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Full Length Research Article

DETERMINISTIC MODEL OF ANTI-BANDITRY DYNAMICS IN NIGERIA WITH INCORPORATION OF VULNERABLE COMMUNITY (IES)

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ABSTRACT
The banditry-induced insecurity in Nigeria has negatively impacted the human and socio-economic well-being of the citizenry. Consequently, Nigerian governments at different levels have deployed both kinetic and non-kinetic approaches to stem the tide of banditry. But these efforts have not brought out the desired results in terms of total safety. In light of this, this work studies the dynamics of anti-banditry controls and its overbearing effect on the vulnerable communities using mathematical models and analysis. This effort hints of two equilibrium points, namely banditry-free and anti-banditry induced equilibrium points. The stability analysis suggests the continuous growth of the community in the absence of banditry and banditry-induced chaotic dynamics when there is bandit groups' proliferation. To determine the optimal strategy to stem the chaotic dynamics obtained at the anti-banditry induced equilibrium point, sensitivity analysis is performed on model parameters, and this effort revealed the need for blockage of banditry access to weapons and checkmating banditry collaborators/informants within the ravaging communities as effective optimal anti-banditry measures.

Keywords: Banditry: Anti-banditry: Vulnerable Community: Mathematical Model: Stability Analysis: Sensitivity Analysis.

INTRODUCTION
Banditry refers to criminal activities carried out to accumulate wealth through armed robbery, kidnapping, cattle rustling, and village raids with impunity among other atrocities (Abdullahi, 2022). In Nigeria's case, banditry has reached alarming heights to the extent of setting up fortified enclaves in the hinterlands and on the frontiers, from where they plot and carry out their operations (Bildirici & Gokmenoglu, 2020) see also in (Chukwuma *et al.,* 2022). Many bandit groups exist in different hide-outs across North-western and North-central Nigeria (Barnett *et al.,* 2022). Though, there is no significant evidence suggesting either collaboration or the existence of any central authority, but there exist similarities in terms of their operations and recruitment approaches (Ojewale, 2021). The victims of the banditry attack were initial communities with material valuables but recently (Dami, 2021), they have had the effrontery to attack military bases, schools, and even a ~~t~~moving train (Saidu, 2022) also in (Sanchi *et al.,* 2022). The banditry activities have led to the loss of lives, goods and services leading to astronomical increases in prices of local products (Ghosh, 2019) also in (Ladan & Matawalli, 2020).

Since the insurgence of banditry, Nigerian governments both at the state and federal levels have adopted both kinetic and non-kinetic approaches to tame the activities of banditry groups. The kinetic approach involves the use of coordinated military deployment under several codes (Lamidi, 2024). Though, this approach has recorded success in the destruction of several hideouts, and killed or arrested hundreds of bandits, the attacks have continued in the affected areas till date (Abubakar *et al.,* 2021). The adoption of Air bombardment of bandit hide-outs by the Nigerian Air Force recently caused a greater causality on the part of bandit groups as deaths were recorded of some prominent bandit leaders and family members (Newton & Tucker, 2023). Though, there are evidences on successes recorded through air stikes on bandit groups hideouts but the accidental attacks on non-bandit communities still call for concise analyis before such strikes are activated (Ojewale, 2023). Also, as the air bombardment takes effect, there is evidence of persistent attacks on government infrastructure and kidnapping along major roads, urban residences and railways. (Maigari, 2021).

The non-kinetic approach involves dialogues, settlement, and peace missions to persuade bandit groups to either sheath their weapons or free their captives, or peace accord. These efforts, saw some states paying compensation to some bandit leaders or offering amnesty to those who have renounced banditry or kidnapping. Though, dialogues, settlement, and peace missions did not end banditry but they did reveal the underlying behind the banditry attacks which in most cases were economical (Okoli & Ugwu, 2019). Other non-kinetic approaches includes shutting down telecommunication services, labeling banditry groups as terrorists, and outlawing ransom. These approaches are beautiful strategies but did not hinder the increase in banditry activities like kidnapping and village raids (Tahir & Bernard, 2021).

Mathematical modeling has been a major tool in studying underpinning factors in dynamical phenomenon in biomedical sciences including cancerous cells dynamics as in (Ibrahim(a) *et al*., 2022) also in (Alblowy, A. H. et al. , (2023)) and infectious diseases as in (Abdo *et al.,* 2022) and recently in the analysis of fuel-subsidy dynamics in Nigeria as in (Ibrahim(b) et al., 2023). Mathematical Modeling of anti-banditry is one method to provide a structured methodology to gain insight into the various phenomenological dynamics entailed in activities of banditry, banditry groups' atrocities, and anti-banditry efforts of governments at all levels. Many mathematical efforts have been channeled to gain understanding on the activities of anti-state forces like insurgency and terrorism as seen in (Abidemi & Akanni, 2022) and in the citations therein. Akaani et al. in (Akanni & Abidemi, 2023) proposed a model to study the potential spread of illicit drug use and banditry in Nigeria population. Their findings suggest that stringent control measure on the reduction of illicit drugs by the people is sacrosanct to the reduction of banditry in the population.

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In light of the continuous search for optimal anti-banditry measures by government at different levels, the use of the mathematical model to identify the key parameter for the anti-banditry measures in the dynamics of banditry scourge is sacrosanct to cost-effective measure against banditry.

In this work, a model depicting the anti-banditry measures based on the sociological framework incorporating all players and their mechanisms for or against banditry dynamics is proposed.

MATERIALS AND METHODS

Model formulation and Analysis

The following assumptions are made for anti-banditry model dynamics:

i. That banditry groups' activities aimed at invading vulnerable community(ies) and form forces to resist attack from government (Abdulrasheed, A., 2021).
ii. That the Government authority develops various strategies including direct actions, negotiation, and other incentives to engage banditry groups so as to secure the civilian community (Alumona & Onwuanabile, 2023).
iii. That government forces collaborate with the community to beef up anti-banditry drive to protect vulnerable communities (OGYE & Smah, 2024).
iv. That banditry groups also infiltrate the vulnerable community to aid their activities (Abdulkadir & UMAR, 2024).
v. That the vulnerable community is prone to loss of resources and people due to banditry (Ladan & Matawalli, 2021).

Let x represent the conglomeration of anti-banditry measures of the government, y represent the assemblage of a banditry group, and z represent the colony of banditry threatened community. The rate of changes of $x, y,$ and z are defined by the system below:

$$\frac{dx}{dt} = \alpha x(t) + \beta x(t)y(t) + \upsilon_1 z(t)x(t) - \theta x(t),$$

$$\frac{dy}{dt} = \sigma y(t) + \gamma x(t)y(t) + \upsilon_2 z(t)y(t) - \vartheta y(t),$$

$$\frac{dz}{dt} = \phi z(t) + \upsilon_3 z(t)x(t) - \upsilon_4 z(t)y(t) - \rho z(t).$$

(1)

with initial conditions:

$$x(0) > 0,\ \ y(0) > 0,\ \ z(0) > 0,\ \ for\ \ t \in [0, t].$$

The first equation in System (1) is the rate of change in government's intensified effort against banditry with the first term depicting enlisting rate of anti-banditry forces (α), the second term is the rate at which government forces engage banditry groups (β), the third term is the collaborative rate between community and anti-banditry force (υ_1) and the last term depicts loss suffered by the anti-banditry force (θ).

The second equation of (1) is the rate of change in the activities of banditry groups with the first representing their proliferation rate (σ), the second term refers to the rate at which banditry group resists anti-banditry force attack (γ), the third is the rate at which banditry groups co-opt vulnerable residents to advance their activities (υ_2) and the last term refers to the loss suffered by bandits (ϑ).

The third equation in (1) represents the colony of vulnerable communities with the first term describing the community progress rate (ϕ), the second is the rate at which anti-banditry forces collaborate with the community in securing their colony (υ_3), the third term is the rate of depletion in colony due to banditry group attack (υ_4) and the last term represent non-banditry community loss (ρ). The schematic diagram of System (1) is given in Fig.1.

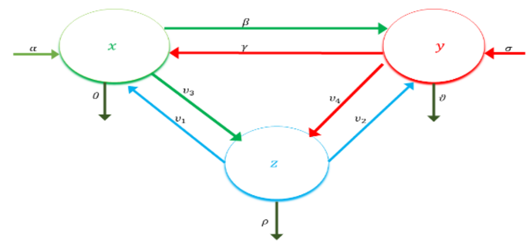

Figure 1: *A schematic diagram of anti-banditry dynamics.*

Model Analysis

Existence of Non-negative Solution

The existence of non-negativity of the solution System (1) is necessary to ensure that the result obtained is dynamically meaningful and conforms to the model assumptions. Thus, the assumption for the existence of non-negativity of solution of System (1) is given in Lemma 1 below.

Lemma 1 *Suppose $x(0)$, $y(0)$ and $z(0)$ are non-negative initial conditions for System (1). Then, there exist positive solutions for $x(t)$, $y(t)$, and $z(t)$ of System (1) for $t \in [0, t]$.*

Proof: Solving for $x(t)$, $y(t)$ & $z(t)$ in (1), we first consider and re-arrange the first equation in System (1) to solve $x(t)$ as follows:

$$\frac{dx}{dt} - [\alpha + \beta y(t) + \upsilon_1 z(t) - \theta]x(t) = 0. \qquad (2)$$

Equation (2) can be solved using the integrating factor defined as

$$I_x(t) = \exp(-[\alpha + \beta y(t) + \upsilon_1 z(t) - \theta]t). \qquad (3)$$

Multiplying equation (2) through by $I_E(t)$ yields

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$$\frac{dx}{dt}\exp(-[\alpha+\beta y(t)+\upsilon_1 z(t)-\theta]t)-[\alpha+\beta y(t)+\upsilon_1 z(t)-\theta]x(t)\exp(-[\alpha+\beta y(t)+\upsilon_1 z(t)-\theta]t)=0.$$

$$\equiv \frac{d}{dt}[\exp(-[\alpha+\beta y(t)+\upsilon_1 z(t)-\theta]t)x(t)]=0. \quad (4)$$

Integrating Equation (4) with respect to "t" in the interval $[0,t]$ yields

$$x(t)=[C_1+x(0)]\exp-[\alpha+\beta y(t)+\upsilon_1 z(t)-\theta]t, \quad (5)$$

where C_1 is an integral constant. Obviously, $x(t)$ is non-negative if $x(0)>0$. The non-negativity of the second and the third equations in System (1) is proved using the same process and thus given as folows:

$$y(t)=[C_2+y(0)]\exp-[\sigma+\gamma x(t)+\upsilon_2 z(t)-\vartheta]t,$$

$$z(t)=[C_3+z(0)]\exp-[\phi+\upsilon_3 x(t)-\upsilon_4 y(t)-\rho]. \quad (6)$$

Therefore, System (1) has non-negative solution for $x(t), y(t)$ and $z(t)$ for $t\in[0,t]$, if the assumptions in Lemma (1) hold.

Existence of Equilibrium Points
Theorem 1 Assume, there exist positive invariant points x^*, y^*, z^* for System (1), then there exists a banditry-free equilibrium point $(x^*,0,z^*)$ when $y^*=0$. And an anti-banditry induced equilibrium point (x^*,y^*,z^*) of System (1) when $y^*\neq 0$.

Proof: Suppose the assumptions in Theorem (1) hold, then x^*, y^* and z^* of System (1) satisfy the following:

$$\alpha x^*+\beta x^*y^*+\upsilon_1 x^*z^*-\theta x^*=0,$$

$$\sigma y^*+\gamma x^*y^*+\upsilon_2 y^*z^*-\vartheta y^*=0,$$

$$\phi z^*+\upsilon_3 z^*x^*-\upsilon_4 z^*y^*-\rho z^*=0.$$

Setting $y^*=0$ in (1) yields a banditry-free equilibrium point as given below:

$$(x^*,0,z^*)=\left(\frac{[\rho-\phi]}{\upsilon_3},0,\frac{[\alpha-\theta]}{\upsilon_1}\right). \quad (7)$$

When $y^*\neq 0$ in Equation (1), and solve for x^*, y^*, & z^* gives

$$x^*=\frac{\vartheta-\sigma-\upsilon_2 z^*}{\gamma},$$
$$y^*=\frac{\theta+\upsilon_1 z^*-\alpha}{\beta},$$

$$z^*=\frac{\beta\gamma[\phi-\rho]+\gamma\upsilon_4[\theta-\alpha]-\beta\upsilon_3[\vartheta-\sigma]}{\beta\upsilon_3\upsilon_2-\gamma\upsilon_4\upsilon_1}. \quad (8)$$

Therefore, Equation (8) forms the anti-banditry-induced equilibrium point. Hence, there exists a positive banditry-free equilibrium point $(x^*,0,z^*)=(\frac{[\rho-\phi]}{\upsilon_3},0,\frac{[\alpha-\theta]}{\upsilon_1})$ when $\rho>\phi$ and $\alpha-\theta$. Also, there exists a positive anti-banditry-induced equilibrium point

$$(x^*,y^*,z^*)=\left(\frac{\vartheta-\sigma-\upsilon_2 z^*}{\gamma},\frac{\theta+\upsilon_1 z^*-\alpha}{\beta},\frac{\beta\gamma[\phi-\rho]+\gamma\upsilon_4[\theta-\alpha]-\beta\upsilon_3[\vartheta-\sigma]}{\beta\upsilon_3\upsilon_2-\gamma\upsilon_4\upsilon_1}\right)$$

when $\phi>\rho$, $\vartheta>\delta$ & $z^*>0$.

Local Stability Analysis
We linearize System (1) and obtain a characteristic equation in its simplest form as:

$$D(\lambda,\tau)=(\lambda-(\alpha+\beta y^*-\theta-\upsilon_1 z^*))[(\lambda-(\sigma-\vartheta+\gamma x^*+\upsilon_2 z^*))(\lambda-(\phi+\upsilon_3 x^*-\upsilon_4 y^*-\rho))+\upsilon_2 y^*\upsilon_4 y^*]-\beta x^*[\gamma y^*(\lambda-(\phi+\upsilon_3 x^*-\upsilon_4 y^*-\rho))-\upsilon_3 z^*\upsilon_2 y^*]-\upsilon_1 x^*\left[-\gamma y^*\upsilon_4 y^*-\upsilon_3 z^*\left(\lambda-(\sigma-\vartheta+\gamma x^*+\upsilon_4 z^*)\right)\right]=0. \quad (9)$$

Hence, the long-term behaviours of equilibrium points are defined by the following theorems:

Theorem 2 The banditry-free equilibrium point for System (1) is asymptotically stable if and only if $\vartheta>\delta+\frac{\upsilon_4[\alpha-\theta]}{\upsilon_1}$, otherwise, it is unstable.

Proof: Substituting $(x^*,0,z^*)=(\frac{[\rho-\phi]}{\upsilon_3},0,\frac{[\alpha-\theta]}{\upsilon_1})$ into Equation (9), the resulting cubic polynomial of (9) becomes

$$D(\lambda,\tau)=\lambda^3-[\sigma-\vartheta+\gamma[\frac{[\rho-\phi]}{\upsilon_3}]+\upsilon_2[\frac{[\alpha-\theta]}{\upsilon_1}]]\lambda^2+\upsilon_1[\frac{[\rho-\phi]}{\upsilon_3}]\upsilon_3[\frac{[\alpha-\theta]}{\upsilon_1}]\lambda$$
$$+\upsilon_1\left[\frac{[\rho-\phi]}{\upsilon_3}\right]\upsilon_3\left[\frac{[\alpha-\theta]}{\upsilon_1}\right]\left(\sigma-\vartheta+\gamma\left[\frac{[\rho-\phi]}{\upsilon_3}\right]+\upsilon_4\left[\frac{[\alpha-\theta]}{\upsilon_1}\right]\right)=0. \quad (10)$$

The simplified version of Equation (10) is

$$\lambda^3-f_1\lambda^2+f_2\lambda+f_3=0, \quad (11)$$

where

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$$f_1 = \sigma - \vartheta + \gamma[\frac{[\rho-\phi]}{v_3}] + v_2[\frac{[\alpha-\theta]}{v_1}],$$
$$f_2 = v_1[\frac{[\rho-\sigma]}{v_3}]v_3[\frac{[\alpha-\theta]}{v_1}],$$
$$f_3 = v_1[\frac{[\rho-\phi]}{v_3}]v_3[\frac{[\alpha-\theta]}{v_1}](\sigma - \vartheta + \gamma[\frac{[\rho-\phi]}{v_3}] + v_4[\frac{[\alpha-\theta]}{v_1}]). \quad (12)$$

By the Routh-Hurwitz criterion, Equation (11) has all negative roots if and only if

$$f_1 > 0, \quad f_3 > 0, \quad \& \ f_1 f_2 - f_3 > 0. \quad (13)$$

The conditions in Equation (13) can only hold if Equation (11) has all positive coefficients. This is only obtainable if and only if $\vartheta > \sigma + \frac{v_4[\alpha-\theta]}{v_1}$. However, this is indeed not realistic because the banditry-free equilibrium point has $y = 0$, which implies $\sigma = \vartheta = 0$. Hence, the banditry-free equilibrium point is always unstable. Sociologically, this implies that:

Peace in a banditry-free community can only be sustained if there is no basis for the emergence of banditry activity in terms of proliferation in such communities.

Theorem 3 If there exists a positive anti-banditry-induced equilibrium point for the System (1), then the anti-banditry-induced equilibrium point is always unstable for a positive.

Proof: Substituting into Equation (11), $(x^*, y^*, z^*) = (\frac{\vartheta-\sigma-v_2 z^*}{\gamma}, \frac{\theta+v_1 z^*-\alpha}{\beta}, \frac{\beta\gamma[\phi-\rho]+\gamma v_4[\theta-\alpha]-\beta v_3[\vartheta-\sigma]}{\beta v_3 v_2-\gamma v_4 v_1})$
and the resulting cubic polynomial becomes

$$D(\lambda,\tau) = \lambda^3 - (\phi + v_3[\frac{\vartheta-\sigma-v_2 z^*}{\gamma}] - v_4[\frac{\theta+v_1 z^*-\alpha}{\beta}] - \rho)\lambda^2 + [v_2[\frac{\theta+v_1 z^*-\alpha}{\beta}]v_4[\frac{\theta+v_1 z^*-\alpha}{\beta}] + v_1[\frac{\vartheta-\sigma-v_2 z^*}{\gamma}]v_3 z^* - \beta[\frac{\vartheta-\sigma-v_2 z^*}{\gamma}][\gamma[\frac{\theta+v_1 z^*-\alpha}{\beta}]]]\lambda + \beta[\frac{\vartheta-\sigma-v_2 z^*}{\gamma}][\gamma\left[\frac{\theta+v_1 z^*-\alpha}{\beta}\right]\left(\phi + v_3\left[\frac{\vartheta-\sigma-v_2 z^*}{\gamma}\right] - v_4\left[\frac{\theta+v_1 z^*-\alpha}{\beta}\right] - \rho\right) + v_3 z^* v_2[\frac{\theta+v_1 z^*-\alpha}{\beta}]] + v_1[\frac{\vartheta-\sigma-v_2 z^*}{\gamma}]\gamma[\frac{\theta+v_1 z^*-\alpha}{\beta}]v_4[\frac{\theta+v_1 z^*-\alpha}{\beta}] \quad (14)$$

The simplified version of Equation (14) is

$$\lambda^3 - h_1\lambda^2 + h_2\lambda + h_3 = 0 \quad (15)$$
,

where

$$h_1 = \phi + v_3[\frac{\vartheta-\sigma-v_2 z^*}{\gamma}] - v_4[\frac{\theta+v_1 z^*-\alpha}{\beta}] - \rho,$$
$$h_2 = v_2[\frac{\theta+v_1 z^*-\alpha}{\beta}]v_4[\frac{\theta+v_1 z^*-\alpha}{\beta}] + v_1[\frac{\vartheta-\sigma-v_2 z^*}{\gamma}]v_3 z^* - \beta[\frac{\vartheta-\sigma-v_2 z^*}{\gamma}][\gamma[\frac{\theta+v_1 z^*-\alpha}{\beta}],$$
$$h_3 = \beta[\frac{\vartheta-\sigma-v_2 z^*}{\gamma}][\gamma[\frac{\theta+v_1 z^*-\alpha}{\beta}](\phi + v_3[\frac{\vartheta-\sigma-v_2 z^*}{\gamma}] - v_4[\frac{\theta+v_1 z^*-\alpha}{\beta}] - \rho) + v_3 z^* v_2[\frac{\theta+v_1 z^*-\alpha}{\beta}]] + v_1[\frac{\vartheta-\sigma-v_2 z^*}{\gamma}]\gamma[\frac{\theta+v_1 z^*-\alpha}{\beta}]v_4[\frac{\theta+v_1 z^*-\alpha}{\beta}]. \quad (16)$$

By the Routh-Hurwitz criterion, Equation (14) has all negative roots if and only if

$$h_1 > 0, \quad h_3 > 0, \quad \& \ h_1 h_2 - h_3 > 0. \quad (17)$$

The conditions in Equation (17) can only hold, if Equation (15) has all positive coefficients, and this can only be obtained if and only if:

$$\phi < \rho, \quad \& \quad v_3[\frac{\vartheta-\sigma-v_2 z^*}{\gamma}] < v_4[\frac{\theta+v_1 z^*-\alpha}{\beta}]. \quad (18)$$

Since $\phi < \rho$ contradicts the condition for the positivity of equilibrium points, Hence, anti-banditry-induced equilibrium point is always unstable for a positive variant.

Sensitivity Analysis

The stability analysis for anti-banditry induced equilibrium point suggests an unstable scenario, since

$$\phi < \rho \ \& \ v_3\left[\frac{\vartheta-\sigma-v_2 z^*}{\gamma}\right] < v_4\left[\frac{\theta+v_1 z^*-\alpha}{\beta}\right] \quad (19)$$

connote the collapse of the banditry ravaging the community due to the inability of government forces to protect the ravaging community. In order to determine the appropriate and possible optimal strategy to eliminate banditry activity, we employed the approach used in (Ibrahim (b) et al., 2020) to perform sensitivity analysis on model parameters to identify which of the model parameters is key to ending the activity of the banditry groups. It is obvious that the anti-banditry-induced equilibrium point:

$$(x^*, y^*, z^*) = \left(\frac{\vartheta-\sigma-v_2 z^*}{\gamma}, \frac{\theta+v_1 z^*-\alpha}{\beta}, \frac{\beta\gamma[\phi-\rho]+\gamma v_4[\theta-\alpha]-\beta v_3[\vartheta-\sigma]}{\beta v_3 v_2-\gamma v_4 v_1}\right)$$

centers on "z^*". Where z^* ~~is~~ literally depicts a state where the threatened community becomes stable due to anti-banditry efforts of government forces represented by $\frac{\beta\gamma[\phi-\rho]}{\beta v_3 v_2+\gamma v_4}$ to protect the

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vulnerable communities by using community collaborators to reduce bandit invasion of community depicts by $\frac{\beta\upsilon_3[\vartheta-\sigma]}{\beta\upsilon_3\upsilon_2+\gamma\upsilon_4}$, such that that the rate at which government forces neutralizes banditry attack on the community is greater than the rate at which banditry groups deplete the government forces in the community describes by $\frac{\gamma\upsilon_4[\theta-\alpha]}{\beta\upsilon_3\upsilon_2+\gamma\upsilon_4}$.

Obviously, if z^* decreases banditry activities drop and vulnerable communities are secured and flourish. However, if z^* increases banditry activities increase and the communities become more threatening. We, therefore carry out the sensitivity analysis on z^* to assess the crucial role of each model parameter in the increase or the decrease in banditry activities. This is aimed at determining the optimal control strategy against banditry groups.
Using the concept in (Ibrahim (b) *et al.*, 2020) and (Chitnis *et al.,* 2008), the normalized forward sensitivity index of a variable for a parameter is the ratio of the relative change in the variable to the relative change in the parameter. Therefore, the normalized forward sensitivity index of z^* with respect to the model parameters p is defined as

$$(z^*)_p^n = \frac{\partial z^*}{\partial p} \times \frac{p}{z^*}. \qquad (20)$$

where
$$p = (\alpha, \beta, \theta, \sigma, \gamma, \vartheta, \upsilon_1, \upsilon_2, \upsilon_3, \upsilon_4,\ \phi\ \&\ \rho). \qquad (21)$$

Differentiating z^* with respect to each parameters p yields

$$(z^*)_\alpha^n = \frac{-\alpha\gamma\upsilon_4}{\beta\gamma[\phi-\rho]+\gamma\upsilon_4[\theta-\alpha]-\beta\upsilon_3[\vartheta-\sigma]}$$

$$(z^*)_\beta^n = \frac{\beta[(\beta\upsilon_3\upsilon_2-\gamma\upsilon_4\upsilon_1)(\gamma[\phi-\rho]-\upsilon_3[\vartheta-\sigma])-(\beta\gamma[\phi-\rho]+\gamma\upsilon_4[\theta-\alpha]-\beta\upsilon_3[\vartheta-\sigma])\upsilon_2\upsilon_3]}{(\beta\upsilon_3\upsilon_2-\gamma\upsilon_4\upsilon_1)(\beta\gamma[\phi-\rho]+\gamma\upsilon_4[\theta-\alpha]-\beta\upsilon_3[\vartheta-\sigma])}$$

$$(z^*)_\theta^n = \frac{\theta\gamma\upsilon_4}{(\beta\gamma[\phi-\rho]+\gamma\upsilon_4[\theta-\alpha]-\beta\upsilon_3[\vartheta-\sigma])}$$

$$(z^*)_\sigma^n = \frac{\sigma\beta\upsilon_3}{\beta\gamma[\phi-\rho]+\gamma\upsilon_4[\theta-\alpha]-\beta\upsilon_3[\vartheta-\sigma]}$$

$$(z^*)_\vartheta^n = \frac{-\vartheta\beta\upsilon_3}{\beta\gamma(\phi-\rho)+\gamma\upsilon_4(\theta-\alpha)-\beta\upsilon_3(\vartheta-\sigma)}$$

$$(z^*)_{\upsilon_1}^n = \frac{\upsilon_1\gamma\upsilon_4}{\beta\upsilon_2\upsilon_3-\gamma\upsilon_1\upsilon_4}$$

$$(z^*)_{\upsilon_2}^n = \frac{-\upsilon_2\beta\upsilon_3}{\beta\upsilon_2\upsilon_3-\gamma\upsilon_1\upsilon_4}$$

$$(z^*)_\gamma^n = \frac{\gamma[[\beta(\phi-\rho)+\upsilon_4(\theta-\alpha)[\beta\upsilon_2\upsilon_3-\gamma\upsilon_1\upsilon_4]]+[\beta\gamma(\phi-\rho)+\gamma\upsilon_4(\theta-\alpha)-\beta\upsilon_3(\vartheta-\sigma)]\upsilon_1\upsilon_4]}{(\beta\upsilon_3\upsilon_2-\gamma\upsilon_4\upsilon_1)(\beta\gamma[\phi-\rho]+\beta\upsilon_3[\vartheta-\sigma]-\gamma\upsilon_4[\theta-\alpha])}$$

$$(z^*)_{\upsilon_3}^n = \frac{-\upsilon_3[\beta(\vartheta-\sigma)[\beta\upsilon_2\upsilon_3-\gamma\upsilon_1\upsilon_4]-[\beta\gamma(\phi-\rho)+\gamma\upsilon_4(\theta-\alpha)-\beta\upsilon_3(\vartheta-\sigma)]\beta\upsilon_2]}{(\beta\upsilon_3\upsilon_2-\gamma\upsilon_4\upsilon_1)(\beta\gamma[\phi-\rho]+\gamma\upsilon_4[\theta-\alpha]-\beta\upsilon_3[\vartheta-\sigma])}$$

$$(z^*)_{\upsilon_4}^n = \frac{\upsilon_4[[\gamma(\theta-\alpha)](\beta\upsilon_3\upsilon_2-\gamma\upsilon_4\upsilon_1)-(\beta\gamma[\phi-\rho]+\gamma\upsilon_4[\theta-\alpha]-\beta\upsilon_3[\vartheta-\sigma])\gamma\upsilon_1]}{(\beta\upsilon_3\upsilon_2-\gamma\upsilon_4\upsilon_1)(\beta\gamma[\phi-\rho]+\gamma\upsilon_4[\theta-\alpha]-\beta\upsilon_3[\vartheta-\sigma])}$$

$$(z^*)_\phi^n = \frac{\phi\beta\gamma}{\beta\gamma[\phi-\rho]+\gamma\upsilon_4[\theta-\alpha]-\beta\upsilon_3[\vartheta-\sigma]}$$

$$(z^*)_\rho^n = \frac{-\rho\beta\gamma}{\beta\gamma[\phi-\rho]+\gamma\upsilon_4[\theta-\alpha]-\beta\upsilon_3[\vartheta-\sigma]}$$

(19)

The sensitivity indices obtained inEquation (19) reveal the contributory effect of each parameter of the model to the security of vulnerable community "z" from banditry activities. The results in Equation (19) indicate increasing or decreasing function for each of the parameters in System (1) depicted by positive and negative signs respectively. Substituting values in Table 1. into Equation (19), we obtain the sensitivity index for each parameter of System (1) as given in Table 2. It is obvious both in analytical computation and numerical estimation of sensitivity index that the leading improving parameters for government forces are attacking rate β and their community collaborative rate υ_1. For Banditry, the leading reducing parameters for banditry activities are their weaponry for resisting government force $[\gamma]$ and their ability to co-opt vulnerable residents for the advancement of their activities $[\upsilon_4]$. For the vulnerable community, the leading promoting parameters are their progress rate $[\phi]$ and the collaborative rate with government forces in securing their community $[\upsilon_3]$.

RESULTS AND DISCUSSION

Table 1: Model Estimated Parameter Values

Parameter	Estimated Value	Source
α	0.4	Estimated from (COAS, (2024))
β	2.9	Assume
θ	0.1	Estimated from (Isamotu, (2024))
σ	0.2	Assumed based on (Ojewale, 2021)
γ	1.5	Assume
ϑ	0.5	Estimated from (Obiejesi, (2023))
υ_1	0.1	(S., (2023))
υ_2	0.5	Assume
ϕ	0.4	Assume
υ_3	0.2	(Oyewole, S. et al., (2022))
υ_4	0.35	Estimated from (Faruk, (2022))
ρ	0.2	Estimated from (Idowu, (2023))

Table 2: Sensitivity index values of model's parameter

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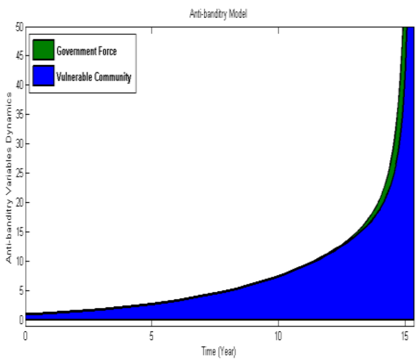
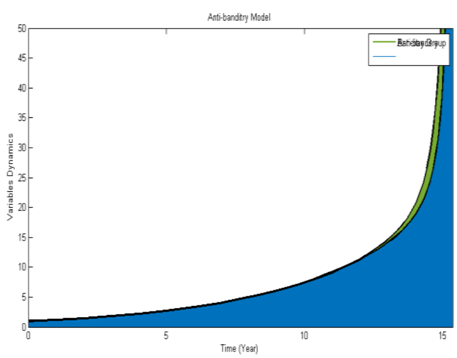
Numerical Simulation
Here, we perform numerical simulations of the system (1) using Matlab DDE223 solver with the model parameter values assumed based on sociological considerations as summarized in Table 1. We equally validate our analytical results for sensitivity analysis by implementing the sensitivity indices values on leading sensitive parameters to determine the optimal anti-banditry control strategies against activities of bandit groups in Nigeria when reducing or increasing functions of sensitive model parameters are factored into the anti-banditry approach of the government.



(a)



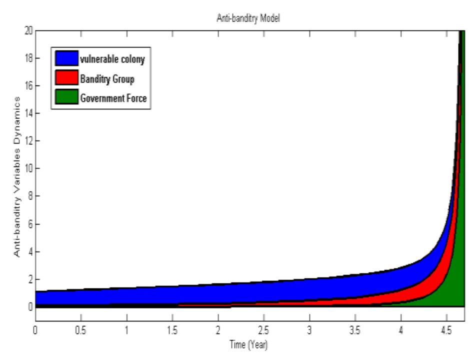
(b)

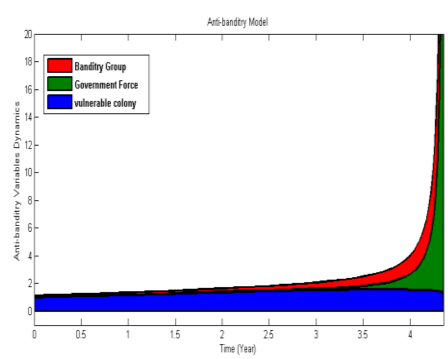
Figure 2: *Time evolution of model variables for the equilibrium points of System (1). (a) Banditry-free equilibrium* point, (b)Anti-banditry induced equilibrium point .

Figure 2 depicts the simulation of System (1) around equilibrium points. (a) The simulation of System (1) around banditry-free equilibrium point based on the conditions in Theorem 2. The dynamics are characterized by the continuous growth of the community in the absence of banditry attacks. The reflective growth of government forces is also featured in the dynamics. This indeed verifies the analytical result obtained which predicted the unstable stability for the banditry-free equilibrium point. (b) the numerical simulation of System (1) around anti-banditry induced equilibrium point based on the conditions in Theorem 3. It shows stagnation in the community's time evolution curve which is relatively caged by the banditry group curve. However, there are initial overlaps of time evolution curves of banditry and government force which indicates engagements. However, the engagement leads to the stagnation of the community under more influence of the banditry groups.

Parameter	Sensitivity index	Sign
α	0.16	Negative
β	0.14	Positive
υ_1	3.12	Positive
θ	0.16	Positive
σ	0.06	Positive
γ	2.18	Negative
ϑ	0.16	Negative
υ_2	2.11	Positive
ϕ	1.9	Positive
υ_3	2.2	Negative
υ_4	3.29	Positive
ρ	0.29	Negative

(a).





(b)

Figure 3: *sensitivity index to reduce the activities of banditry groups in System (1) (a) When recruited into the government force* $[\alpha]$ *is increased by* 0.16. *(b)When community intelligent gathering* υ_1 *is increased by* 0.3125.

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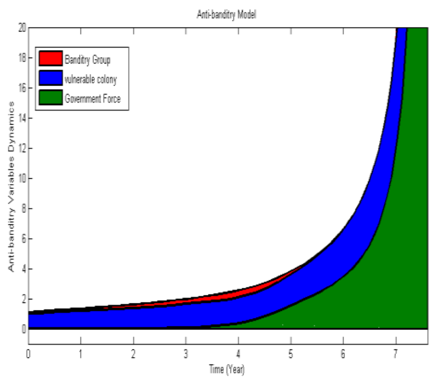
Fig. 3, is the numerical simulation of System (1) with sensitivity indices to enhance government force efficiency against banditry activities. (a) when government forces recruitment $[\alpha]$ is heightened by 0.16. The community time evolution curve is seen progressing in the midst of banditry, and the banditry curve is seen fainting for a longer period. But as time progresses there is a resurgence of banditry curve piercing through community and government force curves, to overlap the community curve (see Fig. 3a). (b) When anti-banditry community intelligence rate $[\upsilon_1]$ is increased by 3.125. The engagement of banditry by the government forces leads to a stagnated community under government protection (see Fig. 3b).



(a)

(b)

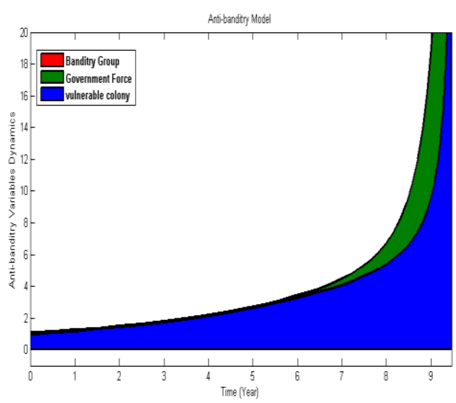
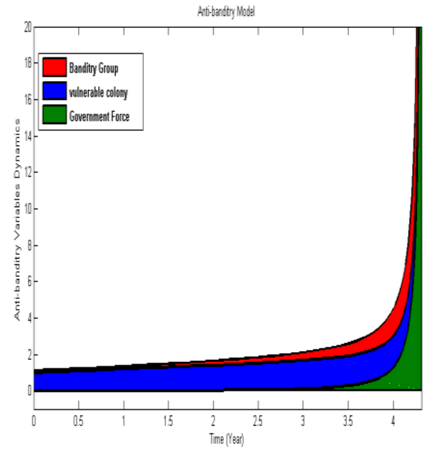
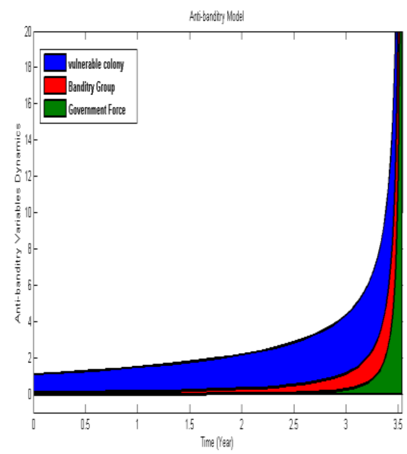


Figure 4*: sensitivity index to reduce the activities of banditry groups in System (1)* (a) When their weaponry for resisting government force $[\gamma]$ is reduced by 2.179. (b)When their ability to co-opt vulnerable resident for the advancement of their activities $[\upsilon_4]$ is reduced by 3.29.

Fig. 4, is the numerical simulation of System (1) with sensitivity indices to reduce the activities of banditry groups. (a) When their weaponry for resisting government force $[\gamma]$ is reduced by 2.179. The dynamics produce a progressing community with initial reflective traces of banditry activities. The government force is progressively saturated, and militaristic in protecting the community. (b) When the banditry's ability to co-opt vulnerable residents for the advancement of banditry activities $[\upsilon_4]$ is reduced by 3.29. The dynamics are characterized by the overwhelming community growth, moderate government forces appearance, and the total disappearance of banditry group curves. Both community and government forces and the banditry curve are relatively vanished.



(a)



(b)

Figure 5: *Sensitivity index to aid security in banditry ravaging vulnerable community in System (1) (a) When the rate $[\upsilon_3]$ of synergy between community and government forces in community protection is increased by 2.2. (b)When the rate community depletion $[\upsilon_4]$ is reduced by 3.29.*

Fig. 5, is the numerical simulation of System (1) with sensitivity indices to aid security in banditry ravaging vulnerable community(s). (a) When the rate $[\upsilon_3]$ of synergy between community and government forces in community protection is increased by 2.2. The dynamics produce a trapped community in the midst bandit-government force duel. The increase in both the banditry and government curves is also prominent in this dynamics (see Fig 6a). (b) When the rate community depletion $[\upsilon_4]$ is reduced by 3.29. The dynamics produce a pronounced community development and growth with a moderate government force present. The disappearance of the banditry curve is also noticed in this dynamics (see Fig. 6b).

In this work, a mathematical model of banditry group and government forces engagement as well as their impact on the vulnerable community is proposed and analyzed. The existense of non-negativity solution to the proposed model was obtained so as to ensure that model assumptions are in conform to reality of anti-bandity dynamics.. Desirable in anti-banditry dynamics is the peaceful state where all the engaging variables come to rest and the ravaging community regains peace. In order to attain this peaceful state, we obtain two equilibrium points, namely, the banditry-free equilibrium point which arises when there is no case of banditry. The anti-banditry-induced equilibrium point arises when government anti-banditry effort brings about peace in the banditry-threatened community(ies). The analysis of these equilibrium points predicted an unstable scenario for both the banditry-free equilibrium point and the anti-banditry-induced equilibrium point. To determine, the viable control strategy to stem banditry group activities, sensitivity analysis is performed to assess the role of model parameters in either promoting or reducing banditry activities. This effort reveals increasing and reducing parameters for all the proposed model parameters.

The numerical solutions not only validate all the analytical results but also provide a graphical view of the model variable dynamics based on the analytic results obtained. For, the simulation of System (1) around the banditry-free equilibrium point, the analytic prediction of unstable stability is verified as the pronounced community growth with a light government force is observable (see Fig. 2a). Also, the simulation of System (1) around the anti-banditry induced equilibrium point verified the analytic result prediction of unstable stability as a banditry-induced community stagnation and continuous engagement are evident (see Fig. 2b). The sensitivity indices in Table 2. provides insight into possible control strategy to enhance anti-banditry forces' performance, eliminate banditry activities, and strengthen the security around the vulnerable communities. For the enhancement of anti-banditry force, recruitment of personnel and intelligent gathering are the leading enhancing parameters. Recruitment of more personnel into government forces will only improve security in the ravaging community but will not bring banditry activity to an end (see Fig 3a). The increment in intelligent gathering leaves no meaningful changes in the dynamics (see Fig. 3b). These had been witnessed in Nigeria recently as reported in [6].

For the elimination of banditry group activities, reduction in getting weaponry and co-opting vulnerable for the banditry advancement are the leading eliminating parameters. Cutting off of banditry weaponry source will bring about growth of the community, and a reduction in the activities but the overwhelming presence of government characterized these dynamics (see Fig. 4a). Reducing the exploitation of vulnerable by banditry groups will return the community to prosperity, the complete eradication of banditry and moderate government presence (see Fig. 4b). Also, for strengthening the security around the vulnerable communities, improve synergy between government force and community, and reduction in banditry depletion of the ravaging community are the leading security strengthening parameters. Improving synergy between government force and community will not bring about the elimination of banditry, but rather a community enclosed in the battlefield (see Fig. 5a). Reduction in banditry depletion of the community will create a developing community enveloping a government force-tamed banditry group (see Fig. 5b).

Conclusion

We have been able to propose a model of anti-banditry dynamics and incorporate the role of banditry and anti-banditry forces on the viability and tranquility of the ravaging community. Our findings highlight that, for continuous peace in a banditry-free community, the government forces should make use of intelligence gathering from the community such that there is no proliferation of banditry groups in such a community. Also, the findings highlight that once there is an emergency of a banditry group in any community, the community will suffer great regression unless a precise effort is undertaken. The potent and optimal strategy according to our findings to eliminate banditry group in Nigeria are the blockage of bandits' access to weapons, and checkmating bandit collaborators and informants within the ravaging community(ies). Also, our finding highlights the need to put up a clear strategy in banditry-free communities against banditry proliferation as a potent preventive measure.

Conflict of Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this article.

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REFERENCES

A. A. Ibrahim et al. ((2022)). Global stability and thermal optimal control strategies for hyperthermia treatment of malignant tumors. *Mathematics*, 10 (13):2188.

Abdo, M. S. et al. ((2022)). Vaccination-based measles outbreak model with fractional dynamics. *Abhath Journal of Basic and Applied Sciences* , 1(2):6–10.

Abdulkadir, T. a. ((2024)). The role of informants in the accentuation of armed banditry in Northwestern Nigeria: A case study of Zamfara state. . *Wukari International Studies Journal.*, 8(3):1–9.

Abdullahi. ((2022)). Armed banditry, coercive approach and human security in the northwest Nigeria, . *International Journal of Research and Innovation in Social Sciences*, 329-338.

Abdulrasheed, A. ((2021)). Armed banditry and human security in north western Nigeria: The impacts and the way forward. *Journal of Humanities Social and Management Sciences Edwin Clark University*, 1(1).

Abidemi, A. a. ((2022)). Dynamics of illicit drug use and banditry population with optimal control strategies and cost-effectiveness analysis. . *Computational and Applied Mathematics*, 41(1):53.

Abubakar, S. G. ((2021)). Strategic management of internal security challenges: Evaluating the effect of banditry and kidnapping in Northwestern Nigeria and the role of the military.

Akanni, J. a. ((2023)). Relationship between illicit drug users and bandits in a population. *Mathematical modelling approach, Appl. Math* , 17(3):475–492.

Alblowy, A. H. et al. . ((2023)). [2Optimal control strategies for sglt2 inhibitors as a novel anti-tumor agent and their effect on human breast cancer cells with the effect of time delay and hyperglycemia . *Computers in Biology and Medicine* , 166:107552.

Alumona, I. M. ((2023)). The state and counter-banditry in Nigeria: Strategies, challenges and prospects. Banditry and Security Crisis in Nigeria. 222–238.

Barnett, J. et al. ((2022)). Northwestern Nigeria: A Jihadization of banditry, or a "banditization" of jihad?. . *CTC Sentinel* , 15(1):46–67.

Bildirici, M. a. ((2020)). [2] The impact of terrorism and fdi on environmental pollution: Evidence from Afghanistan, Iraq, Nigeria, Pakistan, Philippines, Syria, Somalia, Thailand and Yemen. . *Environmental Impact Assessment Review* , 81:1063.

Chitnis, N. et al. ((2008)). Determining important parameters in the spread of malaria through the sensitivity analysis of a mathematical model. . *Bulletin of mathematical biology*, 70(5):1272.

Chukwuma A. U. et al. ((2022)). Journalism and reportage of insecurity: Newspaper and television coverage of banditry activities in northern Nigeria. *Newspaper Research Journal*, 43(3):343–357.

COAS. ((2024)). Army to deploy additional 5,937 soldiers to fight banditry.URL https://punchng.com/army-to-deploy-additional-5937-soldiers-to-fight-banditry-coas.

Dami, C. D. ((2021)). Impacts of terrorism, banditry and kidnapping on human security in Nigeria. *Saudi Journal of Humanities and Social Sciences*, 6(8):299–305.

Faruk, B. M. ((2022)). The impact of armed banditry and kidnapping on socio-economic activities: Case study of selected local government areas in Katsina state, Nigeria . 12 (1) .

Ghosh, S. ((2019)). Uncertainty, economic growth its impact on tourism, some country experiences. *Asia Pacific Journal of Tourism Research*, 24(1):83–107.

Ibrahim A. A. et al. ((2023)). Deterministic model and analysis of fuel subsidy in nigeria commodity market dynamics. . *Journal of Mathematical Analysis and Modeling* , 4(2):27–43.

Ibrahim, A. A. et al. ((2020)). Stability and Sensitivity Analysis of Tumor-induce immune Suppression with Time Delay. *Journal of Advance Research in Dynamical & Control Systems.* , 12(07). Special Issue 2022.

Idowu, B. a. ((2023)). Nigerian security conundrum: An analysis of patterns and trends of banditry in northwest Nigeria. *European Journal of Theoretical and Applied Sciences 1 (2023) 215–224. doi:10.59324/ejtas.2023.*, 1(3).24.

Isamotu, I. ((2024)). Nigerian military has killed 9,300 bandits, terrorists. . *URL https://dailytrust.com/over-9000-terrorists-bandits-killed-in-one-year-defence-ministe*.

Ladan, S. I. ((2020)). Impacts of banditry on food security in katsina state, Nigeria. *Direct Research Journal of Agriculture and Food Science* , 8(12):439–447.

Ladan, S. I. ((2021)). Impacts of banditry on food security in Katsina state, Nigeria. *Emerging Challenges in Agriculture and Food Science* , 1-9.

Lamidi, K. O. ((2024)). Nigerian state and the war against armed banditry, in: Armed Banditry in Nigeria: Evolution, Dynamics, and Trajectories. *Springer*, 221–250.

Maigari, M. A. ((2021)). Negotiating with the bandits and endless security challenges in katsina state, Nigeria (2019-2020). *JSW (Jurnal Sosiologi Walisongo)* , 5(1):47–60.

Newton, C. a. ((2023)). The efficacy of airpower in counterinsurgency, Security Studies [14] Newton, C. and Tucker, C. (2023). The efficacy of airpower in counterinsurgency,. *Security Studies* , 1–33.

Obiejesi, K. ((2023)). Nigerian military records successes in Zamfara but insecurity persists in several rural areas. *https://www.icirnigeria.org/nigerian-military-records-successes-in-zamfara-but-insecurity-persists-in-several-rural-areas/*.

OGYE D. o and Smah, D. ((2024)). Military-civilian relationship in fight against armed banditry in north-west, Nigeria. *International Journal of Peace and Conflict Studies* , 9(1):37–49.

Ojewale, O. ((2023)). Theorizing and illustrating plural policing models in countering armed banditry as hybrid terrorism in northwest Nigeria. *Cogent Social Science*, 9(1):2174486.

Ojewale, O. ((2024)). Northwest nigeria has a banditry problem. what's driving it? *The Global Observatory*.

Ojewale, O. ((2021)). The increasing nexus between bandits and terrorists in Nigeria's Northwest, [5] Ojewale O. (2021). The increasing nexus between bandits and terrorists in Nigeria's Northwest. *Africa at LSE*.

Okoli, A. C. ((2019)). Of marauders and brigands: Scoping the threat of rural banditry in nigeria's north west . *Brazilian Journal of African Studies*, 4(8):201–222.

Oyewole, S. et al. . ((2022)). Wings over flies: Air campaigns against armed banditry in North-west Nigeria,. *The RUSI Journal* , 167(4-5):92–103.

S., A. T. ((2023)). Fighting from the bottom up: Community mobilization as a viable strategy in combating banditry in Nigeria. *Conflict Trends*, (1):25–31.

Saidu, M. T. ((2022)). Dynamics of unemployment and economic growth on terrorism in Nigeria-evidence from a linear quantile modeling approach .

Sanchi, I. e. ((2022)). [8]Implications of rural banditry on educational development in Nigeria: A critical review. . *Direct Research Journal of Social Science and Educational Studies*, 10(5):77–87.

Tahir, T. A. ((2021)). Curbing armed banditry in nigeria: A panacea for good governance. . *Zamfara Journal of Politics and Development* , 2(2):11–11.