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MATHEMATICAL MODEL TO INVESTIGATE EFFECT OF VALVES ON TRANSIENT HYDROGEN-NATURAL GAS MIXTURE PRODUCTION IN HIGH-PRESSURE, HIGH-TEMPERATURE GAS PRODUCING WELLS

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ABSTRACT

High pressure, high temperature (HPHT) hydrogen mixture gas well is a well with temperature over $150^o\,C$ at the bottom and require equipment working pressure of $69\,MPa$. In this situation, a mathematical model to investigate the effect of valves on transient high-pressure hydrogen-natural gas mixture production is important to producing gas industries. Many work in Natural gas producing wells were done to find lasting solutions to problems that led to the premature closure of these wells. To date, no proven solution has been accepted to cover all of the issues encountered by Natural gas producing industries especially the effect of valves closing in high-pressure high-temperature hydrogen-natural gas-producing wells. In this paper, numerical computation has been carried out on mathematical models for the transient flow of hydrogen natural gas mixture (H2NG) in a Natural gas producing well. The model consists of partial differential equations of the conservation of mass and momentum with transient conditions. A transient state occurred during operation when control valves suddenly opened and closed at the wellhead and environmental formation. The model was solved using Steger Warming Flux Vector Splitting Method. The method has been proven to be unconditionally stable and has been applied in pipeline gas transportation network. The results obtained on the flow parameter characteristics namely the pressure and temperature are presented which shows good agreement with existing work. New results of density, velocity and sound wave propagation were also presented. We conclude that the work has provide a technical reliance to gas producing industries. It will served as a way forward for solution of problems encountered during hydrogen–Natural gas production with disturbances at the wellhead and environmental formation effect in high pressure and high temperature (HPHT) gas wells.

Keywords: *HPHT Gas Wells, Wellhead Valve, Transient flow, Hydrogen-Natural gas mixture, FSM.*

INTRODUCTION

A Natural gas producing well that has the characteristics of having a bottom hole temperature of $150^o\,C$ equipped with equipment working pressure of $69\,MPa$ is referred to as High Pressure, High Temperature Natural gas producing wells. Challenges are most encountered during production of hydrogen-Natural gas mixture because transients occurs due to disturbances of the operational valves which causes pressure oscillations in the well. For this reason, developing a mathematical model to investigate the effect of the wellhead valves on transient high pressure, high temperature hydrogen-Natural gas mixture production is necessary to producing gas industries to avoid premature closure of wells. World energy requirement is continuously increasing; thereby causing environmental pollution due to energy generation through the existing sources, hence the need for production of natural gas which has less carbon emission compared to other sources despite the difficulties in its production. In an attempt to find solution of transient flow in the gas wells and to avoid premature closure of producing wells, authors such Mbaya and Amin (2015) developed a model for unsteady flow of gas and heat transfer in a producing gas with the aim to determine the effect of the formation temperature on the flowing fluid. Mbaya and Amin (2018) revisit the work of 2015 and introduced energy equation in the governing equation to account for the heat exchanges between the flowing fluid and the thermal conductivities of formation. They reported that heat transfer in the radial direction is transient but steady within the wellbore. Baba et al., (2020) developed a model to study the effect of hydrogen presents on transient flow perimeters and to improve on accuracy of transportation with reduction on computational time and reported that the presence of hydrogen in natural gas leads to decrease on heat flux in flow environment.

Jie *et al.,* (2020) developed a model for wellbore temperature and pressure distribution along the depth of a well and compare the result obtained with actual wellbore data of Tarim X gas well and XX well in Southwest China and the calculation shows that the result of the model is less than 3% of the field measured data which verifies the accuracy of the model. Bo *et al.,* (2021) reported that tubing leakage is one of the main reasons that cause annular pressure in HPHT gas wells. Their work further discussed the relationship between leakage rate and sustained annular pressure and fluid temperature distribution. Hassan and Kabir, (2018) reported that the perfect balance between theory and practice is to aid understanding of fluid flowing in the wellbore. They recommend that probing pressure traverse to be included in various multiphase fluid-flow situations

Bai *et al.,* (2014) developed a model for the determination of wellhead and bottom-hole pressure based on the principles of fluid dynamics considering fluid temperature to be constant neglecting the effect of the earth temperature on the temperature of the flowing fluid. Jiuping *et al,* (2013) developed a couple systems of partial differential equations for the variation of pressure, temperature, velocity and density at different time and depth in high pressure, high temperature well for two phases. The solution considers splitting techniques with Eulerian Generalized Riemann Problems (GRP) schemes. Haris *et al.,* (2022) reported that continuous energy demand driven by extensive growth in economic development and population places an ever-increasing burden on the existing fossil fuel utilization that represent a substantial percentage of this increasing energy demand but also creates challenges associated with increased greenhouse gas (GHG) emissions. Their work provides critical analysis of the state-of-the-art in blue and green hydrogen production. They use conventional and renewable energy sources, utilization of hydrogen, storage, transportation, distribution with key challenges and opportunities in the commercial deployment of such systems. Fazil *et al.,* (2023) in a review on latest trends in Hydrogen production highlights the recent advances in Hydrogen production, storage, and transportation. They further reported that world's production of fossil fuels, on the other hand, is predicted to reach a plateau soon and then decline. The percentage of diverse humanistic applications' power use, such as building power consumption were regarded as the principal energy usage, accounting for 51 % of overall energy consumption.

Oyeintonbra, et al, 2019) and with an effective well intervention program was designed and executed by joint team of local and global Experts. Currently, 5 of the 7 originally planned wells have been restored post well intervention and are producing above expectation.

Ramanand *et al.*, (2024), reported that the oil and gas drilling industry is now moving toward clear high-density completion fluids at HPHT reservoir conditions and presented that completion fluid is used to complete an oil and gas well which is used to ease final operations before the start of production processes. The results of their work show that alkaline pH value of 7.18 and solid free fluid system provide a suitable completion fluid to keep corrosion rates acceptably low. They further reported that high density of the completion fluid is an essential parameter for pressure maintenance during well control events.

Sidharh et al., (2023) presented a review that highlights the current state in the industry, future position, and strategies for the researchers to follow in dealing with HPHT gas production wells. They further explained that in high-performance HPHT drilling fluids thermal stability, rheology, filtration loss control, lubricity, salt tolerance, and environmental formation affect gas production. They discovered that the presence of sulfonate, methyl, phenyl, furan, lactam, and pyrrolidine functional groups enhance thermal stability, while amide, hydroxyl, cellulose derivatives, and acrylate functional groups improve rheological performances.

Baojiang et al., (2025), reported that extracting hydrogen from underground natural deposits may present a more economically viable alternative, provided that adequate gas capture technologies are implemented. They highlight that integrating the development of natural hydrogen with the production of hydrogen from renewable sources could further mitigate the challenges encountered in hydrogen production.

From the available literatures so far authors were so concerned with storage and transportation of hydrogen-Natural gas but its mixture transient production in high temperature, high pressure gas well and problem associated to its production has not been done especially on the transient situation cause by the disturbances at the wellhead and environmental formation. In this paper, a one dimensional mathematical model for the transient flow of hydrogen natural gas mixture $\left(H_2NG\right)$ in High Pressure, high Temperature (HPHT) producing gas well has been developed. The model consist of partial differential equations of the conservation of mass and momentum with transient condition to account for the possible disturbances at the wellhead due to sadden valves opening and closing and well formation geometry in HPHT gas wells system. The model was solved using Steger Warming Flux Vector Splitting Method.

Model Assumptions

In developing the model, the following assumptions were made;

1. The flow has isothermal condition and in HPHT Natural gas producing well system
2. The well is vertical, effect of inclination is not considered
3. The sadden closure of valves at the wellhead and the formation geometry are considered to be the cause of the transient.
4. The mixture has homogenous condition due to its gaseous state, therefore one dimensional flow equations will be sufficient in description and analysis.
5. Effect of inflow from reservoir is not considered

Governing Equations

The equation governing production of hydrogen-Natural gas mixture in the wellbore which is surrounded by environmental formation are the Euler equation of continuity and momentum in one dimension.

Equation of Continuity

$$\frac{\partial w_1}{\partial t}+\frac{\partial w_2}{\partial L}=0 \qquad 1$$

Equation of Momentum

$$\frac{\partial w_2}{\partial t}+\frac{\partial w_2^2}{w_1\partial L}=-\frac{\partial p}{\partial L}-\frac{fw_2^2}{w_1 2D} \qquad 2$$

Where $w_1=\rho$, $w_2=\rho u$, D is diameter and L is depth of the well. The governing equation (1) and (2) can be written in convective flux form as:

$$\frac{\partial W}{\partial t}+\frac{\partial E(W)}{\partial L}=-H(W) \qquad 3$$

$$W=\begin{pmatrix} w_1 \\ w_2 \end{pmatrix}=\begin{pmatrix} q_1=\rho \\ q_2=\rho u \end{pmatrix} \qquad 4a$$

$$E(W)=\begin{pmatrix} w_2 \\ \dfrac{w_2^2}{w_1}+a^2w_1 \end{pmatrix} \qquad 4b$$

$$H(W)=\begin{pmatrix} 0 \\ -\dfrac{w_2 f\left|w_2\right|}{2w_1D} \end{pmatrix} \qquad 4c$$

Where w_1 is the fluid density, u is fluid velocity, f is a friction factor, D is the well diameter, g is gravitational acceleration and θ is the inclination angle. In (4b) a^2 is the speed of sound given by $a^2=\frac{\gamma p}{w_1}$, w_1 is the hydrogen-Natural gas density and w_2 is the mass flow rate and u axial velocity.

Equation for Hydrogen-Natural Gas Mixture.

Density equation in high pressure, high temperature producing gas well was developed and in cooperated in the governing equation, it is a function of hydrogen and gas which depend on mass ratio per unit volume. Form bulk production in the reservoir, M , V are mass and volume of the mixture define by $M = M_h + M_g$, $V = V_h + V_g$ where $M_h, M_g, V_h, \text{ and } V_g$ are mass and volume of hydrogen and natural gas respectively. During production we assumed that the mixture flow is in polytrophic process such that,

$$\frac{p}{\rho^n_{\ g}} = \frac{p_0}{\rho^n_{\ g0}} \qquad 5$$

where $p,\ p_0,\ \rho_g,\ \rho_{g0}$ and n are motion pressure, initial pressure, motion mixture density of gas, initial gas density and n polytrophic index respectively. We present the mixture density ρ as

$$\frac{1}{\rho} = \frac{V}{M} = \frac{V_g + V_h}{M} \qquad 6$$

$$\frac{M_g V_g}{M M_g} + \frac{M_h V_h}{M M_h} = \left(\frac{M_g}{M}\right)\left(\frac{V_g}{M_g}\right) + \left(\frac{M_h}{M}\right)\left(\frac{V_h}{M_h}\right) \qquad 7$$

Taking reciprocal equation (7) becomes

$$\frac{1}{\rho} = R_m \times \frac{1}{\rho_h} + \left(\frac{M - M_h}{M}\right) \times \frac{1}{\rho_g} \qquad 8$$

$$\frac{1}{\rho} = \frac{R_m}{\rho_h} + \frac{1 - R_m}{\rho_g} \qquad 9$$

From the reservoir the bulk hydrogen of elasticity K_h is defined by

$$K_h = \frac{\rho_h dp}{d\rho_h} \qquad 10$$

Integration and evaluate integral (10) the hydrogen density becomes

$$\rho_h = \rho_{h_0} e^{(p - p_0)/K_h} \qquad 11$$

Substituting equations (11) and (5) into (9)

$$\frac{1}{\rho} = \frac{R_m}{\rho_{h0}}\left(\frac{p_0}{p}\right)^{\frac{1}{n}} + \frac{1 - R_m}{\rho_{g0}} e^{-(p - p_0)/K_g} \qquad 12$$

$$\frac{1}{\rho} = \frac{R_m}{\rho_{h0}}\left(\frac{p_0}{p}\right)^{\frac{1}{n}} + \frac{1 - R_m}{\rho_{h0}} e^{(p_0 - p)/K_g} \qquad 13$$

$$\rho = \left[\frac{R_m}{\rho_{g0}}\left(\frac{p_0}{p}\right)^{\frac{1}{n}} + \frac{1 - R_m}{\rho_{h0}} e^{(p_0 - p)/K_l}\right]^{-1} \qquad 14$$

Equation (14) gives density of gas well hydrogen-Natural gas mixture

Method of Solution

It was earlier mentioned that, Steger-Warming flux vector Splitting method (FSM) has been considered in this work as the numerical scheme because literature has shown that it does not have problem of numerical instability (Toro, 2008). In delta formulation, the finite difference form of the method is having five (5) components as;

$$M + N + P + R = Y \qquad 15$$

where;

$$M = \left(\frac{\Delta t}{\Delta L} A^+_{j-1}\right) Q_{j-1}, \qquad 16a$$

$$N = \left(I + \frac{\Delta t}{\Delta L}\left(A^+_j - A^-_j\right) - \Delta t B_j\right) \qquad 16b$$

$$P = \left(\frac{\Delta t}{\Delta L} A^+_{j+1}\right) \Delta Q_{j+1} \qquad 16c$$

$$R = -\frac{\Delta t}{\Delta L}\left(E^+_j - E^+_{j-1} + E^-_{j+1} - E^-_j\right) \qquad 16d$$

$$Y = \Delta t H_j \qquad 16e$$

The subscript *j* indicates the spatial grid point while the superscript indicates the time level and

$$\Delta Q + Q^n = Q^{n+1} \qquad 17$$

In equation (16) I is an identity matrix, and A , B are Jacobian matrix defined by

$$A = \frac{\partial E}{\partial W} \text{ and } B = \frac{\partial H}{\partial W} \qquad 18a,b$$

Where, A^+ and A^- are positive and negative part of the Jacobian matrix A and E^+ , E^- are positive and negative parts E respectively defined by

$$A^+ = \begin{bmatrix} \dfrac{c^2 - u^2}{2c} & \dfrac{u + c}{2} \\ \dfrac{(u + c)^2 (c + u)}{2c} & \dfrac{(c + u)^2}{2c} \end{bmatrix} \qquad 19a$$

$$A^- = \begin{bmatrix} \dfrac{u^2 - c^2}{2c} & \dfrac{c - u}{2} \\ \dfrac{(u + c)(c - u)^2}{2c} & \dfrac{(c - u)^2}{2c} \end{bmatrix} \qquad 19b$$

$$E^+ = \begin{bmatrix} \dfrac{w_1 (u + c)}{2} \\ \dfrac{w_1 (u + c)^2}{2} \end{bmatrix}, E^- = \begin{bmatrix} \dfrac{w_1 (u - c)}{2} \\ \dfrac{w_1 (u - c)^2}{2} \end{bmatrix} \qquad 20a,b$$

When equation (16) is applied to each grid point, a block tridiagonal system is formed. It is solved at each time step which resulted to ΔQ and next Q can be calculated using equation (17).

Initial Condition

During production and flow in pipeline condition, when field data is not available the steady state solution served as its initial conditions. The initial condition is the therefore the solution to the following equations

$$\frac{\partial w}{\partial L}=0 \qquad 21$$

$$\frac{\partial w_2^2\left(L,0\right)}{\rho\partial L}=-\frac{\partial p}{\partial L}-\frac{fw_2\left|u\right|}{w_1 2D} \qquad 22$$

Boundary Condition

The boundary condition is specifically based on the operational valves at wellhead and bottom of the well. The two valves closes automatically and the boundary are given below as;

At the start of production (bottom of the well) $L=0,$ $\rho\left(L,t\right)=\rho_0\left(t\right)$, $\frac{\partial u}{\partial L}=u_0\left(t\right)$ and at the wellhead, $\rho u\left(L,t\right)=\rho u_L\left(t\right)$ and $\frac{\partial u}{\partial L}=u_L\left(t\right)$ where, ρ, $\rho u=M_{h\,g}$ are the density at the bottom and wellhead gas mass flux respectively. We also consider the flamable limit of the hydrogen-Natural gas to be between $4.1-75$ in air with minimum igniting capacity of $0.2\ \left(M/J\right)$

RESULTS AND DISCUSSION

The method used has similarity agreement with existing work of Jiumping *et al*., (2013) as shown in Figure 1 and Figure 2 tested on pressure and temperature distribution in a producing gas well at a distance of $7000\ ft$ from reservoir to wellhead. At the start hydrogen mixture pressure increases rapidly as in figure 1; similarly temperature of the flowing mixture increases when it mixt up with temperature of the flow geometry as in figure 2.

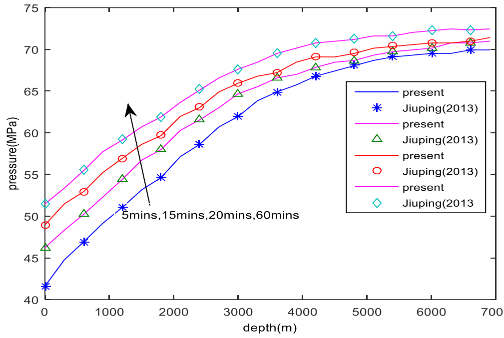

Figure 1: Mixture Pressure of Hydrogen-Natural Gas

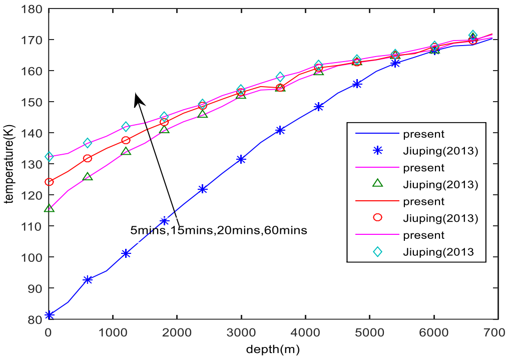

Figure 2: Temperature of mixture of Hydrogen-Natura Gas

At suden closure of the automatic valve at wellhead the mixture velocity and density were observed to drop significantly down the wellbore as shown in figure 3 and figure 4.

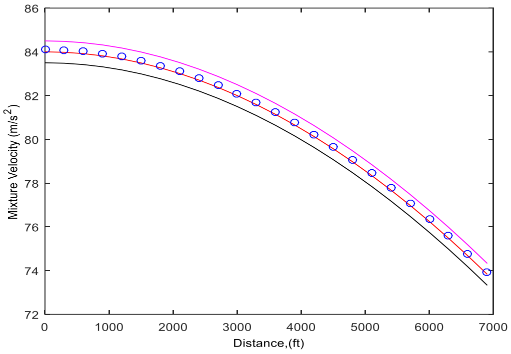

Figure 3: Mixture Velocity

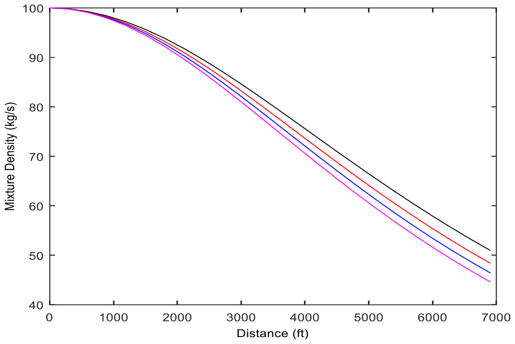

Figure 4: Mixture Density

When the thermal conductivities of the earth was considered, the hydrogen-Natural mixture velocity becomes lighter and it droped fastly doward at automatic closure of the well at wellhead. It was also observed that the combussion temperature increases in the well but unexpectedly droped down as shown in figure 6. The sound wave was steady at start of the production but changes rapidly when the valve was closed at wellhead as in figure 7.
The behaviour of the perforation pressure at different diameter was tested during at the time when the valves at the wellhead closed automatically as in figure 8. The pressure was cloudy which form pressure surge in the reservoir. The max flux was observed to be stable during production but becomes unstable at closing of the valves at wellhead as in figure 9.

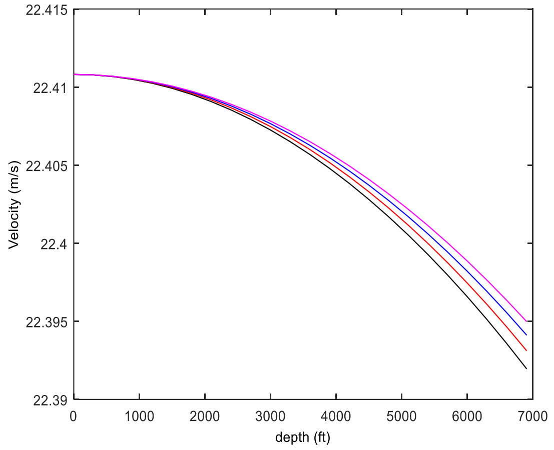

Figure 5: Velocity of Hydrogen at thermal Conductivity

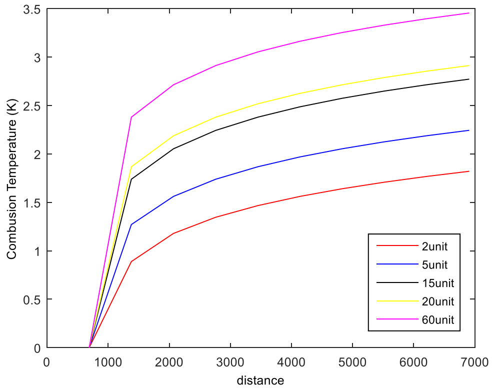

Figure 6: Combusion Temperature

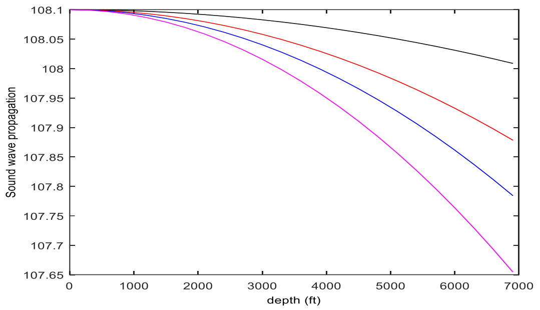

Figure 7: Effect sudden closure of valve on sound wave progation.

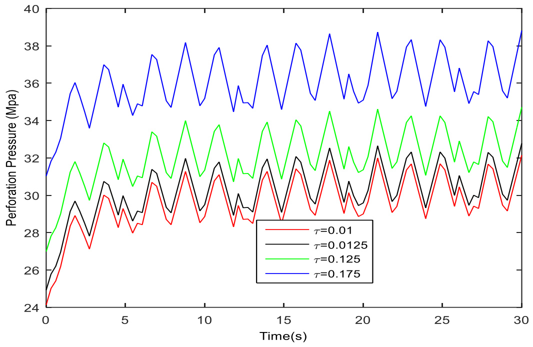

Figure 8: Perforation pressure in reservoir

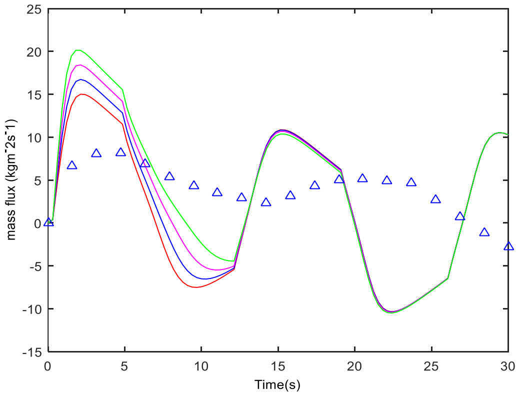

Figure 9: Behaviour of Max flux during closin

CONCLUSION

We have developed a model for hydrogen-Natural mixture in a high pressure, high temperature gas producing well. The model was tested on pressure and temperature distrubution during production period when there was a sadden closure of valve at the wellhead and was in good agreement with the existing work of Jiuping *et al.,* (2013). Other flow parameters were tested and effect of sudden closure of valve at the wellhead also affect them greatly. We also observed that the pressure build up is proportion to long reaction time and advised that it is much better to use small reaction time valve during hydrogen-Natural gas production. The impact of the sound wave at the point of sadden closure form a aback pressure at the early stage but stablised later after few minutes.

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