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Full Length Research Article

MAGNETOHYDRODYNAMIC FLOW OF THIRD-GRADE NANOFLUID WITH CONVECTIVE BOUNDARY CONDITIONS

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ABSTRACT

The research focuses on boundary layer flow of an incompressible third grade nanofluid over a stretching surface under impact of convective boundary condition with presence of thermophoresis, Brownian motion, Newtonian heating, viscous dissipation and chemical reaction. Governing nonlinear equation of velocity, temperature and nanoparticle concentration modelled the problems are solved via shooting method and sixth-order Runge-Kutta. Results of velocity, temperature and nanoparticle concentration profiles are plotted and discussed for various values of fluid parameters such as Prandtl number, Lewis number, thermal biot parameter, chemical reaction parameter, Eckert number and thermophoresis and Brownian motion parameters. Numerical computations are performed. The results show that the change in temperature and nanoparticle concentration distribution functions is similar when we use higher values of material parameters. It is seen that the temperature and thermal boundary layer thickness are increasing functions of chemical reaction parameter *Kr*. An increase in thermophoresis and Brownian motion parameters tends to an enhancement in the temperature. Also increase in Eckert number resulted to rise in fluid temperature

Keywords: Runge-Kutta, third grade fluid, thermophoresis, Lewis number, Non-Newtonian

INTRODUCTION

A nanofluid is a combination of nanoparticles in a base fluid to enhance the thermal performance of the fluid . The nanofluid is an engineered fluid prepared by suspending the nano-sized (1–100 nm) particles of metals/non-metals and their oxide with a base/conventional fluid. The suspension of high thermal conductivity metals/non-metals and their oxides nanoparticles enhance the thermal conductivity and heat transfer ability, etc. of the base fluid. Seth and Ansari (2022) investigated chemical reaction and Newtonian heating effects on MHD third-grade nanofluid flow over a non-linear stretching sheet .The nanofluids are widely used to enhance heat transfer in industrial cooling and heating applications, as smart fluids, in nuclear reactors, for extraction of geothermal and other energy sources, in space and defence; in mass transfer applications; in the automotive application as coolants, brake fluid, nanoparticles mixed in fuel, and as other automotive fluids; in electronic application such as for the cooling of microchips and microscale fluidic application; in biomedical applications such as in nano drug delivery, cancer therapy, in nano-cryosurgery, sensing, and imaging; in other applications such as in detergent in mechanical applications such as for friction reduction, magnetic sealing; in energy application such as energy storage, solar absorption etc. The selection of base fluid plays a crucial role in the preparation of nanofluids. The properties such as viscosity, thermal conductivity, etc. of nanofluid rely on the properties of the base fluid, and the selection of base fluid should be according to the application of nanofluid. The widely used various types of base fluids are water, deionized water, distilled water, ethylene glycol, oil, acetone, ethanol, glycerol, ethylene glycol/water, etc. for various applications. Nanofluids are categorized as single or mono-nanofluids and hybrid nanofluids. In a mono-nanofluid, only one type of nanoparticle is mixed in the base fluid, whereas in hybrid nanofluids, more than one nanoparticle is mixed with the base fluid. Motivated by such facts, Choi(1995) discovered in his investigation that the presence of nanoparticles in base fluid increases thermal characteristics. Mathematical analysis of nanofluids with thermophoresis and Brownian motion effects was studied by Buongiorno (2006). Rashidi and Freidoonimehr, (2019) investigated thermophoresis effect on nanofluid flow over a porous stretching sheet. Multiscale properties of multi component flow of nanofluid and method were examined by Zhou *et al*. (2010). Seth and Ansari (2021) investigated thermophoresis and viscous dissipation effects on MHD nanofluid flow over a non-linear stretching sheet Makinde and Aziz (2010) investigated the convective thermal condition effect in boundary layer flow of viscous nanofluid over a stretching sheet. Makinde and Olawale (2020) examined viscous dissipation and thermophoresis effects on nanofluid flow over a stretching surface Turkyilmazoglu (2013) analyzed the unsteady flow of viscous fluid past a vertical flat plate in the presence of different types of nanoparticles. Second law analysis in steady flow of magneto-nanofluid induced by a porous disk was examined by Rashidi *et al*.(2013). Sheikholeslami and Rashedi (2019) examined numerical investigation of nanofluid thermophoresis inside a rotating system. Turkyilmazoglu and Pop (2013) explored the properties of heat and mass transfer in unsteady natural convection flow of viscous nanofluid in the presence of thermal radiation effect. Khan and Azam (2019) examined viscous dissipation effects on MHD nanofluid flow over a non-linear stretching sheet Analytical treatment of magneto hydrodynamic flow of nanofluid in a porous channel was provided by Sheikholeslami *et al*. (2013). Mustafa et al (2013) numerically investigated the two-dimensional stagnation point flow of nanofluid due to an exponentially stretching sheet. Ibrahim and Makinde (2013) examined the effects of thermal and concentration stratification in mixed convection flow of nanofluid past a vertical flat plate. Kumar and Sood, (2022) examined the effects of thermophoresis and viscous dissipation on MHD flow of nanofluid over a stretching sheet. Rotating flow of nanofluid in the presence of an applied magnetic field was examined by Sheikholeslami *et al*. (2014). Hayat *et al*. (2014) analyzed the effect of convective heat and mass conditions in peristaltic flow of nanofluid. Seth and Ansari (2020) investigated Chemical reaction and Newtonian heating

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effects on MHD third-grade nanofluid flow over a non-linear stretching sheet.

Flows of non-Newtonian fluids are quite prominent in many industrial and engineering processes. There are certain materials like shampoos, muds, soaps, apple sauce, sugar solution, polymeric liquids, tomato paste, condensed milk, paints, blood at low shear rate, which show the characteristics of non-Newtonian fluids. The behavior of such materials cannot be explored by a single constitutive relationship because of their diverse properties. Hence, different fluid models are developed in the past to describe the exact nature of non-Newtonian materials. The fluid model under consideration is a subclass of differential type non-Newtonian fluid namely the third grade. The third grade fluid model exhibits shear thickening and shear thinning characteristics. Some studies on the third grade fluid are seen in Abelman *et al*.(2009), Sajid *et al*.(2008), Sahoo and Do (2010), Makinde and, Chinyoka (2011), Abbasbandy and Hayat (2011), Hayat and Abbasi(2011), Aziz and Mahome (2013), Hatami (2014). Kumar and Sood(2022) applied numerical method to investigate third-grade nanofluid flow with Newtonian heating, chemical reaction, and thermophoresis. Makinde and Olawale (2018) examined third-grade nanofluid flow with Newtonian heating and chemical reaction. Makinde and Olawale (2020) examined the viscous dissipation and thermophoresis effects on nanofluid flow over a stretching surface. Journal of Thermal Analysis and Calorimetry
The aim here is to explore the characteristics of nanoparticles in boundary layer flow of the third grade fluid in the presence of viscous dissipation, chemical reaction, heat absorption and thermal radiation subjected to convective boundary conditions. Effects of thermophoresis and Brownian motion are also incorporated into the investigation. Newtonian thermal condition is utilized for heat transfer analysis. Mathematical modelling is performed under boundary layer assumptions. Similarity variables are employed to convert the partial differential equations into the ordinary differential equations. Numerical analysis method via shooting method with six order Runge Kutta scheme is explored to provide numerical solutions to dimensionless velocity, temperature and concentration models. Graphs and tables are presented to examine the impacts of physical parameters on the temperature and concentration fields. Liao (2012), Turkyilmazoglu(2012), Rashidi *et al*. (2012), Shehzad *et al*.(2013), Abbasbandy *et al.* (2013), Hayat(2013) and Shehzad *et al*.(2014) have all employed analytical approach to solve similar problems. Ghalambaz, and Noghrehabadi (2020) applied numerical method to investigate thermophoresis and viscous dissipation effects on nanofluid flow over a stretching sheet. Ghalambaz and Noghrehabadi (2020) investigated third-grade nanofluid flow with Newtonian heating and chemical reaction in a porous medium. Hayat and Nadeem (2020) investigated thermophoresis and Brownian motion effects on nanofluid flow past a stretching sheet. Abdelsalam,and Abdelrahim, (2020) studied Effects of chemical reaction and Newtonian heating on third-grade nanofluid flow over a porous stretching sheet. Abdelsalam, and Abdelrahim, (2020). Viscous dissipation effects on nanofluid flow over a porous stretching sheet.

Governing problems

We consider the two-dimensional incompressible flow of the third grade fluid generated by a stretching surface in the presence of viscous dissipation, thermal radiation magnetic effect and convective boundary conditions. The sheet is stretched with the velocity $u_w(x) = cx,$ where c denotes a constant. Momentum, heat and mass transfer characteristics are considered in the presence of thermophoresis, Brownian motion, Newtonian heating, heat absorption and magnetic effects. The considered flow is hydrodynamic due to which the influence of Joule heating is not taken into account. Following Sahoo and Do (2010), Hayat et al. (2014), and Shehzad *et al.* (2015), governing boundary layer equations for third grade the nanofluid with viscous dissipation are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad \text{s} \tag{1}$$

$$\begin{aligned} u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} &= \frac{\partial^2 u}{\partial y^2} + 6\frac{\alpha_1}{\rho}\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x\partial y}\left(u\frac{\partial^3 u}{\partial x\partial y^2} + v\frac{\partial^3 u}{\partial y^3} + \frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2}\right) + 4\frac{\alpha_3}{\rho}\left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^2 u}{\partial y^2} \\ &+ 2\frac{\alpha_2}{\rho}\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x\partial y} + \left(u\frac{\partial^3 u}{\partial x\partial y^2} + v\frac{\partial^3 u}{\partial y^3} + \frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2}\right) - 5\frac{\alpha_3}{\rho}\left(\frac{\partial u}{\partial x}\right)^3\frac{\partial^2 u}{\partial x^2} \end{aligned} \tag{2}$$

$$\begin{aligned} u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} &= \alpha\frac{\partial^2 T}{\partial y^2} + r\left(D_B\frac{\partial C}{\partial y}\frac{\partial T}{\partial y} + \frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right) + \frac{v}{c_p}\left(\frac{\partial u}{\partial y}\right)^2\right) - D_B\frac{\partial^2 T}{\partial y^2} \\ &\frac{\alpha_1}{\rho c_p}\left(u\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x\partial y} + v\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2} + 2\frac{\alpha_3}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^4\right) + \frac{Q_0}{(\rho C_p)}(T - T_\infty) - \frac{1}{(\rho C_p)}q_r \end{aligned} \tag{3}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_B\frac{\partial^2 C}{\partial y^2} + \frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2} + D_B\frac{\partial^2 T}{\partial y^2} - Kr(C - C_\infty) \tag{4}$$

The appropriate boundary conditions for the present flow problems are

$$u = u_w(x) = cx, v = 0, \frac{\partial T}{\partial x} = h_s T, C = C_w \;\; at \;\; y = 0 \quad (5)$$

$$u \to 0, v \to 0, T \to T_\infty, C \to C_\infty \;\; as \;\; y \to \infty \quad (6)$$

where u and v denote the velocity components parallel to the x- and y-directions, α_1, α_2 and α_3 are the material parameters, $v = \left(\mu / \rho\right)$ is the kinematic viscosity, u is the dynamic viscosity, ρ is the density of fluid, T is the fluid temperature, α is the thermal diffusivity, $r = (\rho c)_p / (\rho c)_f$ is the ratio of nanoparticle heat capacity to the base fluid heat capacity, D_B is the Brownian diffusion coefficient, D_T is the thermophoretic diffusion coefficient, C is the concentration, c_p is the specific heat at constant pressure, T_∞ and C_∞ are the ambient temperature and concentration away from the sheet and h_s is the heat transfer coefficient, kr is the constant rate of chemical reaction, Q_0 is the coefficient of internal heat generation, q_r is the radiative heat flux
Letting

$$u = cxf'(\eta), y = -\sqrt{cvf(\eta)}, \eta = y\sqrt{\frac{c}{v}}, \theta(\eta) = \frac{T - T_\infty}{T_\infty}, \phi(\eta) = \frac{C - C_\infty}{C_\infty} \quad (7)$$

Eq. (2) is satisfied automatically and Eqs. (3)-(6) take the forms:

$$f''' + \beta_1\left(ff'' - f'^2 + 2ff'' - ff'''\right) + 6\varepsilon_1\varepsilon_2\left((3\beta_1 + 2\beta_2)f''^2 + 6\varepsilon_1\varepsilon_2 f'''f''^2\right) = 0 \quad (8)$$

$$\theta'' + \Pr ff''^2\left(\theta' - prEcf''^2 + Ec\beta_1 f'\right) - \Pr Ec\beta_1 ff'f''' + 2\Pr Ec\varepsilon_1\varepsilon_2 f''^4 +$$
$$\Pr N_b\theta'\phi' + \Pr N_t\theta'^2 - Q\theta + Ra\theta'' + 2Ec\beta_1 f' - \Pr EcN_b\theta'\phi' = 0 \quad (9)$$

$$\phi'' + \Pr Lef\phi' + (N_t / N_b)\theta'' - Kc\phi + Lef\phi' - Nt\phi = 0 \quad (10)$$

$$f(0) = 0, f'(0) = 1, \theta' = -\gamma(1 + \theta(0)) = 0 \quad (11)$$

$$f'(\infty) \to 0, \theta(\infty) \to 0, \phi(\infty) \to 0 \quad (12)$$

Where $\beta_1 = \frac{c\alpha_1}{\mu}, \beta_2 = \frac{c\alpha_2}{\mu}, \varepsilon_1 = \frac{c\alpha_3}{\mu}$ are the material parameters for third grade fluid, $\varepsilon_2 = \frac{cx^2}{v}$ is the local Reynolds number, $\Pr = \frac{v}{\alpha}$ is the Prandtl number, $Ec = \frac{u_w^2}{(c_p T_\infty)}$ is the Ecket number, $N_b = \frac{(\rho c)_p D_B (C_w - C_\infty)}{(\rho c)_f v}$ is the Brownian motion parameter, $N_t = \frac{(\rho c)_p D_T}{(\rho c)_f v}$ is the thermophoresis parameter and $Le = \frac{\alpha}{D_B}$ is the Lewis number, $Q = \frac{v^2 Q_0}{\alpha\rho C_p}$ is the heat source parameter, $Kc = \frac{vKr}{U_0^2}$ is the chemical reactions, $Ra = \frac{16\alpha T_\infty^3}{3k^*\alpha\rho_f C_p}$ is the thermal radiations

Table 1: Numerical Value of $f'(\eta)$, $\theta(\eta)$ and $\phi(\eta)$ for different values of $\beta_1, \beta_2, \varepsilon_1$ and ε_2

β_1	β_2	ε_1	ε_2	$f'(\eta)$	$\theta(\eta)$	$\phi(\eta)$
0	0.1	0.1	0.3	0.14032	1.10772	0.92032
0.2	0.1	0.1	0.3	0.1l051	1.23011	0.86541
0.3	0.1	0.1	0.3	0.29456	1.36408	0.74026
0.1	**0**	0.1	0.3	**1.23784**	**1.45317**	**0.64215**
0.1	**0.2**	0.1	0.3	**1.32195**	**1.54219**	**0.54265**
0.1	**0.2**	0.1	0.3	**1.45065**	**1.65342**	**0.50421**
0.1	0.1	0	0.3	1.59852	1.78497	0.45629
0.1	0.1	0.3	0.3	1.65782	1.79198	0.41190
0.1	0.1	0.5	0.3	1.77832	1.87836	0.3554
0.1	0.1	**0.2**	**0.4**	**1.79812**	**1.88913**	**0.26291**
0.1	0.1	**0.2**	**0.4**	**1.87402**	**1.9819**	**0.19319**
0.1	0.1	**0.2**	**0.4**	**1.90166**	**1.99984**	**0.13150**

Numerical Solution
The system of highly non linear differential equations (8), (9) and (10) subjected to boundary conditions (11) and (12) are solved by a numerical approach via shooting method with the six-order Runge-Kutta method for different moderate values of the flow, heat and mass transfer parameters. The effective Broyden technique is adopted in order to improve the initial guesses and to satisfy the boundary conditions at infinity. Maple software is used to code and simulate the above numerical procedure.

NUMERICAL RESULT AND DISCUSSION
Table1: Convergence of numerical solution for different order of approximate when

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$\beta_1 = \beta_2 = 0.3, \varepsilon_1 = 0.3, \Pr = 1.1,\ Le = 0.7,\ \gamma = 0.1,\ Nt = 0.2, Nb = 0.4, Ec = 0.6,\ h_f = -0.8$ *and* $h_\theta = h_\phi = -0.8,\ Kc = 0,\ Q = 0,\ Ra = 0$

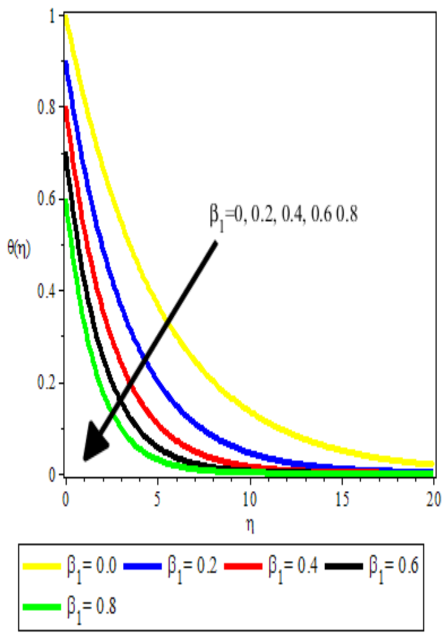

Figure 1 Influence of β_1 on temperature $\theta(\eta)$

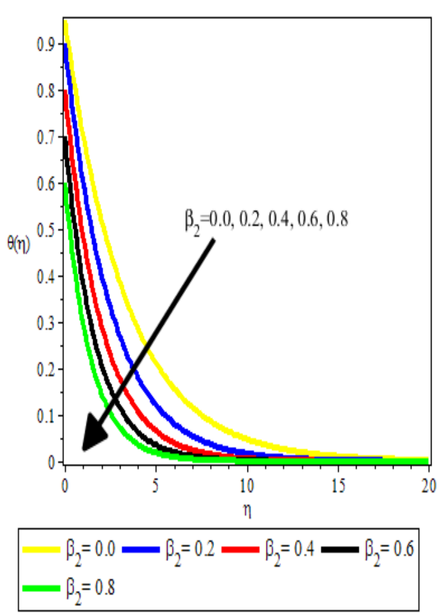

Figure 2 Influence of $\beta 2$ on temperature $\theta(\eta)$

We plot the solutions of dimensionless temperature $\theta(\eta)$ and nanoparticle concentration $\phi(\eta)$ for the multiple values of material parameters β_1 and β_2, Prandtl number Pr, Lewis number Le, Newtonian heating parameter γ, thermophoresis and Brownian motion parameters Nt and Nb and Eckert number Ec.

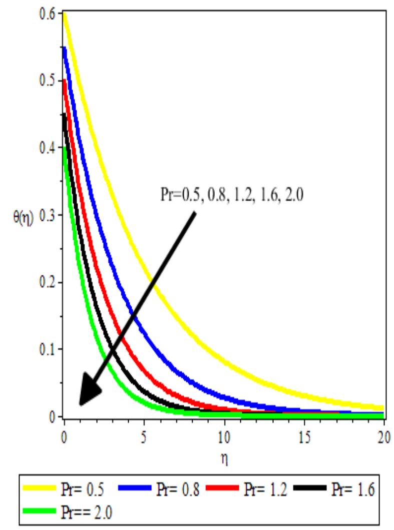

Figure 3 Influence of Pr on temperature $\theta(\eta)$

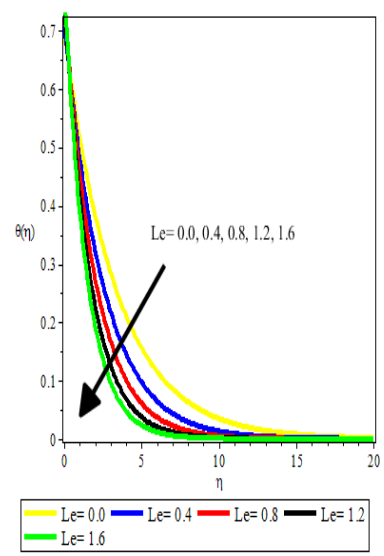

Figure 4 Influence of Le on temperature $\theta(\eta)$

Figures $1-8$ are sketched to examine the temperature distribution function $\theta(\eta)$ corresponding to different values of β_1, β_2, Pr, Le, γ, Nt, Nb and Ec. Figures 1 and 2 outline that an increase in material parameters tends to a decrease in the temperature and thermal boundary layer thickness. Here, the material parameters depend on normal stresses and viscous forces. Normal stresses are increased and viscous forces are decreased when we increase the values of material parameters. This change in normal stresses and viscous forces tends to a reduction in the temperature and thermal boundary layer thickness. An increase in Prandtl number Pr shows a decrease in temperature and thermal boundary layer thickness (see Fig. 3). Prandtl number is inversely proportional to the thermal diffusivity of

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fluid. Thermal diffusivity is weaker for higher Prandtl fluids and stronger for lower Prandtl fluids. Weaker thermal diffusivity corresponds to lower temperature and stronger thermal diffusivity shows higher temperature. Here, thermal diffusivity is responsible for change in temperature.

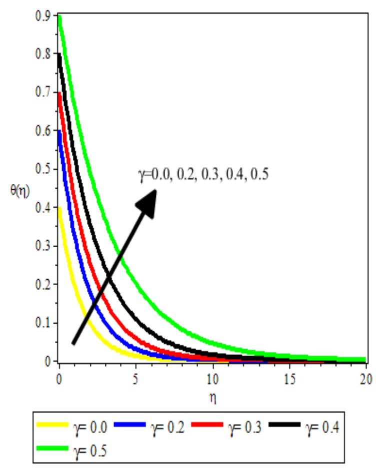

Figure 5 Influence of γ on temperature $\theta(\eta)$

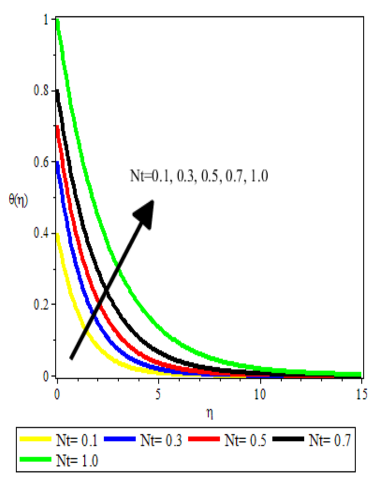

Figure 6 Influence of Nt on temperature $\theta(\eta)$

Figure 5 clearly shows that an increase in Lewis number leads to a reduction in the temperature and thermal boundary layer thickness. Lewis number involves the diffusion coefficient. Increasing values of Lewis number corresponds to decrease in diffusion coefficient. This smaller diffusion coefficient tends to a lower temperature. Effects of Newtonian heating parameter on the temperature are examined in Fig. 5. Temperature is increased when we increase the values of Newtonian heating parameter. It is also seen that the temperature at the wall is an increasing function of Newtonian heating parameter. Newtonian heating parameter is directly proportional to the conjugate heat transfer coefficient. Conjugate heat transfer coefficient increases when we increase the Newtonian heating parameter due to which the temperature rises. It is obvious from Figs. 6 and 7 that temperature and thermal boundary layer thickness are enhanced when we use higher values of thermophoresis and Brownian motion parameters. Further, we examine that the temperature at the wall for $Nb = 1.0$ is slightly greater than $Nt = 1.0$. Figure 8 depicts that the temperature and thermal boundary layer thickness are enhanced when we increase the values of Eckert number Ec.

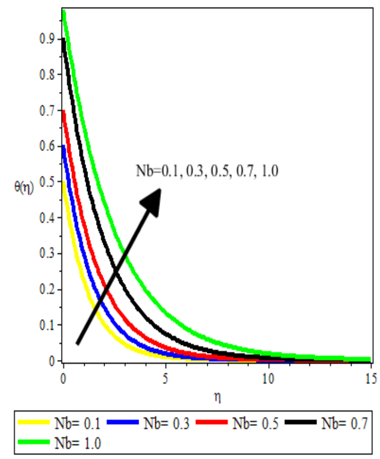

Figure 7 Influence of Ec on temperature $\theta(\eta)$

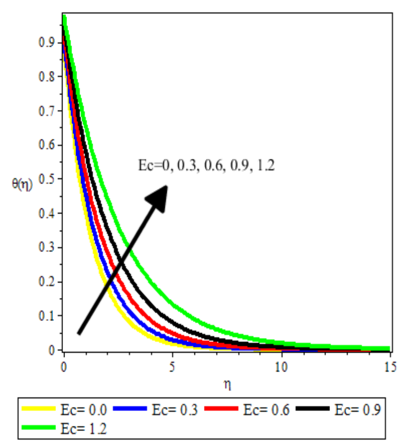

Figure 8 Influence of Nb on temperature $\theta(\eta)$

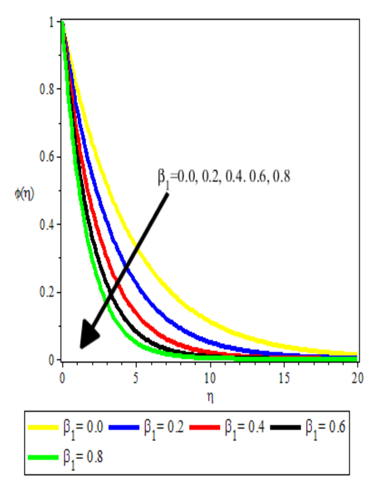

Figure 9 Influence of $\beta 1$ on nanoparticle concentration $\phi(\eta)$

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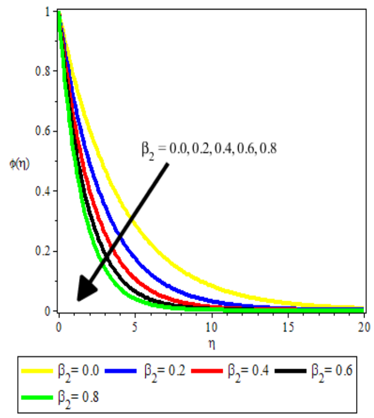

Figure 10 Influence of *β*2 on nanoparticle concentration $\phi(\eta)$

To analyze the variations in nanoparticle concentration distribution function $\phi(\eta)$ for various values of material parameter β_1 and β_2, Prandtl number Pr, Lewis number Le, thermophoresis and Brownian motion parameters Nt and Nb and Eckert number Ec, we have drawn Figs. 9−15. From Figs. 9 and 10, we observe that the nanoparticle concentration and its related boundary layer thickness are lower for higher values of material parameters. We note that the material parameters have similar trends for temperature and nanoparticle concentration but the reduction in temperature is more pronounced in comparison to nanoparticle concentration. Higher values of Prandtl number tends to a decrease in the nanoparticle concentration and boundary layer thickness (see Fig. 11). Figure 12 indicates that an increase in Lewis number leads to a weaker nanoparticle concentration and its associated boundary layer thickness. A comparison of Figs. 6 and 13 shows that temperature and nanoparticle concentration fields are increasing functions of thermophoresis parameter Nt. From Fig. 14, it is examined that an increase in the values of Brownian motion parameter creates a reduction in the nanoparticle

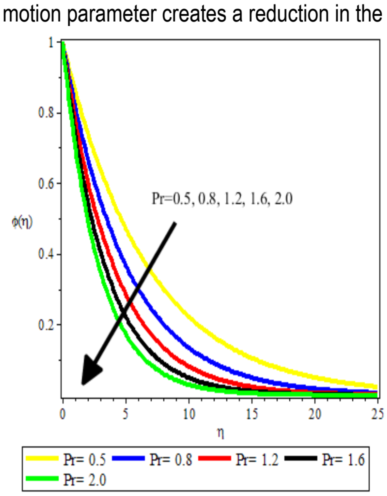

Figure 11 Influence of *Pr* on nanoparticle concentration $\phi(\eta)$

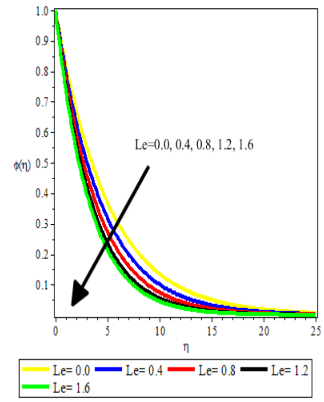

Figure 12 Influence of *Le* on nanoparticle concentration $\phi(\eta)$

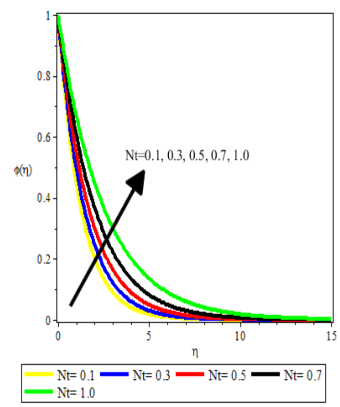

Figure 13 Influence of *Nt* on nanoparticle concentration $\phi(\eta)$

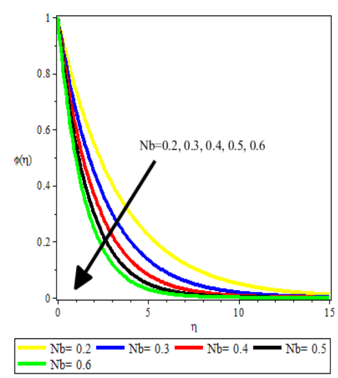

Figure14 Influence of *Nb* on nanoparticle concentration $\phi(\eta)$

Conclusions

We examined the Numerical Analysis of Third grade Nano fluid with convective boundary conditions in the presence of heat absorption and thermal radiation. The models capturing the problem are formulated via continuity, momentum and energy equation. Lie group similarity variable is used to transform the partial differential equation governing the problem to reduce ordinary differential equation, then the resulting system of ODE are solved numerically via shooting method and six order Runge-Kutta scheme. The

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influence of some flow parameters on the temperature and concentration profiles were discussed. Our findings among other reveal that as Eckert number increase, temperature profiles increase. It also reveals that Eckert number that be used to control thermal boundary layer thickness

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