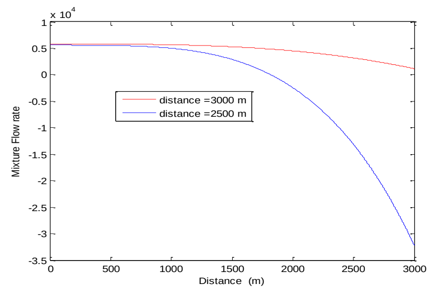

Figure 6: Mixture flow rate at different location of the well.

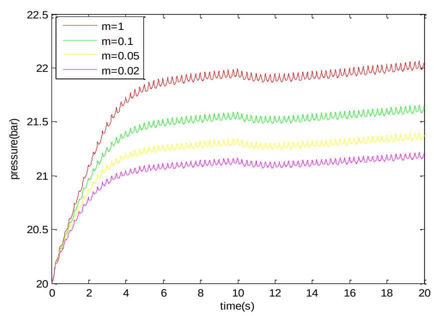

Figure 7: Mixture pressure at different time

Conclusion

We developed a model for two phase flow in a producing gas well and compare our result with existing work which shows good agreement. We observe that both the pressure and the temperature of the mixture fluid increases at constant geothermal gradient while the liquid and gas density when plotted decreases simultaneously. We also plotted the velocity of liquid and gas simultaneously and it was observed that both decreases where gas velocity is slower than that of the liquid but the case is different if the distance is varied the velocity of the liquid is seen to be stable as compared to the gas velocity

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