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Full Length Research Article

OPTIMIZING PERISHABLE INVENTORY SYSTEMS: A DOUBLE-ORDER POLICY WITH WEIBULL-DISTRIBUTED DEMAND

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ABSTRACT

This study develops a new ordering policy for a Fixed Lifetime Inventory System (FLIS) by integrating the three-parameter Weibull distribution to model demand distribution. The proposed double-order inventory model addresses the challenges of perishability, demand uncertainty, and cost optimization by determining replenishment decisions based on the remaining useful lifetime of inventory items rather than stock levels A sensitivity analysis reveals that the Weibull scale parameter α shows an inverse relationship with total inventory cost in the double-order inventory system, while as the shape parameter β increased, it exhibits direction relationship with the cost components, though very minimal. On the other hand, smaller values of the location parameter τ caused a slight decrease to the cost components, while the Economic Order Quantity (EOQ) remain stable. Under the same demand expectation, the model demonstrates superior performance compared to the Poisson based double-order policy, by effectively reducing outdate cost by 40.5%. Practical implications highlight the importance of accurate demand estimation and dynamic inventory control systems for industries dealing with perishable goods. The study contributes to inventory management literature by bridging the gap between Weibull-distributed demand modeling and double-order policies, offering a robust framework for optimizing perishable inventory systems.

Keywords: Inventory, Perishable, Economic Order Quantity (EOQ), Fixed Lifetime Inventory System (FLIS)

INTRODUCTION

According to a survey report published by the Manufacturers Association of Nigeria (MAN) in 2024, the manufacturing sector's unsold goods inventory increased from ₦272 billion in the first half of 2024 to ₦1.24 trillion in the same period the previous year. This staggering increase in unsold inventory indicates inefficiencies in inventory management and reveals deeper economic challenges such as declining consumer purchasing power, inflationary pressures, and disruptions in supply chain (Okojie & Onyema, 2024). For manufacturing firms, unsold goods not only tie up critical working capital but also introduce risks associated with storage, deterioration, and outdate, especially with perishable or time-sensitive inventory.

A FLIS refers to inventory system that deals with items with a defined useful timeframe. Izevbizua and Apanapudor (2019) classified inventory into two types – Regular and Non-regular FLIS. Those dealing with perishable goods or items with fixed lifetime are referred to as regular inventory. Regular inventory systems are those where items must be sold, consumed, or used within a specific period due to deterioration or outdate. Common examples include perishable food items, blood inventory, pharmaceuticals, and certain high-technology goods. On the other hand, non-regular fixed lifetime inventory system deals with reusable items after their useful periods elapsed i.e. after they outdate. Examples include: Hotel rooms, seats on a conference hall, stadium, seats of an airplane, advert space of a daily newspaper.

The fixed lifetime inventory system has gained a lot of attention from researchers over the last decades. This is due to the inherent challenges of managing outdating. Izevbizua and Apanapudor (2018) observed that while effort has been made by several authors to minimize the number of items that outdates in an inventory system, shortages has become a problem to deal with. The author developed a model that involves two set of order y_1 and y_2 . Both orders have one period away from each other and the second order was intended to fill demand that the first order was not able to.

Research on fixed lifetime inventory management often begins with single-period models. Nahmias and Pierskalla (1973) tackled the problem of computing optimal ordering policies for a product with a fixed two-period lifetime under random demand conditions. They constructed a one-period model that adjusted for outdate and stockout costs. This model was subsequently generalized to incorporate ordering and holding costs, providing a more comprehensive approach to inventory optimization. Nahmias (1976) further developed a critical-number policy, which simplified decision-making by approximating optimal ordering thresholds. These contributions laid the groundwork for modeling and optimizing inventory systems with fixed lifetimes.

A large body of literature has been written on the control of inventory of improving and deteriorating objects. Whitin (1957) was the first to study items that were kept in the fashion industry for a certain amount of time. After that, Ghare and Schrade (1963) started the inventory study by creating a basic EOQ model with a fixed deterioration rate. Covert and Philip (1973) then expanded this model to include a variable deterioration rate. The deteriorating function was assumed to be a two-parameter Weibull distribution. The work was expanded by Misra (1975), who included a finite replacement rate. A variant form of the EOQ model is examined in Sarker *et al.* (1997), taking level dependence into account when deterioration is present.

In a very distinct study on an EOQ model, Samanta (2017) proposed a fuzzy inventory model with two parameters for decaying products that adhered to Weibull rule. The creation of a fuzzy-type inventory model is critically examined, and the corresponding linear demand rate function is subject to a fully backlogged two-parameter Weibull deterioration law. Gwanda (2019) created an EOQ model that took into account both improving and degrading products, where the holding costs and demand rate are exponentially growing and linear in time, respectively. An ordering policy that works well for improving retail items with unrestricted money and a steady demand rate was examined by Ahmad & Hudu (2019). The goal of the optimal control study was to determine the best replenishment cycle time while minimizing the total variable cost, or Tvc. Apanapudor and Olowu (2023) used two products with similar usage patterns and shelf lives to develop a model for the total cost function for a fixed lifetime inventory system. The number of outdates decreased as observed from the model's simulation. Izevbizua and Mukoro (2019) presented a model in which two orders enter the inventory system one period apart. In their paper, they emphasized that any demand where the first order is unable to meet is intended to be me by the second order. An economic order quantity (EOQ) model for two competitive items with stock-dependent inventory and an oscillating demand function was put forth by Mondal (2023). The model's related dynamical system was developed, its equilibrium points identified, and its stability characteristics examined.

More recently, Gupta et al. (2025) developed an Economic Production Quantity (EPQ) model that incorporates Weibull-distributed progressive degradation and dynamic holding costs under varying demand patterns. Their work highlights the utility of Maclaurin series approximations to solve differential equations. Kaliraman (2019) and Ibina *et al.* (2023) have expanded the scope of Weibull-based models by incorporating partial backlogging and inventory-dependent demand. Kaliraman's work focuses on a three-parameter Weibull deterioration model with time-dependent demand, while Ibina et al. introduce a modified three-parameter Weibull distribution to capture the effects of both obsolete and future deteriorating items. Their findings underscore the importance of stock-dependent demand in retail settings, where inventory levels influence consumer purchasing behavior. These models provide actionable insights for businesses aiming to balance replenishment strategies with deteriorating inventory.

The integration of time-varying demand and Weibull deterioration has also been a focal point. Ghosh and Chaudhuri (2004) and Singh et al. (2024) examined quadratic demand functions, which are particularly suited for seasonal or fashion products. Their models demonstrate how Weibull distribution parameters can be tailored to reflect real-world deterioration rates, offering a more accurate representation of inventory dynamics. Similarly, Mandal (2024) introduced stochastic demand into the framework, emphasizing the need for adaptive inventory policies in uncertain environments. These studies collectively highlight the versatility of Weibull distribution in addressing diverse market scenarios.

Recent works have also explored the interplay between pricing and inventory control in supply chains. Barman et al. (2023) proposed a bi-objective optimization model for a two-layer supply chain, incorporating Weibull deterioration and price-sensitive demand. Their application of the NSGA-II algorithm provides a novel approach to decentralized decision-making, offering practical solutions for collaborative pricing and inventory scheduling. This research bridges a critical gap in the literature by addressing the competitive dynamics of deteriorating inventory systems.

Moreover, Cholodowicz and Orlowski (2021) advanced the field by developing a hybrid discrete-time perishable inventory model using Weibull distribution. Their work, grounded in system dynamics, incorporates real-world data to validate the model's accuracy, particularly for perishable goods with random lifetimes. This study emphasizes the importance of issuing policies (FIFO, LIFO) and their impact on inventory performance, providing a comprehensive tool for managing perishable products under time-varying demand.

Expanding on these developments, Apanapudor and Olowu (2023) introduced a novel approach by modeling two products with similar usage patterns and shelf lives to derive a total cost function for fixed-lifetime inventory systems. Their simulations demonstrated a reduction in outdates, highlighting the model's effectiveness in minimizing waste for perishable goods. Similarly, in a significant contribution, Zubairu and Gwanda (2024) proposed an EOQ model incorporating Weibull amelioration, where inventory items improve in utility over time.

In summary, recent developments in Weibull-based inventory models have significantly enhanced the ability to address complex real-world challenges. From dynamic demand and partial backlogging to a supply chain coordination, stochastic environments, and ameliorating inventories, these models offer sophisticated tools for optimizing inventory decisions. The continued refinement of these approaches ensures their relevance across industries, paving the way for future research in this evolving domain.

The present study contributes to this body of literature by proposing a double-order inventory model that explicitly incorporates Weibull-distributed demand. The model offers a balanced approach to perishable inventory management by addressing both shortages and outdates, filling a gap in existing frameworks that often prioritize one aspect at the expense of the other. Furthermore, the emphasis on uncertainty of demand aligns with the recent calls for more dynamic and adaptable inventory systems, positioning this work at the forefront of contemporary research.

MODEL DESCRIPTION/ASSUMPTIONS

The proposed model is based on the following assumptions.

(1) Time is separated into discrete periods. The length of a period is arbitrary but constant.

(2) The placement of new order is dependent on the useful lifetime remaining on the inventory at hand. In this work, new orders are placed when there is only one useful period left on the inventory at hand or when demand depletes the inventory at hand.

(3) The arrival of a new order is instantaneous

(4) All items in the new order have the same age and their ages are assumed to start from age zero as they enter the inventory system. Table 3.1 shows the orders and age distribution for a product with m useful lifetime.

(5) The lead time is fixed.

(6) The cycle length begins from when the first order was received and ends when the second order enters the inventory system.

(7) Demand $D \geq 0$

(8) Demand in subsequent periods follows the three parameter Weibull distribution $(D) = 1 - e^{-\left(\frac{D-\tau}{\alpha}\right)^{\beta}}$.

(9) The issuing policy is "first in, first out" (FIFO).

(10) Units in the inventory system expire after period m

(11) Demand in each period is not known but assumed to be independent and identically distributed random variable variables $D_1, D_2, \ldots$ with known distribution.

Table 1: Order and distribution for a product with m lifetime.

$Order$	T_1	T_2	T_3	$\cdots$	T_{m-1}	T_m	T_{m+1}	T_{m+2}
1	$(V-d_1)^+$ $age\ 1$	$\left(V-\sum_{i=1}^{2} d_i\right)^+$ $age\ 2$	$\left(V-\sum_{i=1}^{3} d_i\right)^+$ $age\ 3$	$\cdots$	$\left(y-\sum_{i=1}^{m-1} d_i\right)^+$ $age\ m-1$	$U-d_m$ $age\ m$		
2					$(V-d_1)^+$ $age\ 1$	$\left(y-\sum_{i=1}^{2} d_i\right)^+$ $age\ 2$	$\cdots$	$\left(y-\sum_{i=1}^{m-1} d_i\right)^+$ $age\ m-1$
3								y Age 0

Notations:
(1) U = the amount of items in inventory that has one useful period left i.e., $U = V - \sum_{i=1}^{m-1} D_i$
(2) V = Amount of new items entering the inventory at age 0.
(3) m = the lifetime of inventory items, $m \in \mathbb{Z}^+$
(4) K = Constant ordering cost per unit
(5) H_c= holding cost per unit item
(6) S_c = shortage cost per unit item
(7) θ = outdate cost per unit item
(8) D = Demand in each period, $D \geq 0$.
(9) D_m = Demand at period m.

Derivation of Cost Components
Shortage Cost
Shortages occur when demand exceeds current inventory. In our model, we have assumed that excess demand is lost and is charged as a cost against the inventory manager.
The expected shortage per order is given by

$$Expected\ shortage\ cost = \int_{u+v}^{\infty} [D - (U+V)] f(D) dD \quad (1)$$

with S_c being the shortage cost per unit item, we have the shortage cost as

$$Shortage\ cost = S_c \int_{u+v}^{\infty} [D - (U+V)] f(D) dD \quad (2)$$

Outdate Cost
Outdate period refers to the time in which current inventory items are no longer useful and cannot not satisfies demand. Outdate usually occur when demand at the last useful period of items is less than the number of items in stock.
The expected outdate cost is given as

$$Expected\ Outdate = \int_0^U (U-D) f(D) dD \quad (3)$$

With θ being the outdate cost per unit, the outdate cost is given as

$$Outdate\ Cost = \theta \int_0^U (U-D) f(D) dD \quad (4)$$

Holding cost
The holding cost consists of the cost incurred in holding the items from order 1 in their last useful period m and the new items from order 2 in their first period in stock.
With $H_c > 0$ being the holding cost per unit item, the holding cost is given as

$$Holding\ Cost = H_c \int_0^{U+V} (U+V-D) f(D) dD \quad (5)$$

Ordering cost
ForK_c being the ordering cost per unit item, the ordering cost is given as

$$Ordering\ Cost = K_c \times V \quad (6)$$

For $f(D) = 1 - e^{-\left(\frac{D-\tau}{\alpha}\right)^\beta}$,

$$TC(U,V) = K_c V + H_c \int_0^{U+V} (U+V-D)\left(1 - e^{-\left(\frac{D-\tau}{\alpha}\right)^\beta}\right) dD + S_c \int_{U+V}^{\infty} [D-(U+V)]\left(1 - e^{-\left(\frac{D-\tau}{\alpha}\right)^\beta}\right) dD + \theta \int_0^U (U-D)\left(1 - e^{-\left(\frac{D-\tau}{\alpha}\right)^\beta}\right) dD \quad (7)$$

where $D \geq \tau, \alpha > 0, \beta > 0\ and\ \tau \geq 0$. Let

$$I_1 = \int_0^{U+V} (U+V-D)\left(1 - e^{-\left(\frac{D-\tau}{\alpha}\right)^\beta}\right)$$

$$I_2 = \int_{U+V}^{\infty} [D-(U+V)]\left(1 - e^{-\left(\frac{D-\tau}{\alpha}\right)^\beta}\right) dD \text{, and}$$

$$I_3 = \int_0^U (U-D)\left(1 - e^{-\left(\frac{D-\tau}{\alpha}\right)^\beta}\right) dD$$

Thus, The total inventory cost

$$TC(U,V) = K_c V + H_c I_1 + S_c I_2 + \theta I_3 \quad (8)$$

$EOQ\ (V^*)$ is obtain by minimizing (7) with respect to V.

Partial Backlogging
Suppose a fraction γ (where $0 < \gamma < 1$) of unmet demand is backlogged and fulfilled in future periods with backlogging cost $B_c$per unit. Let remaining fraction $1-\gamma$ of unmet demand is lost with a lost sales cost L_c per unit. Then, to account for partial

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backlogging, we modify the shortage cost in (2). For backlogging cost, we have

$$B_c\gamma \int_{U+V}^{\infty} [D-(U+V)]\left(1-e^{-\left(\frac{D-\tau}{\alpha}\right)^{\beta}}\right)dD \qquad (9)$$

and for lost sales cost, we have

$$L_c(1-\gamma)\int_{U+V}^{\infty} [D-(U+V)]\left(1-e^{-\left(\frac{D-\tau}{\alpha}\right)^{\beta}}\right)dD \qquad (10)$$

Thus, the modified total cost function $TC_{partial}(U,V)$ becomes:

$$TC_{partial}(U,V) = K_cV + H_c\int_0^{U+V}(U+V-D)\left(1-e^{-\left(\frac{D-\tau}{\alpha}\right)^{\beta}}\right)dD + (B_c\gamma + L_c(1-\gamma))\int_{U+V}^{\infty}[D-(U+V)]\left(1-e^{-\left(\frac{D-\tau}{\alpha}\right)^{\beta}}\right)dD + \theta\int_0^U (U-D)\left(1-e^{-\left(\frac{D-\tau}{\alpha}\right)^{\beta}}\right)dD \qquad (11)$$

Let $S_c' = B_c\gamma + L_c(1-\gamma)$. Then we have

$$TC_{partial}(U,V) = K_cV + H_c\int_0^{U+V}(U+V-D)\left(1-e^{-\left(\frac{D-\tau}{\alpha}\right)^{\beta}}\right)dD + S_c'\int_{U+V}^{\infty}[D-(U+V)]\left(1-e^{-\left(\frac{D-\tau}{\alpha}\right)^{\beta}}\right)dD + \theta\int_0^U (U-D)\left(1-e^{-\left(\frac{D-\tau}{\alpha}\right)^{\beta}}\right)dD \qquad (12)$$

NUMERICAL INVESTIGATION

In this section, we simulate the Weibull double order inventory model.

Taking $K_c = ₦250$ per unit, $H_c = ₦0.15$ per unit $S_c = ₦5$ per unit, $\theta = ₦5$ per unit $U = 50$ units and initial Weibull parameters to be $\omega = \{1,1,0\}$, we obtain the following results;

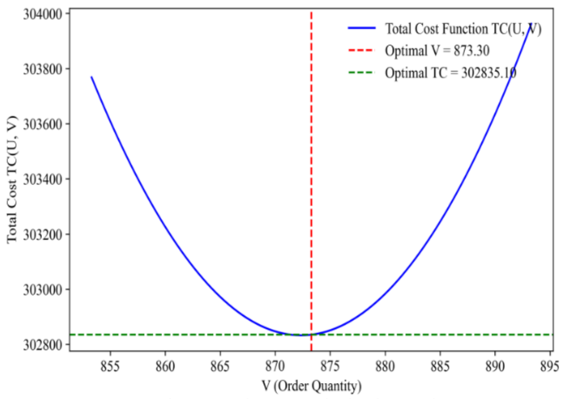

Figure 1: Total Cost vs Optimal Order Quantity

$V = 873.3, K_cV = 250 \times 873.3 = 218{,}325$

$H_cI_1 = ₦63{,}936$, $S_cI_2 = ₦14{,}707$ and $\theta I_3 = ₦6{,}005$

Thus,

$$TC(U,V) = ₦218{,}325 + ₦63{,}936 + ₦14{,}707 + ₦6{,}005 = ₦302{,}973$$

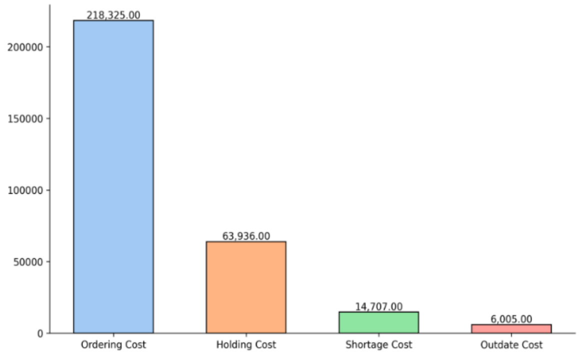

Figure 2: Contribution of Cost Components to Total Cost

Let 70% of unmet demand be backlogged with a cost of N1.5 per unit, then $S_c'I_2 = 7{,}501$ and the total cost function with backlogging is

$$TC_{partial}(50, 873.3) = ₦218{,}325 + ₦63{,}936 + ₦7{,}501 + ₦6{,}005 = ₦295{,}767$$

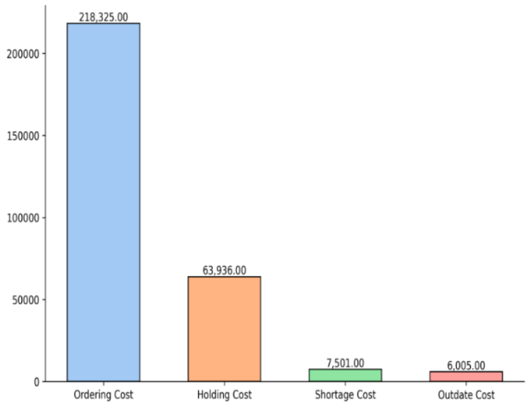

Figure 3: Contribution of Cost Components to Total Cost

Sensitivity Analysis

Table 2 : Variation in scale parameter α. $\beta = 1$ and $\tau = 0$ are fixed.

α	β	τ	EOQ (V)	Ordering Cost	Holding Cost	Shortage Cost	Outdate Cost	Total Cost
0.01	1	0	873.2718	218318	63932	14718	6248	303216
0.05	1	0	873.2718	218318	63932	14718	6238	303206
0.1	1	0	873.2748	218319	63933	14717	6225	303193
0.25	1	0	873.2791	218320	63899	14715	6188	303122
0.5	1	0	873.2864	218322	63865	14712	6126	303025
0.75	1	0	873.2937	218323	63832	14710	6065	302930
0.9	1	0	873.2981	218325	63811	14708	6029	302873
1	1	0	873.301	218325	63798	14707	6005	302835
1.1	1	0	873.3039	218326	63785	14706	5981	302797
1.25	1	0	873.3083	218327	63764	14704	5945	302741
1.5	1	0	873.3155	218329	63731	14701	5886	302647
1.75	1	0	873.3228	218331	63697	14698	5828	302554
2	1	0	873.3301	218333	63664	14696	5770	302462
5	1	0	873.4175	218354	63264	14662	5125	301405
10	1	0	873.5631	218391	62602	14606	4247	299846

Table 3: Variation in shape parameter β. $\alpha = 1$ and $\tau = 0$ are fixed.

α	β	τ	EOQ (V)	Ordering Cost	Holding Cost	Shortage Cost	Outdate Cost	Total Cost
1	0.01	0	848.1183	212030	39420	17059	4006	272515
1	0.05	0	858.2175	214554	45066	15920	4227	279767
1	0.1	0	866.511	216628	51442	15043	4497	287610
1	0.25	0	873.5437	218386	61892	14559	5200	300037
1	0.5	0	873.3301	218333	63665	14696	5809	302502
1	0.75	0	873.3065	218327	63773	14705	5962	302766
1	0.9	0	873.3025	218326	63791	14706	5993	302816
1	1	0	873.301	218325	63798	14707	6005	302835
1	1.1	0	873.2999	218325	63803	14707	6013	302848
1	1.25	0	873.299	218325	63807	14708	6021	302860
1	1.5	0	873.2981	218325	63811	14708	6027	302871
1	1.75	0	873.2978	218324	63813	14708	6030	302875
1	2	0	873.2977	218324	63813	14708	6031	302877
1	5	0	873.2718	218318	63932	14718	6023	302991
1	10	0	873.2718	218318	63932	14718	6014	302983

Table 4: Variation in location parameter τ. $\alpha = 1$ and $\beta = 1$ are fixed

α	β	τ	EOQ (V)	Ordering Cost	Holding Cost	Shortage Cost	Outdate Cost	Total Cost
1	1	0	873.301	218325	63798	14707	6005	302835
1	1	0.01	873.3013	218325	63797	14707	6003	302831
1	1	0.05	873.3025	218326	63791	14706	5992	302815
1	1	0.1	873.304	218326	63784	14706	5979	302795
1	1	0.25	873.3092	218327	63760	14704	5935	302726
1	1	0.5	873.3199	218330	63711	14700	5846	302587
1	1	0.75	873.3335	218333	63648	14694	5731	302407
1	1	0.9	873.3435	218336	63602	14691	5647	302276
1	1	1	873.351	218338	63567	14688	5584	302177
1	1	1.1	873.3593	218340	63529	14684	5514	302067
1	1	1.25	873.3735	218343	63463	14679	5395	301881
1	1	1.5	873.4024	218351	63330	14668	5152	301501
1	1	1.75	873.4395	218360	63159	14654	4840	301013
1	1	2	873.4871	218372	62940	14636	4440	300387
1	1	5	877.5946	219399	43905	13106	-30111	246298
1	1	10	1514.819	378705	-4983163	797552	-5390234	-9197140
1	1	15	873.2718	218318	-4.5E+08	14718	-8E+08	-1.3E+09
1	1	20	873.2718	218318	-6.7E+10	14718	-1.2E+11	-1.9E+11
1	1	25	873.2718	218318	-1E+13	14718	-1.8E+13	-2.8E+13
1	1	30	873.2719	218318	-1.5E+15	14718	-2.6E+15	-4.1E+15

Comparative Analysis with Poison-Based Model

To validate the effectiveness of the proposed Weibull double-order inventory model, a numerical comparison was conducted against an existing Poisson-based double-order FLIS model (Izevbizua & Apanapudor, 2018).

Here, $f(D) = 1 - e^{-\lambda D}$ is the Poisson cumulative density function (cdf) for the demand D. λ is the exponential rate parameter given by $\lambda = \frac{1}{\mu}$. The parameter μ is the Weibull mean with the relation $\mu = \tau + \alpha\Gamma\left(1 + \frac{1}{\beta}\right)$. The model's optimal order quantity $V^*_{poisson}$ is derived by solving its first-order condition numerically, using the Newton-Raphson method with the same convergence criteria as the Weibull model in (7). Table 3 summarizes their comparison for large values of the Weibull parameters ($\alpha = 15, \beta = 15$ and $\tau = 10$).

Table 5: Comparative of the Weibull and Poisson double order model

Costs	Weibull Model	Poisson Model
EOQ (V)	873.985 unit	873.985 unit
Ordering cost	₦218,496	₦218,496
Holding cost	₦60,682	₦60,727
Shortage cost	₦14,446	₦14,446
Outdate cost	₦1,630	₦2,737
Total cost (No backlogging)	₦295,255	₦296,407

DISCUSSION

Here, we discuss the result obtain from the sensitivity analysis conducted on the Weibull parameters and the comparison with the Poisson based model as contained in Tables 2, 3, 4 and 5. We observed that

(i) EOQ, ordering cost, holding cost and shortage cost have nearly constant values across all values of α. However, there is a consistent inverse relationship between α with total inventory costs which driven by heightened outdate risks due to demand clustering near the inventory expiration period.

(ii) as β increases from 0.01 to 0.5, the EOQ (V) rises steadily. This reflects the model's adaptation to increasing demand rates. Beyond $\beta = 0.5$, EOQ stabilizes around 873 units, indicating a saturation point in the system's

(iii) as τ increases from 0 to 2, EOQ increases slightly and total cost decreases graduallyfrom ₦302,835 to ₦300,387. However, beyond $\tau = 2$, the model begins to behave erratically. This indicates a breakdown in the model's validity when τ is too large.

(iv) comparative analysis with the Poisson-based FLIS model validated the Weibull framework's superiority in perishable inventory management. Under identical demand expectations, the Weibull model reduced outdate costs by 40.5% (₦1,630 vs. ₦2,737) while maintaining comparable shortage and holding costs.

Conclusion

The research successfully developed a double-order inventory model for FLIS with Weibull-distributed demand. The findings highlight the importance of accurately estimating demand parameters, particularly the scale and shape parameter, to minimize costs in perishable inventory systems. The model's ability to balance shortages and outdates while adapting to partial backlogging scenarios underscores its practical applicability. By leveraging the flexibility of the Weibull distribution, the study provides a more realistic framework for inventory decision-making compared to traditional static or simplified demand models. The results affirm that demand variability significantly influences inventory performance, and effective management requires dynamic, data-driven policies.

REFERENCES

Ahmad Y. and Hudu D., (2019). Ordering policy for ameliorating items Inventory with linear demand rate and unconstrained. Abacus Mathematics Science Series, 44:25-29.

Apanapudor J.S. Olowu O.O. (2023). The fixed lifetime inventory with two products having the same shelf life. Internation Journal of Statistics and Applied Mathematics. 8(5):237-240.

Barman, A., Chakraborty, A.K., Goswami, A., Banerjee, P. & De, P.K. (2023). Pricing and inventory decision in a two-layer supply chain under the weibull distribution product deterioration: an application of NSGA-II. RAIRO Operations Research, 57, 2279-2300.

Cholodowicz, E. & Orlowski, P. (2021). Development of new hybrid discrete-time perishable inventory model based on Weibull distribution with time-varying demand using system dynamics approach. Computers & Industrial Engineering, 154, 1-13.

Covert R. P. & Philip G. C. (1973). An EOQ Model for Deteriorating Items with Weibull Distributions Deterioration. AIIE Transactions, 5(4), 323-332. doi:10.1080/05695557308974918.

Ghare, P.M. and Schrader, G.F. (1963) A model for an exponential decaying inventory.Journal of industrial Engineering, 14, 238-243.

Ghosh, S. K. & Chaudhuri, K.S. (2004). An Order-Level Inventory Model For A Deteriorating Item With Weibull Distribution Deterioration, Time-Quadratic Demand And Shortages. Advanced Modeling and Optimization, 6(1), 21-35.

Gupta, G.K., Nand, A., Chauhan, N.S. & Shivanand. (2025). An EPQ inventory model with Weibull distribution, dynamic time-dependent holding cost under various demand pattern using Maclaurin series approximations. Discover Applied Sciences. 7, 117 -140.

Gwanda, Y. I., Tahaa. A, Bichi A.B. & Ado. L, (2019), EOQ model for both Ameliorating and Deteriorating items with exponential increasing Demand and linear time dependent holding cost, Global Scientific Journal, 7: 2320-9186

Ibina, E. O., Osisiogu, U. A. & Okoro, C. N. (2023). Optimal Supply Policy For Items With Modified Three-Parameter Weibull Deterioration, Inventory-Dependent Demand, And Partial Backlogging. IOSR Journal of Mathematics, 19(4), 8-18.

Izevbizua O. & Apanapudor J. S. (2019). Total revenue function for non-regular fixed lifetime inventory system. African Journal of Mathematics and Computer Science Research. 12(2), 24 – 30.

Izevbizua. O and Apanapudor S.J. (2018). Fixed Lifetime Inventory System With Two Orders And Stochastic Demand. ABACUS (Mathematics Series), 45(1), 66-72.

Kaliraman N. K. (2019). Inventory Model for Three-parameter Weibull Deterioration and Partial Backlogging. Journal of Advances in Mathematics and Computer Science, 33(5), 1-9.

Mandal, B. (2024). An Inventory Model For Weibull Distributed Deteriorating Items With Stochastic Demand And Time Varying Holding Cost. IOSR Journal of Mathematics, 20(6), 50-57.

Misra, R.B., (1975), optimal production lot-size model for a system with Deteriorating inventory. International Journal of Production Research 13:495-505.

Mondal S. (2023). Impact of periodically varying demand on an generalized EOQ model. Journal of the Calcutta Mathematical Society, 19(1), 85-102

Nahmias, S. (1976). Myopic approximations for the perishable inventory problem. Management Science, 22(9): 1002-1008

Nahmias, S. and Pierskalla W.P. (1973). Optimal ordering policies for a product that perishes in two periods subject to stochastic demand. Naval Research logistic quarterly, 120(2): 207-209

Okojie J. & Onyema J. (2024). Rising unsold inventory weighs on Nigerian manufacturers. Business Day. https://businessday.ng/business-economy/article/rising-unsold-inventory-weighs-on-nigerian-manufacturers/

Samanta, P.N., Das J.P. and Indrajitsingha, S.K. Fuzzy Inventory Model for Two Parameter Weibull deteriorating Items. Transactions on Mathematics, 3(2), 27-36.

Sarker, B.R., Mukhaerjee, S., and Balam, C.V. (1997) an order level lot size inventory model with inventory level dependent demand and deterioration. International Journal of production Economic, 48, 227-236.

Singh, T., Rout, I. & Nayak, A.N. (2024). A Note on a One-Parameter Weibull Distributed Deteriorating Item EOQ Inventory Model with Varying Quadratic Demand and Delay in Payments. American Journal of Operations Research, 14, 151-167.

Whitin, T.M. (1957). Theory of Inventory Management. Princeton University Press, Princeton, N.J., 62-72.

Zubairu T.J. & Gwanda Y.I. (2024). “Economic Order Quantity (EOQ) Model: Weibull Ameliorating Items with Constant Demand Rate”. FUDMA Journal of Sciences. 8(5), 344-349. DOI: https://doi.org/10.33003/fjs-2024-0805-2726