

EXPLORING SMALL ARMS AND LIGHT WEAPONS CONTROL APPROACH TO ANTI-BANDITRY MEASURES USING DISCRETE TIME DELAY MODEL

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ABSTRACT

Uncontrollable access to arms and light weapons by bandit groups across Northwestern Nigeria is adjudged to be responsible for the unsuccessful fight against banditry in Nigeria. This study explores the impact of control of arms and light weapons on the anti-banditry measures using a time delay model and analysis. The conditions for attaining a critical steady state for anti-banditry dynamics are obtained, and the stability analysis of its critical steady state is determined. This result suggests an unstable critical steady state when arms and light weapons control is not enforced. With the enforcement of arms and light weapons control, the critical steady state undergo a supercritical Pitchfork bifurcation characterized by a control-induced regression of activities of bandit groups coupled with the return of progress and prosperity to the banditry-ravaged communities. All analytical results were verified, and the impact of arms and light weapons control is graphically showcased. The work postulates that the ongoing kinetic approach will not bring about a desirable lasting solution as long as arms and weapons proliferations are not controlled. Also, the findings hint at post-banditry government forces' persistence and over-concentration which might degenerate into adversity if not properly planned.

Keywords: Anti-banditry Measures: Vulnerable Community: Nonkinetic Approach: Delay Model: Small arms and light weapons

INTRODUCTION

The criminal activities of bandit groups in the Northwestern states of Nigeria have threatened the lives and livelihoods of various communities, affecting the general well-being of citizens (Faruk et al., 2022) see also in (Abdullahi, 2022). The alarming rate of banditry-induced insecurity is causing unpleasant consequences that affect both social and economic prosperity and peaceful co-existence in the affected areas precipitating economic hardship, loss of lives and increase in poverty index of the affected areas (Edesiri et al., 2016). The target of the banditry attack is initially vulnerable communities with valuable materials (Chukwuma et al., 2022). Recently, they attacked schools, military bases, as well as a moving train (Dami, C. D., (2021)) see also (Sanchi, I. et al., 2022). Banditry activities have led to loss of lives and economic sabotage leading to astronomical increases in prices of local products (Ghosh, 2019) also in (Ladan, et al., 2020). The continuous proliferation of banditry groups in different ungoverned forests and hideouts in the Northwestern states of Nigeria has been a critical threat to the security architecture of the region (Olapaju & Peter, 2021). There are suggestions that there are collaborative networks among these groups in terms of reinforcement and sharing of logistics to advance their criminal activities (Ojo et al., 2023) see also in (Nwangwu & Okoli, 2023). In the last two

decades, the successive Nigerian governments have used kinetic approaches which include ground and air attacks targeting banditry groups in their respective hideouts to stem the tide of banditry-induced insecurity (Lamidi, 2024). The military approach involves the use of coordinated security deployment under several codes (Abubakar et al., 2021). Although remarkable success has been recorded with the kinetic approach in terms of the destruction of some bandit leaders and the arrest of their informants and acolytes, the desirable results in terms of the protection of life and properties of the vulnerable community(ies) are still elusive (Abdulrasheed, 2021). The non-kinetics approach employed by some state governments so far includes initiatives to persuade the bandits leaders to surrender and release their captives, but this approach witnessed a full scale banditry resurgence after initial recession in the banditry activities (Ojewale, 2023). Other non-kinetic approaches by the state include shutting down telecommunication services, labeling bandit groups as terrorist groups, and outlawing ransom. These approaches are beautiful strategies but did not hinder the increase in banditry activities like kidnapping and village raids (Tahi & Bernard, 2021).

Recently, Nigeria military defense headquarters acknowledged the integration of hard and soft power approach in the fight of banditry-induced insecurity through deliberate manipulation of bandit groups' will to fight and public confidence on strategic communication (Yahaya, 2020). Most works on evolution of banditry premise on "ungoverned spaces" theoretical framework to postulate that the inability of the state to adequately secure her territory leaves room for bandit groups to attack vulnerable communities within these ungoverned spaces (Ejiofor, 2022). The continuous access to weapons and ammunition by bandit groups due to uncontrolled proliferation of small arms and light weapons across Nigeria is adjudged to be responsible for unsuccessful fight against banditry in Nigeria (Odeniyi, 2023). The need for institutional mechanism to combat proliferation of small arms and light weapons (SALW) in Nigeria through border control and adequate deterrent penalties is sacrosanct to an effective anti-banditry measures (Afegbua, 2023) see also in (Mwenda, 2021).

The utilization of discrete-time delay model to study the dynamics of a system within a fixed number of sample periods has gained applications in different areas of study: In biology, discrete time delay has been used to study the dynamics of process involved in immune-cancerous cells interaction as in (Ibrahim(a) et al., 2022) see also in (Ibrahim, A. A., Maan, N., Jemon, K., (2020)). Also, the process leading to oil theft is studied using discrete time delay in (Ibrahim(b) et al., 2023). Discrete time delay model is a mathematical representation of dynamical system with aim of

deriving outputs that depend on the input values from previous time steps expressed in a discrete-time form. Many mathematical efforts have been channeled to gain understanding on the activities of anti-state forces like insurgency and terrorism as seen in (Adam et al., 2009) also in (Arney, 2013) and the citations therein. In light of the identified need for small arms and light weapons proliferation control as the optimal non-kinetic approach to banditry in Nigeria as mentioned in (Yahaya, 2020) and (Afegbua, 2023), using discrete time delay model to depict the time lag in access to weapons by the bandit groups will be both cost effective and provide a control design for non-kinetic approach to anti-banditry measures.

In the wake of the unending spates of banditry-induced insecurity in the Northwestern states in Nigeria, this work proposes a delay model to study the impact of small arms and light weapons proliferation control on the dynamics of non-kinetic anti-banditry measures in Nigeria. The choice of a discrete delay model is to incorporate inherent time lag in access to weapon by the bandit groups. This will provide insights on the impact of small arms and light weapons proliferation regulation and design.

MATERIALS AND METHODS

Model formulation and Analysis

Let x represent the conglomeration of anti-banditry measures of the government, y represent the assemblage of a banditry group and z represent the colony of banditry threatened community. The rates of changes of x , y , and z are defined by the model below:

$$\begin{aligned} \frac{dx}{dt} &= \alpha x(t) + \beta x(t)y(t) + v_1 z(t)x(t) - \theta x(t), \\ \frac{dy}{dt} &= \sigma y(t) + \gamma x(t - \tau_2)y(t) + v_2 z(t)y(t) - \vartheta y(t), \\ \frac{dz}{dt} &= \phi z(t) + v_3 z(t)x(t) - v_4 z(t - y(t))y(t - \tau) - \rho z(t). \end{aligned} \quad (1)$$

with initial conditions:

$$x_0 = \phi_1, \quad y_0 = \phi_2, \quad z_0 = \phi_3, \quad \text{for } t \in [-\tau_i, 0].$$

The first equation in (1) is the rate of change in government's intensified effort against banditry with the first term depicting enlisting rate of anti-banditry forces (α), the second term is the rate at which government forces engage banditry groups (β), the third term is the collaborative rate between community and anti-banditry force (v_1) and the last term depicts loss suffered by the anti-banditry force (θ).

The second equation of (1) is the rate of change in the activities of banditry groups with the first representing the proliferation rate (σ), the second term refers to the delay in the rate at which banditry groups resist government force (γ) due to non-access to weapons' control, the third is the rate at which bandit groups co-opt vulnerable residents to advance their activities (v_2) and the last term refers to the loss suffered by bandits (ϑ).

The third equation in (1) represents the colony of vulnerable communities with the first term describing the community progress rate (ϕ), the second is the rate at which anti-banditry forces collaborate with the community in securing their colony (v_3), the

third term is delay in the rate at which bandit groups attack vulnerable (v_4) due to the control of access to arms and weapons and the last term represents non-banditry community loss (ρ).

Model Analysis

Existence of Non-negative Solution

Lemma 1 Given that the initial conditions for System (1) are positive $\forall \xi \in [-\tau, 0]$. Then, there exist positive solutions for $x(t)$, $y(t)$ and $z(t)$ of System (1) $\forall \xi \in [-\tau, 0]$.

Proof: Solving for $x(t)$, $y(t)$ and $z(t)$ in System (1), we have

$$\begin{aligned} x(t) &= [a_1 + x(0)]e^{-[\alpha + \beta y(t) + v_1 z(t) - \theta]t}, \\ y(t) &= [a_2 + y(0)]e^{-[\sigma - v_2 z(t) - \vartheta]t} + \int_0^t \gamma x(t - \tau)y(t)dt, \\ z(t) &= [a_3 + z(0)]e^{-[\phi + v_3 x(t) - \rho]t} - \int_0^t v_4 y(t - \tau)z(t - \tau)dt, \end{aligned} \quad (2)$$

if $a_i \forall i = 1, 2$, are positive constants and assumptions in the lemma (1) hold, then $x(t)$, $y(t)$ and $z(t)$ are non-negative and System (1) has non-negative solutions.

Existence of Steady State

The reason behind the adoption of arms and weapons' control measures in the fight against banditry in Nigeria is to find a lasting solution to banditry-induced insecurity that will bring about the return of peace to the affected communities. This state of peace is mathematical referred to in a dynamical system as invariant points, and obtaining the conditions for arriving at such points is critical in achieving the stability in any dynamical system including anti-banditry dynamics. Therefore, the condition for the existence of steady state for System (1) is captured in Theorem (1) below:

Theorem 1: Suppose there exist positive invariant points x^* , y^* , z^* for System (1), then there exists a critical steady state (x^*, y^*, z^*) for System (1).

Proof: Suppose the assumptions in Theorem (1) hold, then x^* , y^* , and z^* of System (1) satisfy the followings:

$$\begin{aligned} \alpha x^* + \beta x^* y^* + v_1 x^* z^* - \theta x^* &= 0, \\ \sigma y^* + \gamma x^* y^* + v_2 y^* z^* - \vartheta y^* &= 0, \\ \phi z^* + v_3 z^* x^* - v_4 z^* y^* - \rho z^* &= 0. \end{aligned} \quad (3)$$

Solving for x^* , y^* , & z^* in Equation (3), the critical steady state for System (1) becomes

$$(x^*, y^*, z^*) = \left(\frac{\vartheta - \sigma - v_2 z^*}{\gamma}, \frac{\theta + v_1 z^* - \alpha}{\beta}, \frac{\beta \gamma [\phi - \rho] + \gamma v_4 [\theta - \alpha] - \beta v_3 [\vartheta - \sigma]}{\beta v_3 v_2 - \gamma v_4 v_1} \right),$$

when $\phi > \rho$, $\vartheta > \delta$ & $z^* > 0$.

Local Stability Analysis

we linearize System (1) and obtain a characteristic equation of the form:

$$\begin{pmatrix} \lambda - (\alpha + \beta y^* - \theta - v_1 z^*) & \beta x^* & -v_1 x^* \\ \gamma y^* & \lambda - (\sigma - \vartheta + \gamma_2 x^* e^{-\lambda\tau} + v_2 z^*) & v_2 y^* \\ v_3 z^* & -v_4 z^* e^{-\lambda\tau} & \lambda - (\phi + v_3 x^* - v_4 y^* e^{-\lambda\tau} - \rho) \end{pmatrix} = 0 \quad (5)$$

The simplified form of Equation (5) becomes

$$\begin{aligned} D(\lambda, \tau) = & (\lambda - (\alpha + \beta y^* - \theta - v_1 z^*)) \left[(\lambda - (\sigma - \vartheta + \gamma_2 x^* e^{-\lambda\tau} + v_2 z^*)) (\lambda - (\phi + v_3 x^* - v_4 y^* e^{-\lambda\tau} - \rho)) \right. \\ & + v_2 y^* v_4 y^* \\ & - \beta x^* \left[\gamma y^* (\lambda - (\phi + v_3 x^* - v_4 y^* e^{-\lambda\tau} - \rho)) \right. \\ & - v_3 z^* v_2 y^* \\ & \left. \left. - v_1 x^* [-\gamma y^* v_4 z^* e^{-\lambda\tau} - v_3 z^* (\lambda - (\sigma - \vartheta + \gamma_2 x^* e^{-\lambda\tau} + v_2 z^*)) + v_4 z^*] \right] \right] = 0. \end{aligned} \quad (6)$$

Next, we define the long-term behaviors of critical steady state in the case of control and non-control of banditry groups' access to arms and weapons in the theorem below:

Theorem 2 Suppose there exist positive invariants (x^* , y^* , & z^*) depicting the critical steady state for the anti-banditry dynamics defined in System (1), then the critical steady state is:

1. Always unstable when there is lack of effective control of arms and weapons i.e., when $\tau = 0$.
2. Undergoing supercritical Pitchfork bifurcation when $\tau > 0$ characterized with control-induced regression of banditry groups' activity as well as growth of anti-banditry activities which bring about progress and prosperity in the vulnerable communities.

Proof: Substituting into Equation (6), $(x^*, y^*, z^*) = (\frac{\vartheta - \sigma - v_2 z^*}{\gamma}, \frac{\theta + v_1 z^* - \alpha}{\beta}, \frac{\beta\gamma[\phi - \rho] + \gamma v_4[\theta - \alpha] - \beta v_3[\vartheta - \sigma]}{\beta v_3 v_2 - \gamma v_4 v_1})$ with z^* retained for the convenience of the computation, the resulting cubic polynomial becomes:

$$\begin{aligned} D(\lambda, \tau) = & (\lambda) \left[(\lambda - [\sigma - \vartheta + \frac{\beta\gamma[\phi - \rho] + \gamma v_4[\theta - \alpha] - \beta v_3[\vartheta - \sigma]}{\beta v_3 v_2 - \gamma v_4 v_1}]) + [\vartheta - \sigma - v_2 z^*] e^{-\lambda\tau} \right) \\ & (\lambda - (\phi + \frac{v_3}{\gamma} [\vartheta - \sigma - v_2 z^*] - \frac{v_4}{\beta} [\theta + v_1 z^* - \alpha] e^{-\lambda\tau} - \rho)) \end{aligned}$$

$$\begin{aligned} & + \frac{v_2 v_4}{\beta^2} [\theta + v_1 z^* - \alpha]^2 - \frac{\beta}{\gamma} [\vartheta - \sigma - v_2 z^*] \left[\gamma \left[\frac{\theta + v_1 z^* - \alpha}{\beta} \right] (\lambda - (\phi + v_3 x^* - v_4 y^* e^{-\lambda\tau} - \rho)) \right. \\ & \left. + \frac{v_3}{\gamma} [\vartheta - \sigma - v_2 z^*] - \frac{v_4}{\beta} [\theta + v_1 z^* - \alpha] e^{-\lambda\tau} - \rho \right) \\ & - v_3 z^* v_2 \left[\frac{\theta + v_1 z^* - \alpha}{\beta} \right] \end{aligned}$$

$$\begin{aligned} & - v_1 \left[\frac{\vartheta - \sigma - v_2 z^*}{\gamma} \right] \left[-\gamma \left[\frac{\theta + v_1 z^* - \alpha}{\beta} \right] v_4 z^* e^{-\lambda\tau} - v_3 z^* (\lambda - (\sigma - \vartheta + \gamma \left[\frac{\vartheta - \sigma - v_2 z^*}{\gamma} \right] e^{-\lambda\tau} + v_4 z^*)) \right] = 0. \end{aligned} \quad (7)$$

Simplifying (7), we have

$$D(\lambda, \tau) = \lambda^3 - (A_{11} - A_{12} e^{-\lambda\tau}) \lambda^2 - (A_{21} - A_{22} e^{-\lambda\tau}) \lambda + A_{31} + A_{32} e^{-\lambda\tau} = 0, \quad (8)$$

where,

$$\begin{aligned} A_{11} = & \vartheta - \sigma - \frac{v_2}{\beta v_3 v_2 - \gamma v_4 v_1} [\beta\gamma[\phi - \rho] + \gamma v_4[\theta - \alpha] - \beta v_3[\vartheta - \sigma]] + \frac{\phi}{\gamma} [\vartheta - \sigma - \frac{v_2}{\beta v_3 v_2 - \gamma v_4 v_1} [\beta\gamma[\phi - \rho] + \gamma v_4[\theta - \alpha] - \beta v_3[\vartheta - \sigma]]] - \rho \end{aligned}$$

$$\begin{aligned} A_{12} = & [\vartheta - \sigma - \frac{v_2}{\beta v_3 v_2 - \gamma v_4 v_1} [\beta\gamma[\phi - \rho] + \gamma v_4[\theta - \alpha] - \beta v_3[\vartheta - \sigma]]] \\ & - \frac{v_4}{\beta} [\theta + \frac{v_1}{\beta v_3 v_2 - \gamma v_4 v_1} [\beta\gamma[\phi - \rho] + \gamma v_4[\theta - \alpha] - \beta v_3[\vartheta - \sigma]]] - \alpha \end{aligned}$$

$$\begin{aligned} A_{21} = & [\vartheta - \sigma - \frac{v_2}{\beta v_3 v_2 - \gamma v_4 v_1} [\beta\gamma[\phi - \rho] + \gamma v_4[\theta - \alpha] - \beta v_3[\vartheta - \sigma]]] [\theta + \frac{v_1}{\beta v_3 v_2 - \gamma v_4 v_1} [\beta\gamma[\phi - \rho] + \gamma v_4[\theta - \alpha] - \beta v_3[\vartheta - \sigma]]] - \alpha \\ & + \frac{v_1 v_3 [\beta\gamma[\phi - \rho] + \gamma v_4[\theta - \alpha] - \beta v_3[\vartheta - \sigma]]}{\gamma [\beta v_3 v_2 - \gamma v_4 v_1]} [\vartheta - \sigma - \frac{v_2}{\beta v_3 v_2 - \gamma v_4 v_1} [\beta\gamma[\phi - \rho] + \gamma v_4[\theta - \alpha] - \beta v_3[\vartheta - \sigma]]] \end{aligned}$$

$$A_{22} = \frac{v_1 v_4}{\beta} \left[\frac{\beta\gamma[\phi - \rho] + \gamma v_4[\theta - \alpha] - \beta v_3[\vartheta - \sigma]}{\beta v_3 v_2 - \gamma v_4 v_1} \right] [\vartheta - \sigma - \frac{v_2}{\beta v_3 v_2 - \gamma v_4 v_1} [\beta\gamma[\phi - \rho] + \gamma v_4[\theta - \alpha] - \beta v_3[\vartheta - \sigma]]]$$

$$\begin{aligned} & - v_2 \left[\frac{\beta\gamma[\phi - \rho] + \gamma v_4[\theta - \alpha] - \beta v_3[\vartheta - \sigma]}{\beta v_3 v_2 - \gamma v_4 v_1} \right] [\theta + \frac{v_1}{\beta v_3 v_2 - \gamma v_4 v_1} [\beta\gamma[\phi - \rho] + \gamma v_4[\theta - \alpha] - \beta v_3[\vartheta - \sigma]]] - \alpha \end{aligned}$$

$$A_{31} = \frac{\beta}{\gamma} [\vartheta - \sigma - v_2 \left[\frac{\beta\gamma[\phi - \rho] + \gamma v_4[\theta - \alpha] - \beta v_3[\vartheta - \sigma]}{\beta v_3 v_2 - \gamma v_4 v_1} \right]] \left[\frac{\gamma}{\beta} [\theta + v_1 \left[\frac{\beta\gamma[\phi - \rho] + \gamma v_4[\theta - \alpha] - \beta v_3[\vartheta - \sigma]}{\beta v_3 v_2 - \gamma v_4 v_1} \right]] - \alpha \right]$$

$$\begin{aligned} & + v_1 \left[\frac{\beta\gamma[\phi - \rho] + \gamma v_4[\theta - \alpha] - \beta v_3[\vartheta - \sigma]}{\beta v_3 v_2 - \gamma v_4 v_1} \right] - (\phi + \frac{v_3}{\gamma} [\vartheta - \sigma - \frac{v_2}{\beta v_3 v_2 - \gamma v_4 v_1} [\beta\gamma[\phi - \rho] + \gamma v_4[\theta - \alpha] - \beta v_3[\vartheta - \sigma]]] - \rho) \end{aligned}$$

$$A_{31} + A_{32} = [\theta + v_1 z^* - \alpha][[\vartheta - \sigma - v_2 z^*](\phi + \frac{v_3}{\gamma} [\vartheta - \sigma - v_2 z^*] - \rho) + \frac{v_3 v_2}{\beta} z^* + \frac{v_1}{\beta} [v_4 z^* v_3 z^*][\vartheta - \sigma - v_2 z^*]^2 - \frac{v_4}{\beta} [\vartheta - \sigma - v_2 z^*][\theta + v_1 z^* - \alpha] - \frac{v_1 v_3 z^*}{\gamma[\theta + v_1 z^* - \alpha]} [\vartheta - \sigma - v_2 z^*](\vartheta - \sigma - v_4 z^*)]. \quad (17)$$

Therefore,

$$h = A_{31}^2 - A_{32}^2 = [\theta + v_1 z^* - \alpha]^2 [[\vartheta - \sigma - v_2 z^*](\phi + \frac{v_3}{\gamma} [\vartheta - \sigma - v_2 z^*] - \rho) + \frac{v_3 v_2}{\beta} z^* + \frac{v_1}{\beta} [v_4 z^* v_3 z^*][\vartheta - \sigma - v_2 z^*]^2 - \frac{v_4}{\beta} [\vartheta - \sigma - v_2 z^*][\theta + v_1 z^* - \alpha] - \frac{v_1 v_3 z^*}{\gamma[\theta + v_1 z^* - \alpha]} [\vartheta - \sigma - v_2 z^*](\vartheta - \sigma - v_4 z^*)]^2. \quad (18)$$

Obviously from Equation (18) $h > 0$ and consequently, Equation (15) has at most one non-positive root. Therefore, the only way to have a simple positive root in this case is to have two positive real roots. This occurs only if Equation (15) has positive critical point, and the derivative function for the critical point becomes

$$3u^2 + 2f_1 u + g_1 = 0, \quad (19)$$

with roots

$$u_{(2,3)} = \frac{-2f_1 \pm 2\sqrt{f_1^2 - 3g}}{6}. \quad (20)$$

For (20) to have one positive root, g must be negative so that $f^2 - 3g > 0$. Computing g using definitions in (14), we have

$$g = A_{21}^2 - 2A_{31}[A_{11} + A_{12}] - A_{22}^2 \quad (21)$$

$$g = [[\vartheta - \sigma - v_2 z^*][\theta + v_1 z^* - \alpha] + v_1 v_3 z^* [\vartheta - \sigma - v_2 z^*]]^2 - [\frac{v_1 v_4}{\beta} z^* [\vartheta - \sigma - v_2 z^*][\theta + v_1 z^* - \alpha]]^2 - 2[[\vartheta - \sigma - v_2 z^*][\theta + v_1 z^* - \alpha](\phi + \frac{v_3}{\gamma} [\vartheta - \sigma - v_2 z^*] - \rho) + \frac{v_3 v_2 z^*}{\beta} [\theta + v_1 z^* - \alpha] - \frac{v_3 z^* v_1}{\gamma} [\vartheta - \sigma - v_2 z^*][(\vartheta - \sigma - v_4 z^*)][[\vartheta - \sigma - v_2 z^*] + \frac{v_3}{\gamma} [\vartheta - \sigma - v_2 z^* + \phi - \rho] + [\vartheta - \sigma - v_2] - \frac{v_4}{\beta} [\theta + v_1 z^* - \alpha]] \quad (22)$$

Equation (22) suggests g is negative. Hence Equation (15) has two positive real roots. This further reveals that there exists arms and weapons' control-induced supercritical pitchfork bifurcation, where the control of arms and weapons' proliferation induces the regression in the bandit groups' activities and the growth of anti-banditry activities which lead to return of progress and prosperity in the vulnerable communities.

Here, we derive the corresponding value of τ^* at which supercritical pitchfork bifurcation occurs and the boundary condition that determines the optimal path for arms and weapons' control. This is done by solving for τ^* in Equation (11) to obtain

$$\sin[k\tau] = \frac{A_{22}k[[A_{11}+A_{12}]k^2-A_{31}]-A_{32}[k^3+A_{21}k]}{A_{22}A_{22}k+A_{32}^2}, \quad (23)$$

$$\tau^* = \frac{1}{k} \arcsin\left(\frac{A_{22}k[[A_{11}+A_{12}]k^2-A_{31}]-A_{32}[k^3+A_{21}k]}{A_{22}A_{22}k+A_{32}^2}\right). \quad (24)$$

To obtain the transversality condition for determining the optimal path for arms and weapons' control at $\tau = \tau^*$, we differentiate Equation(8) with respect to τ and obtain

$$D(\lambda, \tau) = \lambda^3 - (A_{11} - A_{12}e^{-\lambda\tau})\lambda^2 - (A_{21} - A_{22}e^{-\lambda\tau})\lambda + A_{31} + A_{32}e^{-\lambda\tau} = 0, \quad (25)$$

$$3\lambda^2 - 2A_{11}\lambda - A_{21} - [A_{12} - A_{22}]e^{-\lambda\tau} - \tau[A_{12}\lambda^2 + A_{22}\lambda + A_{32}]e^{-\lambda\tau} \frac{\partial \lambda}{\partial \tau_1} = \lambda[A_{12}\lambda^2 + A_{22}\lambda + A_{32}]e^{-\lambda\tau}. \quad (26)$$

Re-setting Equation (26) to reflect the changes in the form of τ gives

$$\left(\frac{\partial \lambda}{\partial \tau_1}\right)^{-1} = \frac{[3\lambda^2 - 2A_{11}\lambda - A_{21}]e^{\lambda\tau}}{\lambda[A_{12}\lambda^2 + A_{22}\lambda + A_{32}]} - \frac{[A_{12} - A_{22}]}{\lambda[A_{12}\lambda^2 + A_{22}\lambda + A_{32}]} + \frac{\tau}{\lambda}. \quad (27)$$

From Equation (8),

$$e^{-\lambda\tau} = \frac{\lambda^3 - A_{11}\lambda^2 - A_{21}\lambda + A_{31}}{-[A_{12}\lambda^2 + A_{22}\lambda + A_{32}]}. \quad (28)$$

Substituting Equation (28) into (27), we have

$$\left(\frac{\partial \lambda}{\partial \tau_1}\right)^{-1} = \frac{[3\lambda^2 - 2A_{11}\lambda - A_{21}]}{-\lambda[\lambda^3 - A_{11}\lambda^2 - A_{21}\lambda + A_{31}]} - \frac{[A_{12} - A_{22}]}{\lambda[A_{12}\lambda^2 + A_{22}\lambda + A_{32}]} + \frac{\tau}{\lambda}. \quad (29)$$

Hence,

$$Sign\left\{Re \frac{d\lambda}{d\tau_1}\right\}_{\lambda=ui}^{-1} = Sign\left\{\frac{[3\lambda^2 - 2A_{11}\lambda - A_{21}]}{-\lambda[\lambda^3 - A_{11}\lambda^2 - A_{21}\lambda + A_{31}]} - \frac{[A_{12} - A_{22}]}{\lambda[A_{12}\lambda^2 + A_{22}\lambda + A_{32}]}\right\}_{\lambda=ik}. \quad (30)$$

Inserting $\lambda = ik$ into Equation (30) and putting $i^2 = -1$ yields

$$Sign\left\{Re \frac{d\lambda}{d\tau_1}\right\}_{\lambda=ui}^{-1} = Sign\left\{\frac{[3k^2 + 2A_{11}ik + A_{21}]}{[k^4 + A_{11}ik^3 + A_{21}k^2 + A_{31}ik]} + \frac{[A_{12} - A_{22}]}{[A_{12}ik^2 + A_{22}k^2 - A_{32}ik]}\right\}_{\lambda=ik} < 0. \quad (31)$$

Since the sign for the real part of (31) is nonzero, the transversality condition holds. Therefore, a supercritical Pitchfork bifurcation occurs at $\tau = \tau^*$ where the control of arms and weapons' proliferation induces the regression of banditry groups' activity and growth of anti-banditry activities as well as the progress and prosperity of the vulnerable communities.

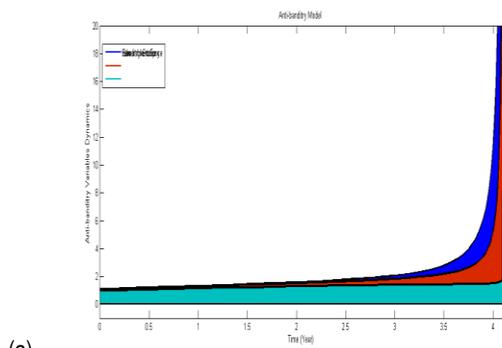
Table 1: Model Estimated Parameter Values

Parameter	Estimated Value	Source
α	0.4	Estimated from (COAS, 2024)
β	2.9	Assumed
θ	0.1	Estimated from (Isamotu, 2023)
σ	0.2	Assumed based on (Ojewale, 2024)
γ	1.5	Assumed
ϑ	0.5	Estimated from (OBIEJESI, 2023)
v_1	0.1	(Akinyetun, 2023)
v_2	0.5	Assumed
ϕ	0.4	Assumed
v_3	0.2	(Oyewole, 2022)
v_4	0.35	Estimated from (Bashir, 2022)
ρ	0.2	Estimated from (Idowu, 2023)

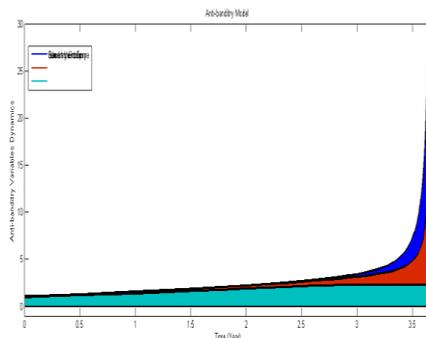
RESULTS AND DISCUSSION

Numerical Simulation

Here, We verify the analytical results in Section 3 by implementing the assumptions in the theorems stated in Section 3 to stimulate System (1) using Matlab DDE223 solver with the model parameter values summarized in Table 1. This will show graphically, the dynamics of System (1) when there exists no arms and weapon control, and when there is a strict arms and weapon control.



(a)

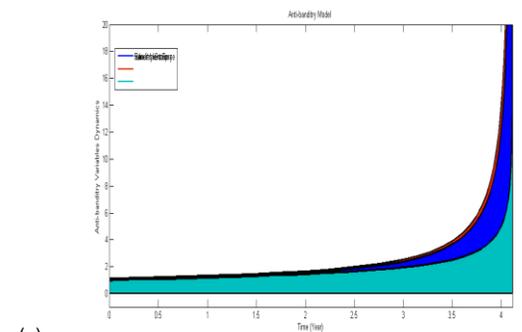


(b)

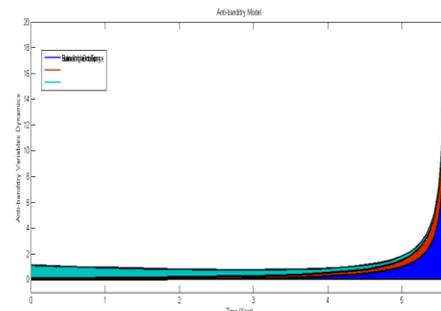
Figure 1: Time evolution of model variables System (1). (a) With involvement of military intelligent gathering and banditry groups informants' collaborations, (b) Without involvement of military

intelligent gathering and banditry groups informants' collaborations.

Figure 1 shows the time evolution of the model variables of System (1) with and without the utilization of intelligence gathering. (a) The simulation of System (1) when intelligence gatherings are used both by the government forces and the bandit groups in the course of engagements. The dynamics are characterized by the trap of the vulnerable communities in the midst of anti-banditry engagement. (b) the numerical simulation of System (1) when intelligence gatherings are used both by the government forces and the bandit groups in the course of engagements. The dynamics is relatively the same as observed in (a). This indeed verifies the analytical result obtained which predicted the unstable stability for the critical steady state when there is no arms and weapon control.



(a)

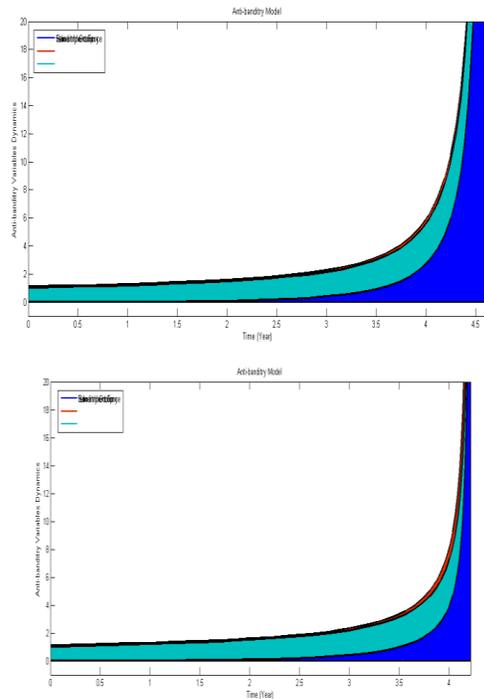


(b)

Figure 2: System (1) dynamics around critical steady state without arms and weapons control (a) When government forces are winning. (b) When bandit groups' assaults are getting untamed.

Fig. 2, is the numerical simulation of System (1) around critical steady state when $\tau = 0$ i.e. when there is no control of arms and weapons proliferation. (a) When government force is triumphing in the war against bandit groups. The dynamics showcased a vulnerable community shielded by the anti-banditry forces against bandit groups. Though, the community witness relative growth, however, the progress is caged by the banditry unrest, but the activity of bandit groups is relatively suppressed by the anti-banditry force. (see Fig. 2a). (b) When bandit groups' assaults are getting untamed, the dynamics exhibits active anti-banditry engagements between banditry groups and anti-banditry force. The activity of the community is seen to be paralyzed with relatively no growth and progress (see Fig. 2b).

(a)



(b)
Figure 3: System (1) dynamics around critical steady state with arms and weapons control (a) When the control impaired the banditry defense against government forces. (b) When the control impaired the banditry groups' invasion of vulnerable communities.

Fig. 3, is the numerical simulation of System (1) dynamics around critical steady state when $\tau \neq 0$ i.e. when there is control of arms and weapons proliferation. (a) When the control affects the banditry group's ability to defense against government forces, the dynamics produces an unperturbed community growth in the midst of the increase of anti-banditry force and the banditry groups curve is seen very tiny with no adversity against vulnerable community (see Fig. 3a). (b) When the control affects the banditry group's ability to invade vulnerable communities, the dynamics is relatively the same with that of Fig. 3a except with much lower anti-banditry force's concentration.

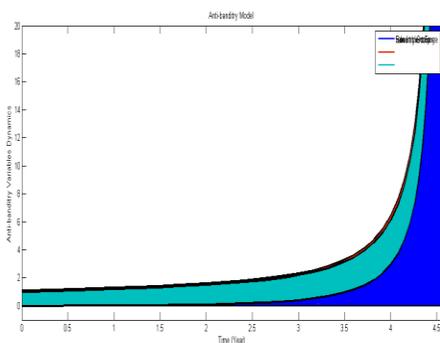


Figure 4: System (1) dynamics around critical steady state with arms and weapons control that impaired both the banditry defense against government forces and the invasion of vulnerable

communities.

Fig. 4, is the numerical simulation of system (1) dynamics around critical steady state when $\tau \neq 0$ i.e. when there is control of arms and weapons control that affects both the banditry defense against government forces and the invasion of vulnerable communities. The dynamics is more of the same with the one obtained when there is control of arms and weapons that affects both the banditry defense against government forces as seen in Fig 3a. Obviously, the banditry groups are scaled down and the safety and progress return to the vulnerable community. However, the persistence and increase of banditry groups as community progress without resurgence of banditry groups needs an optimal decision to arrest likely degeneration of the trend.

This paper seeks to foretell the impact of control of small arms and light weapons on the anti-banditry measures using time delay model. A delay model that factored banditry groups, government forces and vulnerable community as model variables while time delay (τ) is used to depict the control in access to arms and weapons due to law process if implemented is proposed and analyzed. The preliminary analysis for existent of non-negative solution is carried out. Also, the conditions for the existence of positive invariants of the model variables upon which their long time behaviors will be assessed is equally obtained. This produces the critical steady-state for our proposed model. The analysis of the critical steady-state predicted an unstable scenario when there is no control of arms and weapons i.e. when $\tau = 0$, and a arms and weapons' control-induced supercritical pitchfork bifurcation characterized with the bandit groups' scaled down and the return of safety and progress to the vulnerable community when $\tau \neq 0$. This affirms that arms and weapons' proliferation control can induce the regression in the bandit groups' activities, the growth of anti-banditry measures and the return of progress and posterity to the vulnerable communities.

The numerical solutions verified all the analytical results and as well provided extensive perspectives on the dynamics of the model variables in different phases of anti-banditry measures. The dynamics of anti-banditry with and without intelligence gathering demonstrate a similar scenario characterized with the trap of the vulnerable communities in the midst of anti- banditry engagements. This finding is validated by the assertion in (Saheed, 2023) that insecurity will still persist regardless of timely and accurate usage of intelligence by governments' forces in submerging the activities of banditry groups.

To gain pictorial insights into the model variables' dynamics around critical steady state without small arms and light weapons control, the conditions 1 in Theorem 2 is numerically implemented on system (1) to assess the success and failure of kinetic approach on the safety of the vulnerable community(ies). The results showcase a caged community(ies) shielded by the anti-banditry forces when kinetic approach is succeeding. The failure of kinetic approach precipitates paralyzed with relatively no growth and progress made over time. This results align with assertion that "continuous access to weapons and ammunition by banditry groups due to uncontrolled proliferation of small arms and light weapons across Nigeria is adjudged to be responsible for unsuccessful fight against banditry in Nigeria" (Ejiofor, 2022).

To verified the impact of small arms and ligths weapons control on

the dynamics of anti-banditry measures, we simulate System (1) around critical steady state by implementing the condition 2 in theorem 2 on System (1) in the following situations: (1). When the control affects the banditry defense against government forces. (2). When the control affects the banditry groups' invasion of vulnerable communities. (3). When arms and weapons control affects both the banditry defense against government forces and the invasion of vulnerable communities. Obviously in all the three situations, the banditry groups are scaled down, and safety and progress return to the vulnerable community. This validates the postulation of Theorem 2 that the small arms and light weapons' control induces supercritical pitchfork bifurcation leading to regression in the bandit groups' activities, the growth of anti-banditry measures and the return of progress and prosperity to the vulnerable communities.

Conclusion

We have been able to explore the impact of arms and weapons' proliferation control in the dynamics of anti-banditry measures using delay model. Our findings highlight that, the anti-banditry measures to tame the tide of banditry groups through kinetic approach will not bring about desirable lasting solution in as much as arms and weapons proliferation's control is not properly annexed as alternative anti-banditry measure. Also, the findings highlight that, the adoption of arms and weapon proliferation control might precipitates the persistence and over concentration of anti-banditry personnel within the vulnerable communities. This might degenerate into some abuses and recklessness. In order to checkmate the possible degeneration of the persistence and over concentration of the anti-banditry forces, governments should come up with clear cut optimal strategies to arrest such eventuality if arises.

Conflict of Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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