# ANALYSIS OF TEMPERATURE VARIATION IN HYDROMAGNETIZED FLOW IN THE PRESENCE OF JOULE HEATING, VISCOUS DISSIPATION, AND THERMAL RADIATION

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# ABSTRACT:

By incorporating joule heating. viscous dissipation, thermal radiation, variable temperature, we analyse an MHD boundary layer flow over a flat plate, considering the effects of radiation, Joule heating, and viscous dissipation. An appropriate similarity transformation is applied to convert the governing nonlinear coupled PDEs into ODEs, which are solved using the shooting method combined with six order Runge Kutta method. The effect of embedded governing flow parameter such as Prandle number, Eckert number, magnetic parameter on the velocity and temperature profiles are presented graphically and discussed in details. Additionally, the heat transfer rate is computed for various parameter values and presented in tabular form. The results indicate that an increase in the Eckert number leads to a rise in the temperature profile, whereas a higher Prandtl number results in a decrease in the temperature profile. The present results obtained are validated by comparing with similar existing results in literature and there is excellent agreement

# INTRODUCTION

The study of laminar flow over a flat plate has been extensively investigated through both analytical and experimental approaches, yielding significant insights into fluid dynamics under various conditions. A foundational contribution was made by Blasius (1908), who introduced simplifying assumptions to derive a solution for the laminar boundary layer problem. Building on this, White (1991) further explored the subject, providing a comprehensive analysis of laminar flow under simplified conditions. Over the years, researchers have expanded the scope of these studies by incorporating additional physical phenomena. For instance, Das (2012) examined hydromagnetic fluid flow over a flat plate with convective boundary conditions, considering non-uniform heat sources and sinks, radiation effects, partial slip, and temperaturedependent fluid properties. Similarly, Benazir et al. (2016) investigated the unsteady flow of Casson fluid over a vertical cone and flat plate in the presence of a magnetic field, demonstrating that the Casson fluid parameter inversely affects fluid flow.

The role of viscous dissipation in laminar flow has also been a key area of research. Brinkman (1951) pioneered the study of viscous dissipation effects, while Tyagi (1966) analyzed its influence on laminar flow under uniform wall temperature and forced convection in cylindrical tubes. Basu et al. (1985) extended this work by examining viscous dissipation in the Graetz problem, incorporating non-axial conduction. More recently, Kumar and Sivaraj (2013) explored unsteady magnetohydrodynamic (MHD) viscoelastic fluid flow with free convection, focusing on dual dispersion effects, chemical reactions, and the Dufour and Soret effects in non-Darcy porous media. Their findings highlighted the significant impact of

these factors on viscoelastic fluid behavior.

Other studies have investigated the interplay between viscous dissipation, mixed convection, and magnetic fields. Aydin et al. (2007) analyzed four distinct flow scenarios for fluid flow past a vertical plate, considering thermal conditions and free stream direction. Pantokratoras (2005) studied steady laminar flow over a heated vertical plate, emphasizing the role of fluid viscosity dissipation. Mamun et al. (2008) examined incompressible MHD flow along a vertical flat plate, incorporating conduction effects, while Aydın et al. (2009) conducted a computational study of stable laminar MHD flow with mixed convection and heat transfer, accounting for viscous dissipation and Ohmic heating. Additionally, Abo-Eldahab et al. (2005) investigated MHD-free convection flow under the combined influence of Hall currents and ion-slip over a semi-infinite vertical flat plate. Collectively, these studies have significantly advanced our understanding of laminar flow dynamics, particularly in the presence of complex physical and thermal interactions. Sohail et al. (2020) explored the effects of Joule heating, radiation, entropy generation, and thermal propagation on Casson fluid boundary layer flow (BLF) over a linearly stretching surface. In a study on MHD fluid flow over a stretchable porous sheet, Naseem et al. (2021) investigated transport phenomena by considering temperature-dependent thermal conductivity and diffusion coefficients. Their research also incorporated the Dufour and Soret effects, along with thermal radiation. Turkyilmazoglu et al. (2013) analyzed heat and mass transfer in nanofluid flows passing a vertical infinite flat plate, considering the impact of radiation under two distinct thermal boundary conditions. Jha and Samaila (2020) examined the effect of thermal radiation on fluid flow past a flat plate under convective boundary conditions. The dimensionless Eckert number (1987), representing the ratio of boundary layer enthalpy difference to the flow's kinetic energy, was introduced. Modifications to the momentum and energy equations were made by incorporating magnetic effects, radiation, viscous dissipation, and convective thermal transport along a flat plate. Numerous researchers have contributed to the study of fluid flow and heat transfer by examining different physical phenomena. Gangadhar et al. (2017) investigated Magnetohydrodynamic micropolar nanofluid past a permeable stretching/shrinking sheet with Newtonian heating. Kotha et al. (2018) examined the effect of thermal Radiation on Engine Oil Nanofluid Flow over a Permeable Wedge under Convective Heating: Keller Box Method. Rao (2021) et al, studied the bioconvection in a convectional nanofluid flow containing gyrotactic microorganisms over an isothermal vertical cone embedded in a porous surface with chemical reactive species. Kotha (2020) studied internal heat generation on bioconvection of an MHD nanofluid flow due to gyrotactic microorganisms. Ramaiah (2020) et al. examined MHD rotating flow of a Maxwell fluid with Arrhenius activation energy and non-Fourier heat flux model. Gangadhar et al (2021) investigated Nodal/Saddle stagnation point slip flow of an aqueous convectional magnesium oxide-gold hybrid nanofluid with viscous dissipation. Bhattacharyya and Layek examined (2010) chemically reactive solute distribution in MHD boundary layer flow over a permeable stretching sheet with suction or blowing. Gangadhar et al. (2020) investigated unsteady free convective boundary layer flow of a nanofluid past a stretching surface using a spectral relaxation method. Venkata Ramana et al. (2021) investigated heat flux theory on transverse MHD Oldroyd-B liquid over nonlinear stretched flow. Gangadhar et al. (2021) examined thermal slip flow of a three-dimensional Casson fluid embedded in a porous medium with internal heat generation. Kotha et al. (2020) studied Newtonian heating effect on laminar flow of Casson fluids. Narasimha Rao and Gangadhar (2022) carried out research study on Hall and ion-slip effects on MHD natural convective flow past an unbounded vertical porous channel with thermos diffusion. Mandal and Layek (2021) examined Unsteady MHD mixed convective Casson fluid flow over a flat surface in the presence of slip.

In the present study, the MHD boundary layer flow past a flat plate is analyzed, considering the combined effects of Joule heating, viscous dissipation, magnetic effects and thermal radiation under influence of convective boundary conditions for an incompressible Newtonian fluid. The current research work extends the model of Dasale (2015) to include variable thermal radiation and viscous dissipation. The research works further extend the governing model of Nassem (2022) to include the influence of induced magnetic effect . To the best of our knowledge, this particular thermal analysis has not been previously explored for the stated problem.

The structure of this paper is as follows: Section 1 presents the introduction. Section 2 outlines the governing coupled partial differential equations (PDEs), which are transformed into coupled nonlinear ordinary differential equations (ODEs) using appropriate similarity transformations. Section 3 details the methodology employed to solve the equations. Section 4 discusses the results, and Section 5 provides the conclusions

## MODEL FORMULATION

we examine the magnetohydrodynamic boundary layer flow of an incompressible fluid, incorporating thermal radiation, viscous dissipation Joule heating and magnetics effects, over a flat plate maintained at a uniform temperature  $T_{\scriptscriptstyle W}$ , which differs from the

ambient temperature  $T_\infty$  . Figure 1 and Figure 2 presents a schematic illustration of the fluid flow



Figure 1: velocity Profile



(1)

Figure 2: Temperature Profile

**Continuity Equation** 

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0$$

Momentum Equation

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = v\frac{\partial^2 u}{\partial y} - \frac{\sigma}{\rho}B_0^2 u \tag{2}$$

**Energy Equation** 

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha \frac{\partial^2 u}{\partial y} + \frac{v}{C_{\rho}} \left(\frac{\partial u}{\partial y}\right)^2 + \frac{1}{\sigma \rho C_n} B_0^2 u^2 - \frac{1}{\sigma \rho C_n} \frac{\partial}{\partial y} q_r$$
(3)

Using the Rosseland approximation for radiation, we obtain  $q = -\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y}$ , where  $k^*$ 

and  $\sigma^{\hat{}}$  represent the absorption coefficient and the Stefan-Boltzmann constant, respectively. Expanding  $T^4$  in a Taylor series around  $T_{\infty}$  and disregarding higher-order terms, we get...

$$T^{4} = 4T_{\infty}^{3}T - 3T_{\infty}^{4} \tag{4}$$

So Eq. (3) can be rewritten as

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \left(\alpha + \frac{1}{\rho C_p} \frac{16\sigma^*}{3k^*} T_{\infty}^3 \frac{\partial^2 u}{\partial y}\right) \frac{\partial^2 T}{\partial y^2} + (5)$$
$$\frac{v}{C_{\rho}} \left(\frac{\partial u}{\partial y}\right)^2 + \frac{1}{\sigma \rho C_p} B_0^2 u^2$$

Where u(x, y) and v(x, y) denote the velocities, and T signifies the temperature. Additionally, v and  $\alpha$  correspond to the kinematic viscosity and thermal diffusivity of the fluid, respectively. The boundary conditions can be stated as:

$$U = 0, T = T_w(x) = T_\infty + Ax^n, at y = 0,$$
  
and  $u = U, T = T_\infty as y \to \infty$  (6)

A suitable stream function  $\psi = \psi(x, y)$  that inherently satisfies the continuity Equation (1), along with similarity transformations, enables the conversion of the flow equations into nonlinear ODEs.

$$\eta = y \sqrt{\frac{U}{vx}} \psi = \sqrt{Uvx} f(\eta),$$
  

$$\theta = \frac{T - T_{\infty}}{T_{w}(x) - T_{\infty}}$$
(7)

$$u = Uf', v = -\frac{\partial \psi}{\partial x} = \frac{1}{2} \sqrt{\frac{U}{vx}} (\eta f' - f) \bigg]$$
$$u = Uf' \text{ and } v = -\frac{\partial \psi}{\partial x} = \frac{1}{2} \sqrt{\frac{U}{vx}} (\eta f' - f)$$

After applying (7), Equations (2)-(4) transform to

Where 
$$\Pr = \frac{v}{\alpha}$$
 and  $Ec = \frac{U^2}{C_p(T_w - T_{\infty})}$  denote

the dimensionless Prandtl and Eckert numbers, respectively. The heat transfer rate is characterized by the physically significant quantity  $-(1 + Rd)\theta'(0)$ . Consequently, our objective is to determine the influence of the governing parameters Pr, Ec, M, and n on these values.

#### NUMERICAL SOLUTION

The system of coupled system of ordinary differential equations (8) and (9) together with the boundary conditions (10), is numerically solved using the shooting technique, incorporating the sixth-order Runge–Kutta method across a range of moderate fluid flow and heat transfer parameters. The Broyden algorithm is applied to refine the initial guesses and ensure the boundary conditions at infinity are met. The numerical simulations are carried out using the Maple software package.

The procedure for this approach is outlined below:

The higher-order, non-linear differential equations are transformed into a set of first-order simultaneous differential equations.

The resulting initial value problem (IVP) is addressed by applying a fourth-order Runge-Kutta algorithm, Nassem et al. (2022). Appropriate initial guess values are determined using the Newton-Raphson method.

The	computations	were	carried	out

using the MAPLE software function The iterative process is continued until the specified boundary conditions are met



Figure 3: Effect of M on velocity profile

The effect of various physical parameters on temperature and velocity profiles is analyzed in this section using numerical data obtained through the Bvp4c method for different plate temperature conditions. By investigating a specific relationship expressed in the equations above, the plate's temperature is assumed to vary along the x-coordinate. Figure 3 illustrates the impact of the magnetic parameter (M) on the velocity profile. As pressure increases, fluid velocity tends to decline. In figure 4, the temperature distribution under a varying plate temperature is examined for different values of M, while keeping the Prandtl number (Pr), Eckert number (Ec), and Radiation parameter (Rd) unchanged. As depicted in Figure 4, temperature increases as magnetic parameter M increases due to the Lorenz force which its presence generate internal energy which cause rise in fluid temperature. parameter (Rd). As illustrated The temperature distribution under fluctuating plate temperature conditions is analyzed for different values of *n*, while maintaining constant Prandtl number (Pr), Eckert number (Ec), and Radiation in Figure. 5, the temperature decreases with increasing n



Figure 4: Effect of M on temperature profile



Figure 5: Effect of n on temperature profile

Influence of Eckert number on temperature profile is observed in Figure 6. The effects of increasing Eckert numbers on temperature distribution is evident as graphically representation provided in Figure 6. As Ec number increases, the temperature profile rises correspondingly. This increase is due to the supply of thermal energy to the boundary layer flow of the fluid temperature. The thermal energy act as an agent to increase the fluid temperature profile.

The influence of the Prandtl number on temperature distribution is also examined in



Figure 6: Effect of Ec on temperature profile



Figure 7: Effect of Pr on temperature profile

Figure 7 while keeping, M, n and Ec fixed. During this analysis, the temperature distribution decreases as the Prandtl number increase due to the withdrawer of heat from the boundary layer flow. The process provide insight in the significant or usefulness of Pr as cooling agent

Figure 8 describe the relationship between the thermal radiation and velocity profiles. Observation shows that, as radiation parameter increases the temperature profiles increases due to transport of energy flow within the fluid

To determine the Nusselt number, numerical results for  $\theta'(0)$  at the wall are presented in Tables 1–3. For Pr =0.7, numerical values of  $-\theta'(0)$  for different



Figure 8: Effect of Rd on temperature profile

Table 1: Illustrate of $-\theta'(0)$	for Pr= 0.7 and Ec= 0.1 at different
temperature	

n	Dasale (2015)	Nassem et al. (2022)	Present results
1	0.471081	0.47101	0.47102
2	0.576896	0.57678	0.57677
3	0.654579	0.65442	0.65443
4	0.717506	0.71730	0.71731

values of Ec with varying n are computed, as shown in Table 1.For Ec=0.5,  $-\theta'(0)$ 

Table	<b>2:</b> Illu	str	ate of	_	$\theta'(0)$	) fo P	r=0.7	and	Ec=	0.5 a	at diff	erent
tempe	erature	)							-			
									_			

n	Dasale (2015)	Nassem (2022)	et	al.	Present results
1	0.433778	0.43371			0.43372
2	0.544484	0.54437			0.54436
3	0.625153	0.62499			0.62499
4	0.690181	0.68998			0.68999

Table 3: Illustrate of  $-\theta'(0)$  fo n=3 and Ec= 0.5 at different temperature

Ec	Dasale (2015)	Nassem et al. (2022)	Present results			
0.1	0.654579	0.65442	0.65443			
0.3	0.639866	0.63971	0.63972			
0.5	0.625153	0.62499	0.62499			
0.7	0.610440	0.61028	0.61029			

values corresponding to different Prandtl numbers and various

values of n are provided in Table 2. For n=3, the numerical results of  $-\theta'(0)$  for different Prandtl numbers with varying Eckert numbers are given in Table 3. Table 1 reveals that  $-\theta'(0)$  values decrease as Ec increases for each n=1,2,3,4n = 1, 2, 3, 4, indicating that the Nusselt number declines, thereby reducing the rate of heat transfer at the wall. Conversely, as the Nusselt number rises, the heat transfer rate at the wall also increases, as demonstrated in Tables 2 and 3.

## Conclusion

The MHD boundary layer equations are utilized to examine the effects of radiation, viscous dissipation, Joule heating, and convection on heat transfer over a flat plate. Through similarity transformation, the governing boundary layer PDEs are transformed into nonlinear coupled ODEs, and the numerical solutions are obtained by solving these ODEs using the BVP4C method. To analyze radiation, viscous dissipation, and Joule heating, numerical results are generated by computing different values of the Eckert number. As the Eckert number increases, the temperature distribution for a variable temperature also rises, while an increase in the thermal radiation parameter (Rd) enhances thermal diffusivity, leading to a rise in temperature distribution. Additionally, the heat transfer rate at the wall decreases with a higher Eckert number for a fixed Prandtl number (Pr) but increases as Pr grows for a given variable temperature, indicating that the numerical results obtained via the shooting method align with the physical phenomena governing the problem.

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