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Full Length Research Article

ECONOMIC ORDER QUANTITY (EOQ) FOR ITEMS EXHIBITING DELAY IN DETERIORATION WITH PRICE, STOCK, AND RELIABILITY DEMAND UNDER PARTIAL BACKLOGGING

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ABSTRACT
The study explored an advanced EOQ model tailored for items that delay deterioration, such as perishable goods. This model incorporates price, stock level, and reliability as variables affecting demand to optimize profit per unit time under partial backlogging conditions. Differential equations that capture inventory dynamics across stages of no deterioration, active deterioration, and shortage are presented. Numerical simulations using Excel and Maple validate the model, revealing that higher stock-dependent consumption parameters and reliability increase demand and profitability. Results indicate that deterioration negatively impacts profit by reducing product quality, while longer replenishment cycles decrease profitability due to increased spoilage. Enhanced backordering boosts profit by reducing holding costs. This study highlights the importance of integrating price, stock, and reliability in EOQ models for strategic inventory management, balancing costs to maximize operational gains.

Keywords: Economic Order Quantity (EOQ); Deterioration delay; Inventory management; Price and reliability-dependent demand; Partial backlogging

INTRODUCTION
In recent years, inventory management has gained significant attention in both academic and industrial circles. This is largely due to the importance of resource optimization in today's competitive environment, which is a critical responsibility for organizations across the public, private, and government sectors (Bhattarai, 2015). Effective inventory management is essential for organizations aiming to minimize costs, improve operational efficiency, and enhance profitability. The relevance of inventory in this regard cannot be overstated, as it directly impacts financial resources by tying up capital in stock. Consequently, modern management increasingly prioritizes inventory control to optimize investment and improve quality (Bhattarai, 2015).
Inventory refers to materials, goods, or products kept in stock to meet demands as they arise. Among inventory control models, the Economic Order Quantity (EOQ) model stands out as a fundamental approach for determining the optimal order quantity needed to satisfy deterministic demand while minimizing costs. However, many traditional EOQ models assume that items can be stored indefinitely without deterioration. In reality, most inventory items degrade over time. Deterioration, which can be understood as a decline in quality or quantity, is a common phenomenon affecting items such as bread, potatoes, and cakes, which only begin to deteriorate after a certain delay (Dari & Sani, 2013).

This study investigates EOQ models for items exhibiting delayed deterioration, incorporating factors such as price, stock, and reliability into the demand function. Unlike conventional models, this approach accounts for the delayed onset of deterioration and considers demand to be influenced by these factors both before and after deterioration begins. By introducing modifications to existing models, this research aims to provide a more comprehensive understanding of inventory dynamics (Sudip & Mahapatra, 2018).

Businesses maintain inventory for various reasons, including meeting unexpected demands, smoothing seasonal fluctuations, and taking advantage of price discounts. Organizations stock inventory to satisfy customer needs promptly, particularly during unpredictable demand surges. Seasonal fluctuations further necessitate inventory to ensure continuous supply, while bulk purchasing often enables businesses to capitalize on price discounts offered by suppliers (Eiselt & Sandblom, 2010). Additionally, companies use inventory to hedge against price increases and benefit from economies of scale when purchasing or transporting goods (Eiselt & Sandblom, 2010).

However, inventory management involves significant costs, such as ordering costs, holding costs, and shortage costs. Ordering costs encompass expenses related to procurement processes, while holding costs include storage, security, insurance, and losses due to deterioration or obsolescence (Sani, 2014). Shortage costs arise when customer demand exceeds available inventory, potentially leading to lost sales or backorders. Additional considerations such as salvage value, revenue implications, and the discount rate also play a role in inventory decisions (Sani, 2014).

The primary goal of inventory models is to address two critical questions: how much to order and when to order. The answers to

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these questions are expressed as the order quantity and the reorder point, respectively (Sani & Dari, 2013). According to Hadley and Whitin (1963), total inventory cost can be summarized as the sum of purchasing, setup (or ordering), holding, and shortage costs. Inventory models are generally classified into deterministic and stochastic categories, with each offering distinct approaches to managing inventory under varying conditions.

Despite their utility, traditional EOQ models have certain limitations, which have prompted researchers to propose extensions or modifications. For instance, incorporating factors such as reliability, in addition to price and stock, can provide a more accurate representation of demand dynamics. Reliable products often have higher market demand, and considering this aspect alongside stock and price in EOQ models can enhance decision-making (Dari & Sani, 2013).

This research introduces an EOQ model that incorporates delayed deterioration and considers demand as a function of price, stock, and reliability. By addressing these factors, the study aims to formulate a more robust inventory management framework, investigate their effects on demand, and maximize profit per unit time. Ultimately, this approach seeks to identify the optimal inventory management policies to balance costs and maximize organizational efficiency.

MATERIALS AND METHODS

Model Definition

An EOQ model for deteriorating inventory that exhibits delay in deterioration in which the demand depends on price, stock and reliability wherein shortages are allowed and partially backlogged was formulated.

Assumptions and Notation

The assumptions and Notation to the Economic order quantity for items that exhibit delay in deterioration with price, stock and reliability demand under partial backlogging is being given below:

Notation

Q	The order quantity in one cycle
T	The replenishment cycle time
C	The unit cost of the item
B	The ordering cost per order
h_1	The inventory holding cost per unit item
h_2	The shortage cost per unit per time
h_3	The unit cost of lost sales
θ	The rate of deterioration
s	The purchasing cost per unit
p	Selling price per unit, where p>s
T_1	The length of time in which the product exhibits no deterioration
T_2	The Length of time in which the stock level vanishes
$N_1(t)$	The inventory level at time t Є [0, T_1]
$N_2(t)$	The inventory level at time t Є [T_1, T_2], where $T_2>T_1$
$N_3(t)$	The inventory level at time t Є [T_2, T]
N_0	The maximum inventory level
S	The maximum amount of demand backlogged
β	backlogging rate, $0 \le \beta \le 1$
D(N(t); r, p)	demand rate where N(t) is inventory level at time t, r, is the reliability, p is price of the stock

Assumptions:

a) Replenishment rate is infinite
b) Lead time is zero
c) The deterioration rate is constant on the on-hand inventory per unit time and there is no repair of the deterioration item within the cycle.
d) Demand rate is D(N(t); r, p) = k(p) [$xpr^6 + \alpha N(t)$] where $k(p) = \gamma e^{(p\delta/r)}$ is the price factor where x,y,δ > 0 are the parameter. α is the stock dependent consumption rate parameter $0 \le \alpha \le 1$; and r is the reliability.
e) Shortages are allowed and partially backlogged

Model Formulation

The differential equation of the proposed inventory system can be written in the form of mathematical model under consideration based on the above assumptions is as follows:

$$\frac{dN_1(t)}{dt} = -k(p)\left[xpr^y + \alpha N_1(t)\right] \quad 0 \le t \le T_1 \tag{1}$$

$$\frac{dN_2(t)}{dt} + \theta N_2(t) = -k(p)\left[xpr^y + \alpha N_2(t)\right] \quad T_1 \le t \le T_2 \tag{2}$$

$$\frac{dN_3(t)}{dt} = -k(p)xpr^y \quad T_2 \le t \le T \tag{3}$$

Where:
Equation (3.1) depends only on demand and the differential equation that describes the situation before deterioration sets in, equation (3.2) describe the situation in which After deterioration sets in, depletion of inventory will depend on both demand and deterioration and equation (3.3) describes the situation in which the inventory level is on zero when shortages has set in.

Solution to the formulated model

By solving (3.1) using integrating factor, we have

$$\frac{dN_1(t)}{dt} + \alpha k(p)N_1(t) = -k(p)xpr^y \tag{4}$$

$$I.Fe^{\int \alpha k(p)dt} = e^{\alpha k(p)t} \tag{5}$$

$$N_1(t)I.F = \int Q_1(t) \tag{6}$$

$$N_1(t).e^{\alpha k(p)t} = -\int k(p)xpr^y.e^{\alpha k(p)t}dt + E \tag{7}$$

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$$N_1(t).e^{\alpha k(p)t} = -k(p)xpr^y.\int e^{\alpha k(p)t}dt + E \quad (8)$$

$$N_1(t).e^{\alpha k(p)t} = -\frac{k(p)xpr^y}{\alpha k(p)}.e^{\alpha k(p)t} + E \quad (9)$$

$$N_1(t) = -\frac{k(p)xpr^y}{\alpha k(p)} + Ee^{-\alpha k(p)t} \quad (10)$$

$$N_1(t) = Ee^{-\alpha k(p)t} - \frac{k(p)xpr^y}{\alpha k(p)} \quad (11)$$

By applying the boundary condition ($N_1(t) = N_0$ at $t = 0$) in (3.11)

$$N_0 = Ee^{-\alpha k(p)(0)} - \frac{k(p)xpr^y}{\alpha k(p)} \quad (12)$$

$$N_0 = E - \frac{xpr^y}{\alpha} \quad (13)$$

$$E = N_0 - \frac{xpr^y}{\alpha} \quad (14)$$

Substituting (3.14) into (3.11) we have

$$N_1(t) = \left(N_0 + \frac{xpr^y}{\alpha}\right)e^{-\alpha k(p)t} - \frac{xpr^y}{\alpha} \quad (15)$$

By re-writing (3.2), we have,

$$\frac{dN_2(t)}{dt} + (\theta + \alpha k(p))N_2(t) = -k(p)xpr^y \quad (16)$$

By solving (3.16) using integrating factor, we have

$$\frac{dN_2(t)}{dt} + \sigma N_2(t) = -k(p)xpr^y \quad (17)$$

Where $\sigma = \theta + \alpha k(p)$

$$I.Fe^{\int \sigma dt} = e^{\sigma t} \quad (18)$$

$$N_2(t)I.F = \int Q_2(t).I.Fdt + F \quad (19)$$

$$N_2(t)e^{\sigma t} = \int -k(p)xpr^y.e^{\sigma t}dt + F \quad (20)$$

$$N_2(t)e^{\sigma t} = -k(p)xpr^y\int e^{\sigma t}dt + F \quad (21)$$

$$N_2(t)e^{\sigma t} = -\frac{k(p)xpr^y}{\sigma}.e^{\sigma t} + F \quad (22)$$

$$N_2(t) = -\frac{k(p)xpr^y}{\sigma} + Fe^{-\sigma t} \quad (23)$$

$$N_2(t) = Fe^{-\sigma t} - \frac{k(p)xpr^y}{\sigma}$$

where F is a constant (24)

Applying the boundary condition ($N_2(t) = 0$ at $t = T_2$) in

$$0 = Fe^{-\sigma T_2} - \frac{k(p)xpr^y}{\sigma} \quad (25)$$

$$Fe^{-\sigma T_2} = \frac{k(p)xpr^y}{\sigma} \quad (26)$$

$$F = \frac{k(p)xpr^y}{\sigma}.e^{\sigma T_2} \quad (27)$$

By Substituting (3.27) in (3.24) we have,

$$N_2(t) = \left(\frac{k(p)xpr^y}{\sigma}.e^{\sigma T_2}\right)e^{-\sigma t} - \frac{k(p)xpr^y}{\sigma} \quad (28)$$

$$N_2(t) = \frac{k(p)xpr^y}{\sigma}.e^{\sigma(T_2-t)} - \frac{k(p)xpr^y}{\sigma} \quad (29)$$

$$N_2(t) = \frac{k(p)xpr^y}{\sigma}\left(e^{\sigma(T_2-t)} - 1\right) \quad (30)$$

Now, considering continuity of $N_1(t)$ and $N_2(t)$ at point $t=T_1$, that is, $N_1(t)= N_2(t)$, the maximum inventory level for each cycle can be obtained as follows.
Therefore, from (3.15) and (3.30), we have that

$$\left(N_0 + \frac{xpr^y}{\alpha}\right)e^{-\alpha k(p)T_1} - \frac{xpr^y}{\alpha} = \frac{k(p)xpr^y}{\sigma}\left(e^{\sigma(T_2-T_1)} - 1\right) \quad (31)$$

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$$N_0 e^{-\alpha k(p)T_1} + \frac{xpr^y}{\alpha} e^{-\alpha k(p)T_1} - \frac{xpr^y}{\alpha} = \frac{k(p)xpr^y}{\sigma}\left(e^{\sigma(T_2-T_1)}-1\right) \quad (32)$$

$$N_0 e^{-\alpha k(p)T_1} = \frac{k(p)xpr^y}{\sigma}\left(e^{\sigma(T_2-T_1)}-1\right) + \frac{xpr^y}{\alpha} - \frac{xpr^y}{\alpha} e^{-\alpha k(p)T_1} \quad (33)$$

$$N_0 = \frac{k(p)xpr^y}{\sigma}\left(e^{\sigma(T_2-T_1)}-1\right)\left(e^{\alpha k(p)T_1}\right) + \frac{xpr^y}{\alpha}\left(e^{\alpha k(p)T_1}\right) - \frac{xpr^y}{\alpha} e^{-\alpha k(p)T_1}\left(e^{\alpha k(p)T_1}\right) \quad (34)$$

$$N_0 = \frac{k(p)xpr^y}{\sigma}\left(e^{\sigma(T_2-T_1)}-1\right)\left(e^{\alpha k(p)T_1}\right) + \frac{xpr^y}{\alpha}\left(e^{\alpha k(p)T_1}\right) - \frac{xpr^y}{\alpha} \quad (35)$$

$$N_0 = \frac{k(p)xpr^y}{\sigma}\left(e^{\sigma(T_2-T_1)}-1\right)\left(e^{\alpha k(p)T_1}\right) + \frac{xpr^y}{\alpha}\left(e^{\alpha k(p)T_1}-1\right) \quad (36)$$

Now, by substituting the value of N_0 in (3.36) into (3.15) we have,

$$N_1(t) = \left(\frac{k(p)xpr^y}{\sigma}\left(e^{\sigma(T_2-T_1)}-1\right)\left(e^{\alpha k(p)T_1}\right) + \frac{xpr^y}{\alpha}\left(e^{\alpha k(p)T_1}-1\right) + \frac{xpr^y}{\alpha}\right)e^{-\alpha k(p)t} - \frac{xpr^y}{\alpha}$$

$$0 \le t \le T_1 \quad (37)$$

During the shortage time interval $[T_2, T]$ the demand at time t is partially backlogged and the model equation is given in (3.3).

Therefore, by integrating (3.3), we have

$$\int dN_3(t) = -\int k(p)xpr^y dt \quad (38)$$

$$\int N_3(t) = -k(p)xpr^y \int dt + G \quad (39)$$

$$N_3(t) = -k(p)xpr^y t + G \quad (40)$$

By applying the boundary condition $N_3(T_2) = 0$ we have,

$$0 = -k(p)xpr^y t + G \quad (41)$$

$$G = k(p)xpr^y T_2 \quad (42)$$

$$N_3(t) = -k(p)xpr^y t + k(p)xpr^y T_2 \quad (43)$$

$$N_3(t) = -k(p)xpr^y (t - T_2) \quad (44)$$

By putting t=T in (3.44) we have the maximum amount of demand backlogged per cycle to be

$$S = -N_3(t) = k(p)xpr^y (T - T_2) \quad (45)$$

We obtain the order quantity by adding $N_0 + S$ i.e., by adding (3.36) and (3.44)

Therefore,

$$Q = N_0 + S$$

$$= \frac{k(p)xpr^y}{\sigma}\left(e^{\sigma(T_2-T_1)}-1\right)\left(e^{\alpha k(p)T_1}\right) + \frac{xpr^y}{\alpha}\left(e^{\alpha k(p)T_1}-1\right) + k(p)xpr^y (T - T_2) \quad (46)$$

The cost of holding inventory in stock is computed for until it is sold or used, which is inventory carrying cost H.C, and is given by:

$$HC = h\int_0^{T_1} N_1(t)\,dt + h\int_T^{T_2} N_2(t)dt \quad (47)$$

By substituting (3.30), (3.37) into (3.47) we have,

$$HC = h\int_0^{T_1}\left(\left(\frac{k(p)xpr^y}{\sigma}\left(e^{\sigma(T_2-T_1)}-1\right)\left(e^{\alpha k(p)T_1}\right) + \frac{xpr^y}{\alpha}\left(e^{\alpha k(p)T_1}-1\right) + \frac{xpr^y}{\alpha}\right)e^{-\alpha k(p)t} - \frac{xpr^y}{\alpha}\right)dt + h\int_{T_1}^{T_2}\left(\frac{k(p)xpr^y}{\sigma}\left(e^{\sigma(T_2-t)}-1\right)\right)dt \quad (48)$$

$$HC = h\left(\frac{k(p)xpr^y}{\sigma}\left(e^{\sigma(T_2-T_1)}-1\right)\left(e^{\alpha k(p)T_1}\right) + \frac{xpr^y}{\alpha}\left(e^{\alpha k(p)T_1}-1\right) + \frac{xpr^y}{\alpha}\right)\int_0^{T_1} e^{-\alpha k(p)t}dt -$$
$$h\frac{xpr^y}{\alpha}\int_0^{T_1} 1dt + h\frac{k(p)xpr^y}{\sigma}\int_{T_1}^{T_2} e^{\sigma(T_2-t)}dt - h\frac{k(p)xpr^y}{\sigma}\int_{T_1}^{T_2} 1dt \quad (49)$$

$$HC = h\left(\frac{k(p)xpr^y}{\sigma}\left(e^{\sigma(T_2-T_1)}-1\right)\left(e^{\alpha k(p)T_1}\right) + \frac{xpr^y}{\alpha}\left(e^{\alpha k(p)T_1}-1\right) + \frac{xpr^y}{\alpha}\right)\left[-\frac{1}{\alpha k(p)}e^{-k(p)t}\right]_0^{T_1}$$
$$-h\frac{xpr^y}{\alpha}[t]_0^{T_1} + h\frac{k(p)xpr^y}{\sigma}e^{\sigma T_2}\int_{T_1}^{T_2} e^{-\sigma t}dt - h\frac{k(p)xpr^y}{\sigma}\int_{T_1}^{T_2} dt \quad (50)$$

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$$HC = h\left(\frac{k(p)xpr^y}{\sigma}\left(e^{\sigma(T_2-T_1)}-1\right)\left(e^{\alpha k(p)T_1}\right)+\frac{xpr^y}{\alpha}\left(e^{\alpha k(p)T_1}-1\right)+\frac{xpr^y}{\alpha}\right)\left(-\frac{1}{\alpha k(p)}e^{-k(p)T_1}+\frac{1}{\alpha k(p)}\right) \quad (51)$$
$$-h\frac{xpr^y}{\alpha}T_1+h\frac{k(p)xpr^y}{\sigma^2}e^{\sigma T_2}e^{-\sigma(T_2-T_1)}-h\frac{k(p)xpr^y}{\sigma}(T_2-T_1)$$

$$HC = \frac{h}{\alpha k(p)}\left(\frac{k(p)xpr^y}{\sigma}\left(e^{\sigma(T_2-T_1)}-1\right)\left(e^{\alpha k(p)T_1}\right)+\frac{xpr^y}{\alpha}\left(e^{\alpha k(p)T_1}-1\right)+\frac{xpr^y}{\alpha}\right)\left(1-e^{-k(p)T_1}\right)- \quad (52)$$
$$h\frac{xpr^y}{\alpha}T_1+h\frac{k(p)xpr^y}{\sigma^2}e^{\sigma T_2}e^{-\sigma(T_2-T_1)}-h\frac{k(p)xpr^y}{\sigma}(T_2-T_1)$$

Shortage due to stock out is accumulated in the system during the interval [T_2, T]. The optimum level of shortage is present at t=T; therefore, the total shortage cost during this time period is as follows:

$$SC = h_2\int_{T_2}^{T}\left(-N_3(t)\right)dt \quad (53)$$

$$SC = h_2\int_{T_2}^{T}\left(k(p)xpr^y(T-T_2)\right)dt \quad (54)$$

$$SC = h_2\left(k(p)xpr^y(T-T_2)\right)[t]_{T_2}^{T} \quad (55)$$

$$SC = h_2\left(k(p)xpr^y(T-T_2)\right)(T-T_2) \quad (56)$$

$$SC = h_2k(p)xpr^y(T-T_2)^2 \quad (57)$$

Due to stock out during (T_2, T), shortage is accumulated, but not all customers would be willing to wait for the next lot size to come. Therefore, this results in some loss of sale which accounts to loss in profit.

Hence, lost sale cost is calculated as follows:

$$LSC = h_3\int_{T_2}^{T}(1-\beta)k(p)xpr^y dt \quad (58)$$

$$LSC = h_3(1-\beta)k(p)xpr^y[t]_{T_2}^{T} \quad (59)$$

$$LSC = h_3(1-\beta)k(p)xpr^y(T-T_2) \quad (60)$$

The purchase cost denoted by PC is as follows

$$PC = sQ$$
$$= s\left(\frac{k(p)xpr^y}{\sigma}\left(e^{\sigma(T_2-T_1)}-1\right)\left(e^{\alpha k(p)T_1}\right)+\frac{xpr^y}{\alpha}\left(e^{\alpha k(p)T_1}-1\right)+k(p)xpr^y(T-T_2)\right) \quad (61)$$

The total cost is the sum of ordering cost, purchase cost, inventory holding cost, shortage cost and lost sales.

$$K(T,T_2,s) = OC+PC+HC+SC+LSC \quad (62)$$

$$= B+s\left(\frac{k(p)xpr^y}{\sigma}\left(e^{\sigma(T_2-T_1)}-1\right)\left(e^{\alpha k(p)T_1}\right)+\frac{xpr^y}{\alpha}\left(e^{\alpha k(p)T_1}-1\right)+k(p)xpr^y(T-T_2)\right)+$$
$$\frac{h}{\alpha k(p)}\left(\frac{k(p)xpr^y}{\sigma}\left(e^{\sigma(T_2-T_1)}-1\right)\left(e^{\alpha k(p)T_1}\right)+\frac{xpr^y}{\alpha}\left(e^{\alpha k(p)T_1}-1\right)+\frac{xpr^y}{\alpha}\right)\left(1-e^{-k(p)T_1}\right)-$$
$$h\frac{xpr^y}{\alpha}T_1+h\frac{k(p)xpr^y}{\sigma^2}e^{\sigma T_2}e^{-\sigma(T_2-T_1)}-h\frac{k(p)xpr^y}{\sigma}(T_2-T_1)$$
$$+h_2k(p)xpr^y(T-T_2)^2+h_3(1-\beta)k(p)xpr^y(T-T_2) \quad (63)$$

The sales Revenue (denoted by SR) is given as
SR= Selling price * Total demand over the cycle

$$SR = p\left(\int_0^{T_2}\left(k(p)xpr^y + \alpha N_0\right)dt - N_3(t)\right)$$

$$SR = p\left(\int_0^{T_2}\left(k(p)xpr^y + \alpha\left(\frac{k(p)xpr^y}{\sigma}\left(e^{\sigma(T_2-T_1)}-1\right)\left(e^{\alpha k(p)T_1}\right)+\frac{xpr^y}{\alpha}\left(e^{\alpha k(p)T_1}-1\right)\right)\right)dt + k(p)xpr^y(t-T_2)\right) \quad (64)$$

$$SR = p\left(\left(k(p)xpr^y + \alpha\left(\frac{k(p)xpr^y}{\sigma}\left(e^{\sigma(T_2-T_1)}-1\right)\left(e^{\alpha k(p)T_1}\right)+\frac{xpr^y}{\alpha}\left(e^{\alpha k(p)T_1}-1\right)\right)\right)T_2 + k(p)xpr^y(T-T_2)\right) \quad (65)$$

$$SR = p\left(\left(k(p)xpr^y + \frac{\alpha}{\sigma}\left(k(p)xpr^y\left(e^{\sigma(T_2-T_1)}-1\right)\left(e^{\alpha k(p)T_1}\right)\right)+\left(xpr^y\left(e^{\alpha k(p)T_1}-1\right)\right)\right)T_2 + k(p)xpr^y(T-T_2)\right) \quad (66)$$

Let P (T, T_2, s) be the profit rate function, since the profit rate function is the total sales revenue per unit minus Total cost per unit.

$$P(T,T_2,s) = SR - K(T,T_2,s) \quad (67)$$

$$\begin{aligned}P(T,T_2,s) = {} & p\left(\left(k(p)xpr^y + \frac{\alpha}{\sigma}\left(k(p)xpr^y\left(e^{\sigma(T_2-T_1)}-1\right)\left(e^{\alpha k(p)T_1}\right)\right)+\left(xpr^y\left(e^{\alpha k(p)T_1}-1\right)\right)\right)T_2 + k(p)xpr^y(T-T_2)\right) - \\ & \left(\begin{aligned} & B + s\left(\frac{k(p)xpr^y}{\sigma}\left(e^{\sigma(T_2-T_1)}-1\right)\left(e^{\alpha k(p)T_1}\right)+\frac{xpr^y}{\alpha}\left(e^{\alpha k(p)T_1}-1\right)+k(p)xpr^y(T-T_2)\right)+ \\ & \frac{h}{\alpha k(p)}\left(\frac{k(p)xpr^y}{\sigma}\left(e^{\sigma(T_2-T_1)}-1\right)\left(e^{\alpha k(p)T_1}\right)+\frac{xpr^y}{\alpha}\left(e^{\alpha k(p)T_1}-1\right)+\frac{xpr^y}{\alpha}\right)\left(1-e^{-k(p)T_1}\right) \\ & -h\frac{xpr^y}{\alpha}T_1 + h\frac{k(p)xpr^y}{\sigma^2}e^{\sigma T_2}e^{-\sigma(T_2-T_1)} - h\frac{k(p)xpr^y}{\sigma}(T_2-T_1) \\ & +h_2k(p)xpr^y(T-T_2)^2 + h_3(1-\beta)k(p)xpr^y(T-T_2)\end{aligned}\right)\end{aligned} \quad (68)$$

Total profit per unit time is

$$TP(T_2,T,p) = \frac{1}{T}\left(\begin{aligned} & p\left(\left(k(p)xpr^y + \frac{\alpha}{\sigma}\left(k(p)xpr^y\left(e^{\sigma(T_2-T_1)}-1\right)\left(e^{\alpha k(p)T_1}\right)\right)+\left(xpr^y\left(e^{\alpha k(p)T_1}-1\right)\right)\right)T_2 + k(p)xpr^y(T-T_2)\right) - \\ & \left(\begin{aligned} & B + s\left(\frac{k(p)xpr^y}{\sigma}\left(e^{\sigma(T_2-T_1)}-1\right)\left(e^{\alpha k(p)T_1}\right)+\frac{xpr^y}{\alpha}\left(e^{\alpha k(p)T_1}-1\right)+k(p)xpr^y(T-T_2)\right)+ \\ & \frac{h}{\alpha k(p)}\left(\frac{k(p)xpr^y}{\sigma}\left(e^{\sigma(T_2-T_1)}-1\right)\left(e^{\alpha k(p)T_1}\right)+\frac{xpr^y}{\alpha}\left(e^{\alpha k(p)T_1}-1\right)+\frac{xpr^y}{\alpha}\right)\left(1-e^{-k(p)T_1}\right) \\ & -h\frac{xpr^y}{\alpha}T_1 + h\frac{k(p)xpr^y}{\sigma^2}e^{\sigma T_2}e^{-\sigma(T_2-T_1)} - h\frac{k(p)xpr^y}{\sigma}(T_2-T_1) \\ & +h_2k(p)xpr^y(T-T_2)^2 + h_3(1-\beta)k(p)xpr^y(T-T_2)\end{aligned}\right)\end{aligned}\right) \quad (69)$$

Differentiating (3.69) with respect to T_2, T, and p, we have

$$\frac{\partial TP(T_2,T,p)}{\partial T_2}, \frac{\partial TP(T_2,T,p)}{\partial T}, \frac{\partial TP(T_2,T,p)}{\partial p}$$

$$\frac{\partial TP(T_2,T,p)}{\partial T_2}=\frac{1}{T}\begin{pmatrix} P\left(A+A_1\left(e^{\sigma(T_2-T_1)}\right)+A_2\left(e^{\alpha k(p)T_1}-1\right)\right)+P\left(0+A_1\left(A\left(\sigma e^{\sigma(T_2-T_1)}\right)\right)+A_2(0)\right)T_2 \\ +(-A)-A_3\left(A_4\left(\sigma e^{\sigma(T_2-T_1)}\right)e^{\alpha k(p)T_1}\right)+A_5(0)-A+A_6\left(A_4\left(\sigma e^{\sigma(T_2-T_1)}\right)\left(e^{\alpha k(p)T_1}\right)+A_5(0)+0\right)\left(1-e^{-\alpha k(p)T_1}\right) \\ -A_7(0)+A_8(0)-A_9+A_{10}(-2T+2T_2)+A_{11}(-1) \end{pmatrix} \quad (70)$$

That is, (3.70) becomes

$$\frac{\partial TP(T_2,T,p)}{\partial T_2}=\frac{1}{T}\begin{pmatrix} P\left(A+A_1\left(\left(e^{\sigma(T_2-T_1)}\right)\right)+\left(A_2\left(e^{\alpha k(p)T_1}-1\right)\right)\right)+P\left(A_1\left(A\left(\sigma e^{\sigma(T_2-T_1)}\right)\right)\right)T_2 \\ -A-A_3\left(A_4\left(\sigma e^{\sigma(T_2-T_1)}\right)e^{\alpha k(p)T_1}\right)-A+A_6\left(A_4\left(\sigma e^{\sigma(T_2-T_1)}\right)\left(e^{\alpha k(p)T_1}\right)\right)\left(1-e^{-k(p)T_1}\right) \\ -A_9+A_{10}(2T_2-2T)-A_{11} \end{pmatrix} \quad (71)$$

$$\frac{\partial TP(T_2,T,p)}{\partial T}=-\frac{1}{T^2}\begin{pmatrix} P\left(A+A_1\left(A\left(e^{\sigma(T_2-T_1)}-1\right)\right)+\left(A_2\left(e^{\alpha k(p)T_1}-1\right)\right)\right)T_2+A(T-T_2) \\ -A_3\left(A_4\left(e^{\sigma(T_2-T_1)}-1\right)\left(e^{\alpha k(p)T_1}\right)+A_5\left(e^{\alpha k(p)T_1}-1\right)+A(T-T_2)\right) \\ +A_6\left(A_4\left(e^{\sigma(T_2-T_1)}-1\right)\left(e^{\alpha k(p)T_1}\right)+A_5\left(e^{\alpha k(p)T_1}-1\right)+A_5\right)\left(1-e^{-\alpha k(p)T_1}\right) \\ -A_7T_1+A_8e^{\sigma T_1}-A_9(T_2-T_1)+A_{10}(T-T_2)^2+A_{11}(T-T_2) \end{pmatrix}$$
$$+\frac{1}{T}\left(P(0)+A-A_3(A)+0+0+2A_{10}(T-T_2)+A_{11}\right) \quad (72)$$

This Implies that;

$$\frac{\partial TP(T_2,T,p)}{\partial T}=-\frac{1}{T^2}\begin{pmatrix} P\left(A+A_1\left(A\left(e^{\sigma(T_2-T_1)}-1\right)\right)+\left(A_2\left(e^{\alpha k(p)T_1}-1\right)\right)\right)T_2+A(T-T_2) \\ -A_3\left(A_4\left(e^{\sigma(T_2-T_1)}-1\right)\left(e^{\alpha k(p)T_1}\right)+A_5\left(e^{\alpha k(p)T_1}-1\right)+A(T-T_2)\right) \\ +A_6\left(A_4\left(e^{\sigma(T_2-T_1)}-1\right)\left(e^{\alpha k(p)T_1}\right)+A_5\left(e^{\alpha k(p)T_1}-1\right)+A_5\right)\left(1-e^{-\alpha k(p)T_1}\right) \\ -A_7T_1+A_8e^{\sigma T_1}-A_9(T_2-T_1)+A_{10}(T-T_2)^2+A_{11}(T-T_2) \end{pmatrix}$$
$$+\frac{1}{T}\left(A-A_3A+2A_{10}(T-T_2)+A_{11}\right) \quad (73)$$

and similarly,

$$\frac{\partial TP(T_2,T,p)}{\partial p}=\frac{1}{T}\left(T_2\left(A+A_1\left(A\left(e^{\sigma(T_2-T_1)}-1\right)\right)+\left(A_2\left(e^{\alpha k(p)T_1}-1\right)\right)\right)\right) \quad (74)$$

Where For (3.70) to (3.74)

$A=K(p)xpr^y$, $A_1=\dfrac{\alpha}{\sigma}$, $A_2=xpr^y$,

$A_3=B+s$, $A_4=\dfrac{K(p)xpr^y}{\sigma}$,

$A_5=\dfrac{xpr^y}{\alpha}$, $A_6=\dfrac{h}{\alpha k(p)}$,

$A_7 = \frac{h}{\alpha} xpr^y$, $A_8 = \frac{hK(p)xpr^y}{\sigma^2}$

$A_9 = \frac{hK(p)xpr^y}{\sigma}$, $A_{10} = h_2 K(p) xpr^y$ and

$A_{11} = h_3(1-\beta)K(p)xpr^y$.

The model is solved and we arrived at (3.69) using direct integration and integrating factor as a method of solution.

RESULTS

The researchers presented the data used for 'Economic Order Quantity for Items that Exhibit Delay in Deterioration with Price, Stock and Reliability Demand Under Partial Backlogging'. Computational results are performed using Intel(R) Core(TM) i5-3340M CPU @ 2.70GHz 8.00 GB memory, Excel office 2016 and Maple2015.

Numerical Examples

To exhibit delayed deterioration with constant deterioration and Price, Stock and Reliability demand consideration in an EOQ model of items, to investigate the effect of price, stock and reliability on demand of items that exhibit delayed deterioration and to maximize the profit function in order to determine the best inventory management policy, numerical examples were given for illustrations.

In a superstore the demand rate not only depends upon the amount of the stock but also depends upon the reliability as well as the price of the item so that demand rate is D(N(t); r, p) where γ = 100, δ = 1.4, x= 100, p = 6, r = 1, y = 3, α = 0.15 β = 0.6. Let us consider that the item deteriorates at constant rate 0.1 part of the total inventory. Let the shortages cost be #2per unit item and #250 to order the total inventory. Let the cost of each item be #3, selling price is #20; to hold the item it requires #0.6 per unit and the reliability of the item is 1. Now we have to maximize the profit per unit item per unit time for the above situations of inventory system. We consider the following information as input parameters for the proposed inventory model, we have p = 20, θ = 0.1, B = 250 per order, h= 0.6unit, s= 3 per item, per year, T =1, T_1 = 0.5, T_2 = 1.2.

DISCUSSION

Table 1 shows the effect of stock dependent consumption rate parameter against the demand.

Table 1: Stock dependent consumption parameter rate against Demand rate

A	R	P	K(p)	D
0.15	1	20	74.59123	96125655
0.2	1	20	74.59123	3.77E+09
0.25	1	20	74.59123	6.67E+74
0.3	1	20	74.59123	5.75E+12
0.35	1	20	74.59123	2.25E+14
0.4	1	20	74.59123	8.77E+15
0.45	1	20	74.59123	3.43E+17
0.5	1	20	74.59123	1.34E+19
0.55	1	20	74.59123	5.22E+20
0.6	1	20	74.59123	2.04E+22

α = stock dependent consumption rate parameter (0.15 – 0.6), r = is the reliability (1), p = selling price (20), k(p) =The price factor (74,59123), D= demand rate (96125655 – 2.04E+22),

Table 1 shows that as the stock dependent consumption rate parameter increases, the price factor k(p) is constant which increases the demand for goods this is as a result of the attraction brought by products display on shelf, its popularity and variety to the customers, because when there is low stock in the shop, goods are most times treated as though they are not fresh even though on the other hand a customer can think that a large amount of stock means the item is of less demand because the other customers are not buying but when the stock is well optimized, the demand keep increasing and thereby increasing the total profit.

Table 2 shows the effect of price factor on the demand.

Table 2: Table of price factor against Demand Rate

A	R	P	K(p)	D
0.15	1	20	61.07014	96125655
0.15	1	20.5	74.59123	2.69E+09
0.15	1	21	91.10594	9.63E+10
0.15	1	21.5	111.277	5.93E+12
0.15	1	22	135.9141	7.64E+14
0.15	1	22.5	166.0058	2.52E+17
0.15	1	23	202.76	2.69E+20
0.15	1	23.5	247.6516	1.21E+24
0.15	1	24	302.4824	3.22E+28
0.15	1	24.5	369.4528	7.52E+33

α = stock dependent consumption rate parameter (0.15), r = is the reliability (1), p = selling price (20 – 24.5), k(p) =The price factor (61.07014 – 369.4528), D= demand rate (96125655 – 7.52E+33),

Table 2 shows that as the selling price increases, the price factor k(p) increases, of which the demand for goods also increases. The price of a product in the market has both negative and positive aspects. Some customers can buy a product because it is cheap and has a low price while some other customers believes that the product with the higher price is of more quality than the one with a lower price therefore, increasing the demand for the product with the higher price which also increases the total profit.

Table 3 shows the effect of reliability of goods on the demand.

Table 3: Reliability against Demand rate

A	R	P	K(p)	D
0.15	1	20	61.07014	96125655
0.15	1.2	20	69.78062	93304691
0.15	1.4	20	76.75315	96509914
0.15	1.6	20	82.43606	1.03E+08
0.15	1.8	20	87.14545	1.12E+08
0.15	2	20	91.10594	1.22E+08
0.15	2.2	20	94.47986	1.34E+08
0.15	2.4	20	97.3867	1.46E+08
0.15	2.6	20	99.91609	1.6E+08
0.15	2.8	20	102.1364	1.75E+08

α = stock dependent consumption rate parameter (0.15), r = is the reliability (1- 2.8), p = selling price (20), k(p) =The price factor (61.07014 – 102.1364), D= demand rate (96125655 – 7.52E+08). Table 3 shows that as the reliability of goods increases, even though the price factor k(p) increases, but the demand for goods also increases. As it has been discussed above, even though some customers so far, the product is cheap, they do not really care if the product is of good quality or not, they go for it. But we still have more demand from a product that is more reliable even if the price factor of the demand increases. This shows that most customers prefer the product to be reliable and expensive than to be cheap and not reliable. Thus, the reliability of a particular product increases the demand for the product which in returns, increase the total profit.

The analysis of the table 4.4 is given from the equation (3.69). The table 4.4 shows the effect of deterioration of goods against the Total Profit in thousands of Naira

Table 4: Deterioration rate against the Total Profit per time

Θ	B	T_1	T_2	T	TP
0.1	0.6	0.5	1.2	3	131
0.15	0.6	0.5	1.2	3	126
0.2	0.6	0.5	1.2	3	122

0.25	0.6	0.5	1.2	3	118
0.3	0.6	0.5	1.2	3	114
0.35	0.6	0.5	1.2	3	111
0.4	0.6	0.5	1.2	3	107
0.45	0.6	0.5	1.2	3	104
0.5	0.6	0.5	1.2	3	100
0.55	0.6	0.5	1.2	3	097

θ = Deterioration rate (0.1- 0.5), β=backordering rate (0.6), T_1 =The length of time in which the product exhibits no deterioration (0.5), T_2 =The Length of time in which the stock level vanishes (1.2), T= The replenishment cycle time (3), and TP = the total profit per unit time

Table 4 shows that as the deterioration of a product increases at a given time, the Total profit of the product decreases as a result of the reduction in the quantity and quality of the product that can be sold at a given time. As a product deteriorates, the willingness for a customer to pay for the product start dropping once the product is getting to its expiration date, or when the product is no more in good shape due to spoilage, or when the product decreases in its usefulness or in its obsolescence state. During this period, if necessary, actions are not taken for example, discounting the price or quantity of the deteriorated product, the demand rate would drastically be reduced and therefore, causing a decrease in the total profit due to deteriorated product. On the other hand, too much of price discounts or quantity discounts on a product to generate enough sales can bring a decrease to the total profit.

The analysis of the table 4.5 is given from the equation (3.69). Table 4.5 shows the effect of replenishment cycle time against the Total Profit in thousands of Naira

Table 5 Replenishment cycle time against the Total Profit per time

Θ	B	T_2	T_1	T	TP
0.1	0.6	1.2	0	3	264
0.1	0.6	1.2	0	3.2	247
0.1	0.6	1.2	0	3.4	232
0.1	0.6	1.2	0	3.6	219
0.1	0.6	1.2	0	3.8	207
0.1	0.6	1.2	0	4.0	196
0.1	0.6	1.2	0	4.2	186
0.1	0.6	1.2	0	4.4	177
0.1	0.6	1.2	0	4.6	169
0.1	0.6	1.2	0	4.8	161

θ = Deterioration rate (0.1), β=backordering rate (0.6), T_1 =The length of time in which the product exhibits no deterioration (0.5), T_2 =The Length of time in which the stock level vanishes (1.2), T= The replenishment cycle time (3 – 4.8), and TP = the total profit per unit time.

Table 5 shows that as the replenishment cycle time increases, the Total profit of the goods decreases especially when instantaneous deterioration of the product is being considered. As a result of this, the quality of the product is worsened when the replenishment cycle time is increased. When the replenishment cycle time of a product is being increased, it gives room for a product to deteriorate more in such that sometimes, the product might no longer even meet the changing demand of a customer when the product stays long in the store, this may result in decrease in demand and therefore cause a decrease in the total profit.

The analysis of the table 4.6 is given from the equation (3.69). Table 4.6 shows the effect of backlogging rate on the Total Profit in thousands of Naira

Table 6: Backlogging rate against the Total Profit per time

Θ	B	T_2	T_1	T	TP
0.1	0.6	1.2	0	3	14867
0.1	0.7	1.2	0	3.2	14873
0.1	0.8	1.2	0	3.4	14880
0.1	0.9	1.2	0	3.6	14886
0.1	1.0	1.2	0	3.8	14892
0.1	1.1	1.2	0	4.0	14899
0.1	1.2	1.2	0	4.2	14906
0.1	1.3	1.2	0	4.4	14912
0.1	1.4	1.2	0	4.6	14919
0.1	1.5	1.2	0	4.8	14925

θ = Deterioration rate (0.1), β=backordering rate (0.6 – 1.5), T_1 =The length of time in which the product exhibits no deterioration

(0.5), T_2=The Length of time in which the stock level vanishes (1.2), T= The replenishment cycle time (3 – 4.8), and TP = the total profit per unit time.

Table 6 shows that as the backordering rate of a product increases, the Total profit of the product increases also, this is because backordering allows customers to continue purchasing items that are not readily available and so one can keep accumulating sales even when products are not physically available for delivery, this reduces unnecessary inventory cost such as holding cost or cost due to the deterioration of the product, this process therefore enhances the total profit of an organization.

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