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Full Length Research Article

EXISTENCE AND UNIQUENESS SOLUTION OF TRANSIENT NATURAL CONVECTION FLOW PAST A CYLINDER MODEL WITH VISCOUS DISSIPATION AND STRATIFICATION

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ABSTRACT
Stratification is the layering of a fluid due to concentration or temperature differences. The flow of fluid past a cylinder and the study of stratification in parabolic flow are of significant interest due to their role in minimizing energy losses by reducing viscous interactions between adjacent fluid layers and the pipe wall. Thermal stratification causes the temperature to vary more sharply with distance, creating stable layers that resist vertical mixing. In contrast, mass stratification leads to density gradients in the flow direction. A thorough understanding of these dynamics is crucial for developing effective strategies for managing, mitigating, and controlling flow. This study incorporates temperature-dependent diffusion and convection terms into the energy and concentration equations. Parabolic flow is particularly relevant in mass and heat transfer processes involving an infinite vertical plate, which may be subjected to various boundary conditions such as constant or variable temperature, heat flux, or concentration. These problems have attracted considerable attention due to their applications in engineering and industrial systems, including the cooling of electronic devices, solar collectors, chemical reactors, and combustion chambers. To preserve the mathematical integrity of the model, the coupled nonlinear partial differential equations are transformed into a dimensionless form using appropriate nondimensional variables, thereby ensuring consistency and simplifying the analytical process. The existence and uniqueness of solutions to the model equations are rigorously established via the Lipschitz continuity criterion. The results confirm that a unique solution exists under the satisfaction of the Lipschitz conditions.

Keywords: Cylindrical plate, Dimensionless variables, Existence and uniqueness, Fluid flows, Lipschitz continuity, Stratification.

INTRODUCTION

Thermal stratification is a common occurrence in natural environments such as lakes and oceans. The presence of mass stratification can further complicate the behavior of fluid flow. This study aims to explore how thermal stratification influences flow dynamics and its interaction with other stratified bodies. The findings have a wide range of practical applications, including the design of more efficient chemical reactors and heat exchangers, as well as the evaluation of how thermal stratification affects the efficiency of cooling systems in electronic devices (Wang et al, 2023).

Using a porous material and a magnetic field (Mondal et al.,2017) examined the impact of chemical processes and radiation on the transfer of mass in unstable natural convection flow across an infinite perpendicular plate moving at an exponential pace.

(Paul and Deka, 2013) Presented analytical investigation to study the effects of thermal stratification and mass stratification on natural convection heat and mass transfer over moving vertical cylinder. The study of mixed convective fluid flow around cylinders with various cross-sectional shapes cannot be overemphasized. It provides insights into how geometry affects heat and mass transfer in stratified flow conditions as investigated by (Olayemi et al., 2023).

Free convection boundary layer flows over a vertical surface in a doubly stratified fluid-saturated porous medium in the presence of constant suction/injection has been analyzed (Srinivasacharya et. al., 2015).

(Rehman, et al., 2023) Presents heat and mass transfer characteristics of Casson fluid flow over flat and cylindrical surfaces, providing a comprehensive analysis relevant to stratified flow scenarios. (Paul and Deka 2017) presented an analytical investigation of one-dimensional free convective flow past a stationary infinite vertical cylinder with combined effects of thermal and mass stratification (both not temperature dependent) but neglected viscous dissipation energy

Similarly, (Megahed and Abbas, 2022) researched the consequences of non-Newtonian fluid flow over a porous medium on both effects. They also considered how buoyancy-driven flows in a stratified fluid were examined and came up with an analytical solution to describe how fluid would flow past an infinite vertical plate. (A. Selvaraj and E. Jothi, 2021) studied MHD and the absorption of radiation of a stream of fluid passing a vertical plate moving at an exponentially increasing rate, when warmth and mass diffuse exponentially across a porous material, and the influences of the source of heat on these variables.

(Muthucumaraswamy and Visalakshi, 2019) Investigated the impact of heat radiation on the motion of a fluid with a high viscosity and low compressibility through a vertically infinite plate moving at an exponential rate, subject to a homogeneous mass diffusion and changeable temperature.

The present study focuses on analyzing a specific class of Natural convection flow past a cylinder with thermal and mass stratification. We address a fundamental question, whether the model admits a unique solution by employing the Lipschitz continuity approach under theorems described by Ayeni (1978). Through rigorous investigation of the existence and uniqueness of solutions, this research aims to contribute to the advancement of reliable predictive tools for understanding and mitigating fluid flow behavior, embedded with viscous energy dissipation.

MATERIAL AND METHODS

We consider a fluid that is stratified, viscous, and in-compressible, traveling along an accelerating vertical plate with a first-order chemical reaction present. We use a coordinate system in which the y' axis is perpendicular to the plate and the x-axis is taken vertically upward along the plate to study the flow situation. The fundamental equations of momentum, energy, and mass conservation are used to develop the system of equations that describes unsteady flow via a porous material with exponential mass diffusion and fluctuating temperatures across an indefinitely accelerated vertical plate. An existing model is seen in the work of (Paul and Deka 2017) and thus modified to its present form for a more comprehensive framework for modeling convective flows past cylindrical plates.

The temperature-dependent thermal conductivity and diffusion coefficient are in the form:

$$K_T = K_0\left(\frac{T'}{T_0}\right), \ and \quad D_T = D_0\left(\frac{T'}{T_0}\right) \qquad (1)$$

The leading governing equations for this investigation follow thus:

$$\frac{1}{r'}\frac{\partial}{\partial r'}\left(r'u'\right)=0 \qquad (2)$$

$$\rho\frac{\partial u'}{\partial t'}=\frac{\mu}{r'}\frac{\partial}{\partial r'}\left(r'\frac{\partial u'}{\partial r'}\right)+\rho g\beta\left(T'-T_R'\right)+\rho g\beta^*\left(C'-C_R'\right) \qquad (3)$$

$$\rho c_p\frac{\partial T'}{\partial t'}=\frac{k}{r'}\frac{\partial}{\partial r'}\left(K_T r'\frac{\partial T'}{\partial r'}\right)+\mu\left(\frac{\partial u'}{\partial r'}\right)^2-\gamma u' \qquad (4)$$

$$\frac{\partial C'}{\partial t'}=\frac{1}{r'}\frac{\partial}{\partial r'}\left(D_T r'\frac{\partial C'}{\partial r'}\right)-\xi u' \qquad (5)$$

Subject to initial and boundary conditions are formulated as:

$$\left.\begin{array}{lll} u'(r',0)=U_0, & \left.\frac{\partial u'}{\partial r'}\right|_{r'=0}=\xi_0, & u'(R,t')=0 \\ T'(r',0)=T_R', & -k^*\left.\frac{\partial T'}{\partial r'}\right|_{r'=0}=h_f\left(T(0,t')-T_R'\right), & T'(R,t')=T_R' \\ C'(r',0)=C_R', & -D^*\left.\frac{\partial C'}{\partial r'}\right|_{r'=0}=h_m\left(C(0,t')-C_R'\right), & C'(R,t')=C_R' \end{array}\right\} \qquad (6)$$

where,

ρ = fluid density

u' = fluid velocity

t' = time

μ = kinematic viscosity

r' = radial coordinate

g = acceleration due to gravity

β = volumetric coefficient of thermal expansion,

T' = Temperature of the fluid

T_R' = starting temperature at the plate

β^* = volumetric coefficient of expansion with concentration,

C' = Concentration of the fluid

C_R' = fluid concentration at the plate

c_p = Specific heat Capacity

k = thermal diffusivity

K_T = thermal conductivity

γ = thermal stratification parameter

D_T = diffusion coefficient

ξ = mass stratification

U_0 = velocity at the initial point.

ξ_0 = constant

D^* = diffusion coefficient at the solid phase

k^* = thermal conductivity at the solid phase

h_f = the convective heat transfer coefficient at the fluid phase

h_m = mass transfer coefficient at the fluid phase

RESULTS AND DISCUSSION

Method of solution

Ayeni (1978) investigated the issue of the existence and uniqueness of the solution, revealing, among other findings, that these qualities are reasonably well understood. The subsequent system of parabolic equations serves as an illustration:

$$\frac{\partial\phi}{\partial t}=\Delta\phi+f\left(x,t,\phi,u,v\right),\ x\in R^n,\ t>0 \qquad (7)$$

$$\frac{\partial u}{\partial t}=\Delta u+g\left(x,t,\phi,u,v\right),\ x\in R^n,\ t>0 \qquad (8)$$

$$\frac{\partial v}{\partial t}=\Delta v+h\left(x,t,\phi,u,v\right),\ x\in R^n,\ t>0 \qquad (9)$$

$$\phi(x,0)=f_0(x),\ \ u(x,0)=g_0(x),\ \ v(x,0)=h_0(x),\quad x:(x_1,x_2,...,x_n) \qquad (10)$$

(S.1): $f_0(x), g_0(x)\ and\ h_0(x)$ are bounded for $x\in R^n$. Each has at most a countable number of discontinuities.

(S.2): $f, g\ and\ h$ Satisfies the uniform Lipchitz condition.

$$\left|\phi\left(x,t,\phi_1,u_1,v_1\right)-\phi\left(x,t,\phi_2,u_2,v_2\right)\right|\le M\left(\left|\phi_1-\phi_2\right|+\left|u_1-u_2\right|+\left|v_1-v_2\right|\right),\ (x,t)\in G, \qquad (11)$$

Where, $G=\left\{(x,t):x\in R^n, 0<t<\tau\right\}$

Theorem (Ayeni 1978): Let $\left(f_0,g_0,h_0\right)$ and $\left(f,g,h\right)$ satisfy (S.1) and (S.2) respectively, then there exists a solution of problem (7) – (10).

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Dimensionless analysis
Dimensionless variables are been introduced as

$$t=\frac{U_0t'}{R}\quad,\qquad r=\frac{r'}{R}\quad,\qquad u=\frac{u'}{U_0},$$

$$\phi=\frac{C'-C_0}{C'_R-C_0},\qquad \theta=\frac{T'-T_0}{T'_0-T_0}\qquad (12)$$

Using (12), on (2) – (6) gives the dimensionless form of the model equations (13) – (16)

$$\frac{\partial u}{\partial t}=\frac{1}{\text{Re}}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right)-G_{r\theta}(1-\theta)-G_{r\phi}(1-\phi)\qquad (13)$$

$$\frac{\partial\theta}{\partial t}=\frac{1}{Pe}\frac{1}{r}\frac{\partial}{\partial r}\left(r(1+\alpha\theta)\frac{\partial\theta}{\partial r}\right)+\frac{Ec}{\text{Re}}\left(\frac{\partial u}{\partial r}\right)^2-\gamma_\theta u\qquad (14)$$

$$\frac{\partial\phi}{\partial t}=\frac{1}{P_{em}}\frac{1}{r}\frac{\partial}{\partial r}\left(r(1+\alpha\theta)\frac{\partial\phi}{\partial r}\right)-\varepsilon_\phi u\qquad (15)$$

Subject to,

$$u(r,0)=1,\qquad \left.\frac{\partial u}{\partial r}\right|_{r=0}=\xi,\qquad u(1,t')=0$$

$$\theta(r,0)=1,\qquad \left.\frac{\partial\theta}{\partial r}\right|_{r=0}=-Nu\theta+Nu\qquad \theta(1,t)=1$$

$$\phi(r,0)=1,\qquad \left.\frac{\partial\phi}{\partial r}\right|_{r=0}=-Sh_m\phi+Sh_m\qquad \phi(1,t)=1\qquad (16)$$

Where,
Re =Reynold number,
$G_{r\theta}$ = Thermal Grashoff number,
$G_{r\phi}$ = Mass Grashoff number
Pe = Peclet Energy;
γ_θ = Thermal Stratification;
Ec = Eckert number
α = thermal diffusivity
P_{em} = Peclet energy mass;
ε_ϕ = Mass Stratification
Sh_m = Sherwood number

Existence and Uniqueness of Solution
Here, (13) – (15) are written as follows,

$$\frac{\partial u}{\partial t}=\frac{1}{\text{Re}}\left(\frac{\partial^2u}{\partial r^2}\right)+\frac{1}{\text{Re}}\frac{1}{r}\left(\frac{\partial u}{\partial r}\right)-G_{r\theta}(1-\theta)-G_{r\phi}(1-\phi)\qquad (17)$$

$$\frac{\partial\theta}{\partial t}=\frac{1}{Pe}(1+\alpha\theta)\left(\frac{\partial^2\theta}{\partial r^2}\right)+\frac{1}{Pe}\alpha\left(\frac{\partial\theta}{\partial r}\right)^2+\frac{1}{Pe}\frac{1}{r}(1+\alpha\theta)\left(\frac{\partial\theta}{\partial r}\right)+\frac{Ec}{\text{Re}}\left(\frac{\partial u}{\partial r}\right)^2-\gamma_\theta u\qquad (18)$$

$$\frac{\partial\phi}{\partial t}=\frac{1}{P_{em}}(1+\alpha\theta)\left(\frac{\partial^2\phi}{\partial r^2}\right)+\frac{1}{P_{em}}\alpha\frac{\partial\phi}{\partial r}\frac{\partial\theta}{\partial r}+\frac{1}{P_{em}}\frac{1}{r}(1+\alpha\theta)\left(\frac{\partial\phi}{\partial r}\right)-\varepsilon_\phi u\qquad (19)$$

Theorem 1: Let $\left|\frac{\partial\theta}{\partial t}\right|\le a,\ \left|\frac{\partial\phi}{\partial t}\right|\le b,\ \left|\frac{\partial^2\theta}{\partial r^2}\right|\le c,\ \left|\frac{\partial^2\phi}{\partial r^2}\right|\le d\ \ and\ \ r>0$. Then there exists a unique solution $u(r,t),\ \theta(r,t),\ and\ \phi(r,t)$ of equations (17) – (19).

Proof of theorem:- We rewrite the equation (17) – (19) as

$$\frac{\partial u}{\partial t}=\frac{1}{\text{Re}}\frac{\partial^2u}{\partial r^2}+f(r,t,u,\theta,\phi),\quad r\in R^n,\ t>0\qquad (20)$$

$$\frac{\partial\theta}{\partial t}=\frac{1}{P_e}\frac{\partial^2\theta}{\partial r^2}+g(r,t,u,\theta,\phi),\quad r\in R^n,\ t>0\qquad (21)$$

$$\frac{\partial\phi}{\partial t}=\frac{1}{P_{em}}\frac{\partial^2\phi}{\partial r^2}+h(r,t,u,\theta,\phi),\quad r\in R^n,\ t>0\qquad (22)$$

Where

$$f(r,t,u,\theta,\phi)=\frac{1}{\text{Re}}\frac{1}{r}\left(\frac{\partial u}{\partial r}\right)-G_{r\theta}(1-\theta)-G_{r\phi}(1-\phi)\qquad (23)$$

$$g(r,t,u,\theta,\phi)=\frac{1}{Pe}(\alpha\theta)\left(\frac{\partial^2\theta}{\partial r^2}\right)+\frac{1}{Pe}\alpha\left(\frac{\partial\theta}{\partial r}\right)^2+\frac{1}{Pe}\frac{1}{r}(1+\alpha\theta)\left(\frac{\partial\theta}{\partial r}\right)+\frac{Ec}{\text{Re}}\left(\frac{\partial u}{\partial r}\right)^2-\gamma_\theta u\qquad (24)$$

$$h(r,t,u,\theta,\phi)=\frac{1}{P_{em}}(\alpha\theta)\left(\frac{\partial^2\phi}{\partial r^2}\right)+\frac{1}{P_{em}}\alpha\frac{\partial\phi}{\partial r}\frac{\partial\theta}{\partial r}+\frac{1}{P_{em}}\frac{1}{r}(1+\alpha\theta)\left(\frac{\partial\phi}{\partial r}\right)-\varepsilon_\phi u\qquad (25)$$

According to Toki and Tokis (2007), the fundamental solutions of the equation of equation (20) – (22) are;

$$F(r,t)=\frac{r}{2\pi^{\frac{1}{2}}\left(\frac{1}{\text{Re}}\right)^{\frac{1}{2}}t^{\frac{3}{2}}}\exp\left(-\frac{\text{Re}}{4t}r\right)\qquad (26)$$

$$G(r,t)=\frac{r}{2\pi^{\frac{1}{2}}\left(\frac{1}{P_e}\right)^{\frac{1}{2}}t^{\frac{3}{2}}}\exp\left(-\frac{P_e}{4t}r\right)\qquad (27)$$

$$H(r,t)=\frac{r}{2\pi^{\frac{1}{2}}\left(\frac{1}{P_{em}}\right)^{\frac{1}{2}}t^{\frac{3}{2}}}\exp\left(-\frac{P_{em}}{4t}r\right)\qquad (28)$$

Next, it suffices to show that the Lipchitz conditions in Lemma 1 are satisfied. That is if we can show that;

$$\left|f(x,t,u_1,\theta_1,\phi_1)-f(x,t,u_2,\theta_2,\phi_2)\right|\le K_1\left(|u_1-u_2|+|\theta_1-\theta_2|+|\phi_1-\phi_2|\right)\qquad (29)$$

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$$\left|g(x,t,u_1,\theta_1,\phi_1)-g(x,t,u_2,\theta_2,\phi_2)\right|\le K_2\left(|u_1-u_2|+|\theta_1-\theta_2|+|\phi_1-\phi_2|\right) \quad (30)$$

$$\left|h(x,t,u_1,\theta_1,\phi_1)-h(x,t,u_2,\theta_2,\phi_2)\right|\le K_3\left(|u_1-u_2|+|\theta_1-\theta_2|+|\phi_1-\phi_2|\right) \quad (31)$$

Where,

$$K_1=\max\left\{\left|\frac{\partial f}{\partial u}\right|,\left|\frac{\partial f}{\partial \theta}\right|,\left|\frac{\partial f}{\partial \phi}\right|\right\},\ K_2=\max\left\{\left|\frac{\partial g}{\partial u}\right|,\left|\frac{\partial g}{\partial \theta}\right|,\left|\frac{\partial g}{\partial \phi}\right|\right\},\ K_3=\max\left\{\left|\frac{\partial h}{\partial u}\right|,\left|\frac{\partial h}{\partial \theta}\right|,\left|\frac{\partial h}{\partial \phi}\right|\right\} \quad (32)$$

$$\left|\frac{\partial f}{\partial u}\right|=0, \qquad \left|\frac{\partial f}{\partial \theta}\right|=G_{r\theta}, \qquad \left|\frac{\partial f}{\partial \phi}\right|=G_{r\phi} \quad (33)$$

$$\left|\frac{\partial g}{\partial u}\right|=\gamma_\theta, \quad \left|\frac{\partial g}{\partial \theta}\right|=\frac{1}{Pe}\alpha\frac{\partial^2\theta}{\partial r^2}+\frac{1}{Pe}\frac{1}{r}\alpha\frac{\partial\theta}{\partial r}=\frac{\alpha}{Pe}c+\frac{1}{r}a, \quad \left|\frac{\partial g}{\partial \phi}\right|=0 \quad (34)$$

$$\left|\frac{\partial h}{\partial u}\right|=\varepsilon_\theta, \quad \left|\frac{\partial h}{\partial \theta}\right|=\frac{1}{P_{em}}\alpha\frac{\partial^2\phi}{\partial r^2}+\frac{1}{P_{em}}\frac{1}{r}\alpha\frac{\partial\phi}{\partial r}=\frac{\alpha}{P_{em}}d+\frac{1}{r}b, \quad \left|\frac{\partial h}{\partial \phi}\right|=0 \quad (35)$$

Since,

$$G_{r\theta},\ G_{r\phi},\ \gamma_\theta,\ \varepsilon_\theta \ge 0,\ r>0 \ \ and \ \ 0\le\phi\le\phi\max \quad (36)$$

$$K_1=\max\left\{0,\ G_{r\theta},\ G_{r\phi}\right\}=G_{r\theta}+G_{r\phi} \quad (37)$$

$$K_2=\max\left\{\gamma_\theta,\ 0,\ 0\right\}=\gamma_\theta \quad (38)$$

$$K_3=\max\left\{\varepsilon_\theta,\ 0,\ 0\right\}=\varepsilon_\theta \quad (39)$$

Therefore,

$$\left|f(x,t,u_1,\theta_1,\phi_1)-f(x,t,u_2,\theta_2,\phi_2)\right|\le\ K_1\left(|u_1-u_2|+|\theta_1-\theta_2|+|\phi_1-\phi_2|\right) \quad (40)$$

$$\left|g(x,t,u_1,\theta_1,\phi_1)-g(x,t,u_2,\theta_2,\phi_2)\right|\le\ \gamma_\theta\left(|u_1-u_2|+|\theta_1-\theta_2|+|\phi_1-\phi_2|\right) \quad (41)$$

$$\left|h(x,t,u_1,\theta_1,\phi_1)-h(x,t,u_2,\theta_2,\phi_2)\right|\le\ \varepsilon_\phi\left(|u_1-u_2|+|\theta_1-\theta_2|+|\phi_1-\phi_2|\right) \quad (42)$$

Thus,

$$f(r,t,u,\theta,\phi),\ g(r,t,u,\theta,\phi)\ and\ h(r,t,u,\theta,\phi)$$

are Lipschitz continuous. This completes the proof.

Conclusions

In this study, we analytically establish the existence and uniqueness of solutions to the governing model equation for unsteady natural convection flow past an infinite cylinder. A key novelty of our work is the incorporation of temperature-dependent thermal conductivity and diffusion coefficient as well as the incorporation of the viscous dissipation energy explicitly to the energy equation, which to the best of our knowledge, has not been previously integrated in this manner. By explicitly accounting for these modifications, we provide a more comprehensive framework for modeling transient natural convection flows. Our findings offer a solid theoretical foundation for future numerical simulations and reinforce the well-posedness of the model, ensuring its ability to accurately capture the underlying physical phenomena under specified conditions and assumptions.

Author Contributions

UCU conceptualized the article and conducted the literature search. UCU and MOD developed and modified the model, while all authors critically revised the work. MOD and ROO jointly conceived and designed the study framework. UCU performed the problem-solving, with drafting the initial manuscript. MOD and ROO supervised the manuscript throughout the process. UCU, MOD, and ROO carried out the final review and revision of the paper.

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