

# ANALYSIS OF THE EFFECTS OF MASS AND THERMAL STRATIFICATION ON NATURAL CONVECTION FLOW PAST A CYLINDER WITH VARIABLE THERMAL CONDUCTIVITY AND DIFFUSION COEFFICIENT

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## ABSTRACT

This paper presents a comprehensive analysis of the effects of mass and thermal stratification by providing analytical solution for the unsteady, one-dimensional natural convection flow past an infinite vertical circular cylinder in a stably thermally stratified fluid medium, incorporating variable thermal conductivity and diffusion coefficients. The dimensionless coupled linear partial differential equations (PDE) governing the flow are solved using Olayiwola's Generalized Polynomial Approximation Method (OGPAM) for different sets of physical parameters. The effects of these parameters are illustrated and discussed with the aid of graphs. The results show that mass and thermal stratification significantly influence the velocity, temperature, and concentration profiles of the fluid. Specifically, an increase in thermal stratification leads to a rise in the system's temperature as Increased thermal stratification traps heat in the upper layers, reduces convective mixing, and suppresses cooling. Furthermore, as thermal diffusivity increases, fluid velocity decreases across the radial distance due to reduced buoyancy forces, producing a flatter and less pronounced velocity profile this is due to the fact that heat spreads faster, temperature gradients shrink, buoyancy forces weaken, and fluid motion slows down. This creates a lower and flatter velocity profile across the radial distance. Similarly, increase in mass stratification generates a concentration gradient which traps solute in dense layers, reduces mixing and dilution, and resists vertical transport. This causes local buildup and a rise in the overall concentration of the system.

**Keywords:** Cylindrical plate, Diffusion coefficient, Fluid Flow, Mass Stratification, Natural convection, Stable, Thermal stratification, Thermal conductivity, Thermal diffusivity.

## INTRODUCTION

Natural convection flows with heat and mass stratification are frequently encountered in nature. These types of problem over vertical cylinder have wide range of applications in the field of science and technology such as startup of chemical reactors and emergency cooling of nuclear fuel elements. The effect of stratification is important in several heat rejection process and energy storage systems. The effect of stratification is important in several heat rejection process and energy storage systems. Olayemi et al., (2023) investigated the influences heat and mass transfer in stratified flow conditions The study of mixed convective fluid flow around cylinders of different cross-sectional shapes is of great importance, as it offers valuable insights into how geometry.

Ugwu *et al* (2021) studied the effects of MHD flow on convective fluids incorporating viscous dissipation energy though it was a Newtonian fluid. This problem was analysed numerically using method of lines and various fluid parameter and that of the particles were obtained.

Recently, Deka et al., (2019) presented the analytical investigation of one-dimensional unsteady natural convection flow past an infinite vertical cylinder with heat and mass transfer under the effect of constant heat flux at the surface of the cylinder. They have shown that the velocity

and temperature increase unboundedly with time, while the concentration approaches steady state at larger times. Ugwu *et al.* (2022) investigated the unsteady free convective flow of a viscous incompressible electrically conducting fluid over an infinite vertical porous plate under the influence of uniform transverse magnetic field with time dependent permeability and oscillatory suction embedded with viscous energy dissipation using numerical approach (MOL). Muthucumaraswamy et. al., (2013) investigated the influence of a chemical reaction on the behavior of an unsteady flow through an accelerating vertical plate, where the mass transfer was variable and without considering stratification. The purpose of this research is to determine how fluid flow past an accelerated vertical plate impacts the interaction between thermal stratification and chemical reaction. Deka and Neog (2019) studied the unsteady flow of a thermally stratified fluid past a vertically accelerated plate under a variety of conditions. Ugwu et al., (2025) investigated the existence and uniqueness solution of transient natural convection flow past a cylinder model with viscous dissipation, mass and thermal stratification, their findings offer a solid theoretical foundation for future numerical simulations and reinforce the well-posedness of the model, ensuring its ability to accurately capture the underlying physical phenomena under specified conditions and assumptions. Megahed and Abbas, (2022) researched the consequences of non-Newtonian fluid flow over a porous medium on both effects. They also considered how buoyancy-driven flows in a stratified fluid were examined and came up with an analytical solution to describe how fluid would flow past an infinite vertical plate. Ugwu *et al* (2021) studied the MHD effects on convective flow of dusty viscous fluid. The problem was solved numerically under the influence of magnetic field.

This research is significant because it models a realistic scenario where temperature- and concentration-induced stratification influence buoyancy-driven flows around cylindrical structures. It will improve predictions, enhance thermal system design, and contribute new insights to the scientific community.

## MATERIALS AND METHODS

Consider an unsteady, laminar, and incompressible viscous flow past an infinite vertical cylinder of radius  $r_0$  with constant temperature and concentration in presence of thermal and mass stratification.

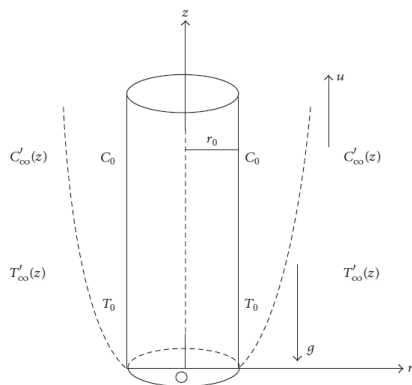


Figure 1 Sketch of the Geometry

The  $z$ -axis of the cylinder is taken vertically upward along the axis of the cylinder and the radial coordinate  $r$  is taken normal to the cylinder as shown in Figure 1. The physical model and coordinate system of the flow problem is shown in Figure 1. Upon commencement of the transient, we consider the fluid moving up from the leading edge ( $z = 0$ ) parallel to the cylinder as a wave, in front of which the velocity, temperature, and concentration are only functions of the time and the radial distance  $r$  from the cylinder. Behind the wave there must be a dependence on the vertical coordinate,  $z$ . The basic premise in this work is that convective effects will begin at a position,  $z$ , as soon as fluid which was initially located at the leading edge rises to this position, regardless of the distance,  $r$ , away from the cylinder at which it first arrives. Since the surface temperature and concentration above the leading edge are uniform with  $z$ , the temperature of the fluid and concentration may be assumed to be independent of  $z$ . In addition, the vertical velocity,  $u$ , must be independent of  $z$  and from the continuity equation, the velocity normal to the plate is seen to be zero, except that the temperature and concentration of the ambient fluid are function of the vertical distance  $z$  only. At time  $t' > 0$ , the uniform temperature

( $T_0$ ) and concentration ( $C_0$ ) are specified at the surface of the cylinder. Viscous dissipation terms have been neglected. All derivatives in the direction parallel to the cylinder are zero, except  $\frac{\partial T'_\infty}{\partial z}$  and  $\frac{\partial C'_\infty}{\partial z}$  termed as thermal stratification and mass

stratification, respectively. Here,  $T'_\infty$  and  $C'_\infty$  are the temperature and concentration of the undisturbed fluid. It is to be noted that initially the fluid may not be stratified, but upon commencement of the transient the fluid gets self-stratifications.

Following Boussinesq's approximation, the one-dimensional equations for continuity, momentum, energy, and concentration are as follows:

$$\frac{1}{r'} \frac{\partial}{\partial r'} (r' u') = 0 \quad (1)$$

$$\frac{\partial C'}{\partial t'} = \frac{1}{r'} \frac{\partial}{\partial r'} \left( D_T r' \frac{\partial C'}{\partial r'} \right) - \xi u' \quad (2)$$

$$\rho \frac{\partial u'}{\partial t'} = \frac{\mu}{r'} \frac{\partial}{\partial r'} \left( r' \frac{\partial u'}{\partial r'} \right) + \rho g \beta (T' - T'_R) + \rho g \beta^* (C' - C'_R) \quad (3)$$

$$\rho c_p \frac{\partial T'}{\partial t'} = \frac{k}{r'} \frac{\partial}{\partial r'} \left( K_T r' \frac{\partial T'}{\partial r'} \right) + \mu \left( \frac{\partial u'}{\partial r'} \right)^2 - \gamma u' \quad (4)$$

The temperature dependent thermal conductivity and diffusion coefficient is in the form:

$$K_T = K_0 \left( \frac{T'}{T_0} \right) \quad (5)$$

$$D_T = D_0 \left( \frac{T'}{T_0} \right) \quad (6)$$

The initial and boundary conditions are formulated as:

$$\left. \begin{aligned} u'(r', 0) &= U_0, & \frac{\partial u'}{\partial r'} \Big|_{r'=0} &= \xi_0, & u'(R, t') &= 0 \\ T'(r', 0) &= T'_R, & -k^* \frac{\partial T'}{\partial r'} \Big|_{r'=0} &= h_f (T(0, t') - T'_R), & T'(R, t') &= T'_R \\ C'(r', 0) &= C'_R, & -D^* \frac{\partial C'}{\partial r'} \Big|_{r'=0} &= h_m (C(0, t') - C'_R), & C'(R, t') &= C'_R \end{aligned} \right\} \quad (7)$$

where

$\gamma$  and  $\xi$  termed as thermal stratification and mass stratification, respectively.

The model formulated would be non-dimensionalized using appropriate dimensionless variables.

where,

$\rho$  = fluid density

$u'$  = fluid velocity

$t'$  = time

$\mu$  = kinematic viscosity

$r'$  = radial coordinate

$g$  = acceleration due to gravity

$\beta$  = volumetric coefficient of thermal expansion,

$T'$  = Temperature of the fluid

$T'_R$  = starting temperature at the plate

$\beta^*$  = volumetric coefficient of expansion with concentration,

$C'$  = Concentration of the fluid

$C'_R$  = fluid concentration at the plate

$c_p$  = Specific heat Capacity

$k$  = thermal diffusivity

$K_T$  = thermal conductivity

$\gamma$  = thermal stratification parameter

$D_T$  = diffusion coefficient

$\xi$  = mass stratification

$U_0$  = velocity at the initial point.

$\xi_0$  = constant

$D^*$  = diffusion coefficient at the solid phase

$k^*$  = thermal conductivity at the solid phase

$h_f$  = the convective heat transfer coefficient at the fluid phase

$h_m$  = mass transfer coefficient at the fluid phase

### Dimensionless analysis

Dimensionless variables are been introduced as

$$t = \frac{U_0 t'}{R}, \quad r = \frac{r'}{R}, \quad u = \frac{u'}{U_0},$$

$$\phi = \frac{C' - C_0}{C'_R - C_0}, \quad \theta = \frac{T' - T_0}{T'_0 - T_0} \quad (8)$$

Using (2), on (2) – (4) and (7) gives the dimensionless form of the model equations (9) – (12)

$$\frac{\partial \phi}{\partial t} = \frac{1}{P_{em}} \frac{1}{r} \frac{\partial}{\partial r} \left( r(1 + \alpha\theta) \frac{\partial \phi}{\partial r} \right) - \varepsilon_\phi u \quad (9)$$

$$\frac{\partial u}{\partial t} = \frac{1}{Re} \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial u}{\partial r} \right) - G_{r\theta} (1 - \theta) - G_{r\phi} (1 - \phi) \quad (10)$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{Pe} \frac{1}{r} \frac{\partial}{\partial r} \left( r(1 + \alpha\theta) \frac{\partial \theta}{\partial r} \right) + \frac{Ec}{Re} \left( \frac{\partial u}{\partial r} \right)^2 - \gamma_\theta u \quad (11)$$

Subject to,

$$\begin{aligned} u(r, 0) = 1, \quad \frac{\partial u}{\partial r} \Big|_{r=0} &= \xi, \quad u(1, t) = 0 \\ \theta(r, 0) = 1, \quad \frac{\partial \theta}{\partial r} \Big|_{r=0} &= -Nu\theta + Nu \quad \theta(1, t) = 1 \\ \phi(r, 0) = 1, \quad \frac{\partial \phi}{\partial r} \Big|_{r=0} &= -Sh_m\phi + Sh_m \quad \phi(1, t) = 1 \end{aligned} \quad (12)$$

Where,

$Re$  = Reynold number,

$G_{r\theta}$  = Thermal Grashoff number,

$G_{r\phi}$  = Mass Grashoff number

$Pe$  = Peclet Energy;

$\gamma_\theta$  = Thermal Stratification;

$Ec$  = Eckert number

$\alpha$  = thermal diffusivity

$P_{em}$  = Peclet energy mass;

$\varepsilon_\phi$  = Mass Stratification

$Sh_m$  = Sherwood number

### Method of Solution

Let  $0 < \alpha < 1$  and  $G_{r\theta} = \alpha a^*$ ,  $G_{r\phi} = \alpha b^*$  such that we

can express the solutions  $u(r, t)$ ,  $\theta(r, t)$  and  $\phi(r, t)$  as

$$\phi(r, t) = \phi_0(r, t) + \alpha\phi_1(r, t) + \dots \quad (13)$$

$$u(r, t) = u_0(r, t) + \alpha u_1(r, t) + \dots \quad (14)$$

$$\theta(r, t) = \theta_0(r, t) + \alpha\theta_1(r, t) + \dots \quad (15)$$

Thus we proceed to obtain the solutions to order 0 and 1 equations. Hence, we have that

$$u_0(r, t) = \zeta r - \zeta r^2 + e^{-A_0 t} (1 - r^2) \quad (16)$$

$$\theta_0(r, t) = \theta_0 \Big|_{r=0} + (Nu - Nu\theta_0 \Big|_{r=0}) r + ((Nu - 1)\theta_0 \Big|_{r=0} + 1 - Nu) r^2 \quad (17)$$

$$\phi_0(r, t) = \phi_0 \Big|_{r=0} + (Sh_m - Sh_m\phi_0 \Big|_{r=0}) r + ((Sh_m - 1)\phi_0 \Big|_{r=0} + 1 - Sh_m) r^2 \quad (18)$$

$$u_1(r, t) = u_1 \Big|_{r=0} - u_1 \Big|_{r=0} r^2 \quad (19)$$

$$\theta_1(r, t) = \theta_1 \Big|_{r=0} - Nu\theta_1 \Big|_{r=0} r + (Nu - 1)\theta_1 \Big|_{r=0} r^2 \quad (20)$$

$$\phi_1(r, t) = \phi_1 \Big|_{r=0} - Sh_m\phi_1 \Big|_{r=0} r + (Sh_m - 1)\phi_1 \Big|_{r=0} r^2 \quad (21)$$

Where,

$$u_0 \Big|_{r=0} = e^{-A_0 t} \quad (22)$$

$$\theta_0 \Big|_{r=0} = M_{11} - M_{15} e^{-A_1 t} - M_{12} e^{-A_0 t} - M_{13} e^{-2A_0 t} \quad (23)$$

$$\phi_0 \Big|_{r=0} = M_{20} + M_{21} e^{-A_0 t} + M_{23} e^{-A_0 t} \quad (24)$$

$$u_1 \Big|_{r=0} = M_{32} + M_{33} e^{-A_1 t} + M_{34} t e^{-A_1 t} - M_{35} e^{-2A_1 t} + M_{36} e^{-A_0 t} - M_{37} e^{-A_1 t} \quad (25)$$

$$\begin{aligned} \theta_1 \Big|_{r=0} &= M_{67} + M_{81} e^{-A_1 t} + M_{55} t e^{-A_1 t} - M_{82} e^{-A_0 t} + M_{69} t e^{-A_1 t} + M_{83} e^{-2A_1 t} \\ &\quad - M_{72} t e^{-2A_0 t} - M_{74} e^{-2A_1 t} - M_{75} e^{-4A_0 t} + M_{76} e^{-3A_0 t} - M_{77} e^{-A_0 t} \\ &\quad - M_{78} e^{-A_0 t} + M_{79} e^{-A_1 t} - M_{80} e^{-A_2 t} \end{aligned} \quad (26)$$

$$\begin{aligned} \phi_1 \Big|_{r=0} &= M_{118} + M_{110} t e^{-A_0 t} + M_{129} e^{-A_0 t} - M_{117} t e^{-A_0 t} \\ &\quad - M_{119} e^{-A_1 t} + M_{121} e^{-2A_0 t} + M_{122} e^{-A_0 t} + M_{123} e^{-A_1 t} \\ &\quad + M_{124} e^{-A_1 t} + M_{125} e^{-3A_0 t} + M_{126} e^{-A_0 t} + M_{128} e^{-A_0 t} \end{aligned} \quad (27)$$

Therefore, substituting equation (22) – (27) in equations (16) – (21), we obtain the values of equation (13) – (15) which is further substituted into equation (9) – (12). We obtain the following results and upon

successful simplification and computation gives the semi-analytical solution to the dimensionless governing model equation.

## RESULTS AND DISCUSSION

The resulting semi-analytical solutions, presented in equations (9) – (12), were computed using MAPLE 2021. The parameter values and fixed dimensionless constants taken into consideration in this work are listed in Table 1 below: where  $\alpha = 1.0$ ,  $Pe = 20$ ,  $P_{em} = 4$ ,  $\Re = 7$ ,  $Ec = 1$ ,  $G_{r\theta} = 1$ ,  $G_{r\phi} = 1$ ,  $N = 1$ ,  $Sh_m = 0.1$ ,  $\xi = 1$ ,  $\varepsilon_\phi = 1$ ,  $\gamma_\theta = -0.2$ ,  $r = 0$

Parameter	Values
Thermal Stratification	0.2, 0.3, 0.4, 0.5, 0.6
Mass Stratification	1.0, 1.5, 2.0, 2.5, 3.0
Thermal diffusivity	1.0, 2.0, 3.0, 4.0, 5.0

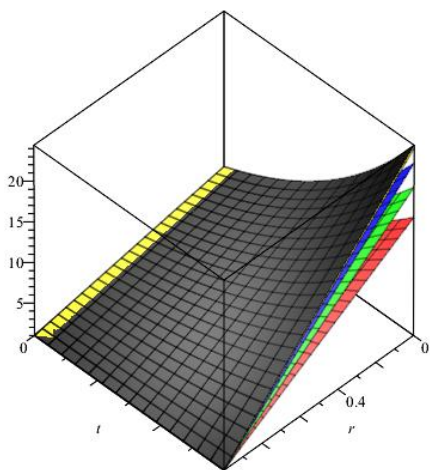


Figure 1: Effect of thermal stratification on temperature

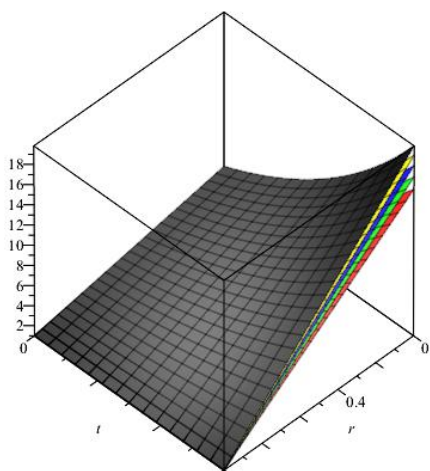


Figure 2: Effect of Mass Stratification on temperature

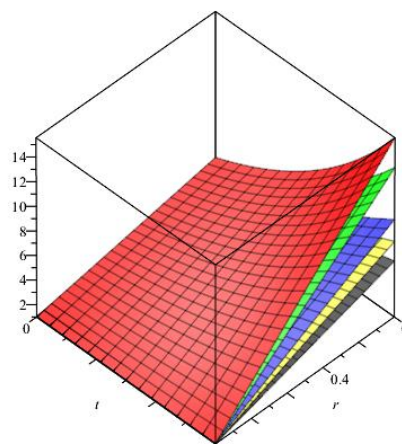


Figure 3: Effect of thermal diffusivity on temperature

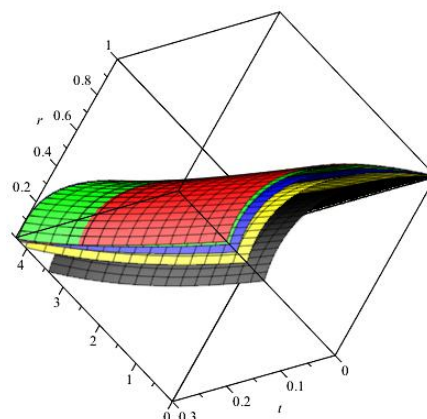


Figure 4: Effect of thermal Stratification on velocity

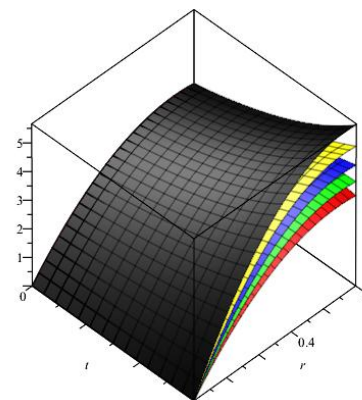
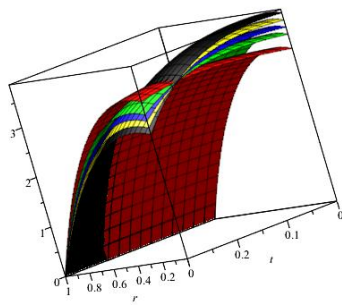
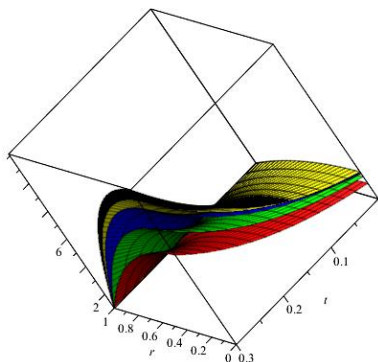


Figure 5: Effect of Mass Stratification on velocity

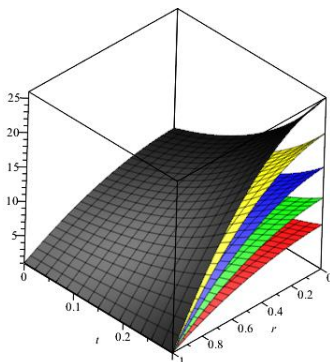




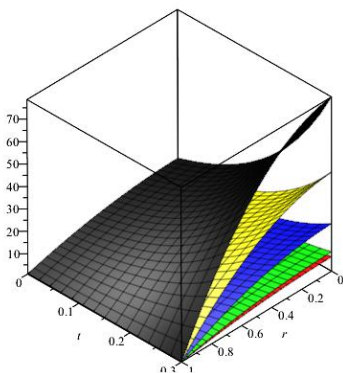
**Figure 6:** Effect of thermal diffusivity on velocity



**Figure 7:** Effect of thermal Stratification on concentration



**Figure 8:** Effect of Mass stratification on concentration



**Figure 9:** Effect of thermal diffusivity on concentration

Fig. 1: illustrates that the thermal stratification causes the temperature to vary more sharply with distance, creating stable layers that resist vertical mixing. Over time, this stabilizes the thermal gradient, slowing down heat transfer and delaying temperature equilibration along the flow. This observation aligns with the findings of (Deka and Paul, 2017)

Fig. 2: illustrates the flow direction (distance), temperature increases persist or even amplify farther downstream due to stronger convective transport influenced by Solutal layering. Faster temperature build-up in the domain as enhanced Solutal stratification boosts flow and heat transport. System reaches higher steady-state temperature over time, depending on stratification strength and fluid properties.

Fig. 3: show that increased thermal diffusivity, weakens thermal gradients, possibly reducing buoyancy forces that drive flow. It also leads to more uniform but cooler temperature distribution. May reduce the strength of natural convection if viscous dissipation and stratification effects are not dominant.

Fig. 4: Illustrates that fluid velocity decreases with distance from the heat source (e.g., cylinder), especially in a *stably stratified* (temperature increasing with height) environment. However, over time, thermal stratification dampens the growth of convective motion, leading to a slower rise or even decay in velocity.

Fig. 5: shows that if an increase in mass stratification leads to an increase in velocity, this suggests a most likely a stable stratification, where the concentration gradient enhances Solutal buoyancy, driving stronger convection. Enhanced Solutal buoyancy increases velocity magnitude, causing the flow to penetrate farther from the surface and maintain higher velocity gradients. (Shah et al, 2024)

Fig. 6: This decline indicates that as heat diffuses further, the temperature gradient (driving force for convection) weakens, reducing velocity. Higher thermal diffusivity ( $\alpha$ ) higher leads to initial velocity due to stronger and faster thermal convection. Over time, velocity decreases for all  $\alpha$  due to loss of driving thermal gradients. Thus, thermal diffusivity enhances the initial momentum of the fluid but leads to earlier decay in convective strength.

Fig. 7: When thermal stratification increases and leads to an increase in concentration, it suggests a coupling between temperature gradients and mass diffusion, a phenomenon often referred to as thermo-diffusion or the Soret effect. When increase in thermal stratification causes increased concentration, it implies that, temperature gradients drive solute migration, enhancing mass accumulation in certain regions. This can also be seen in the findings of (Deka and Paul, 2017)

Fig. 8: shows that an increase in mass stratification leads to increase in concentration as a result of the fact that the stratified medium supports higher solute accumulation near the region of interest (e.g., the surface of the cylinder). Mass stratification suppresses outward diffusion or traps solute in layers, and finally buoyant forces from solute gradients reinforce solute transport toward regions of higher concentration (Shah et al, 2024).

Fig. 9: indicates that faster thermal spreading enhances conditions that promote solute transport and accumulation, possibly through stronger thermal-driven flow. Increasing thermal diffusivity accelerates heat propagation, strengthening convection and mass transport, which raises solute concentration over time and expands its distribution across distance. This is also seen in the work of (Deka and Paul, 2017)

## Conclusions

Ugwu et al., (2025) proved the existence and uniqueness of the equations above via the Lipschitz continuity criterion investigated by Ayeni (1978). The results confirm that a unique solution exists under the satisfaction of the Lipschitz conditions. In this research, we went on to obtain the Numerical simulations were performed to examine the effects of key model parameters, specifically the Mass and thermal stratification, thermal diffusivity. The dimensionless equations were solved over space and time, applying the Perturbation Method to decompose them into zeroth- and first-order components. The results revealed that when the thermal conductivity & diffusion coefficient are variables ( $\alpha > 0$ ) and atmosphere is stable ( $\gamma > 0$ ), the mass and thermal stratification greatly influence the velocity, temperature and concentration of the particle. It is observed that,

- ❖ Temperature increases with increase in thermal and mass stratification, whereas it decreases with increase in thermal diffusivity.
- ❖ Higher thermal diffusivity leads to increase in velocity but later leads to decay with time, and solely leads to increase in concentration and decrease in temperature.
- ❖ Increase in mass and thermal stratification leads to increase in concentration of the system as a result of temperature gradients drive solute migration, enhancing mass accumulation in certain regions.

The proposed model has practical relevance in various industrial engineering applications, such as:

- Heat exchanger design where precise control of stratification enhances energy efficiency.
- Chemical processing units involving mass and heat-sensitive reactions.
- Cooling systems in nuclear and thermal power plants, especially during emergency operations or equipment failure scenarios.

Finally, the analytical approach using the OGPAM has proven effective for capturing these dynamics, offering an accurate, computationally efficient solution framework. The model and insights developed here are particularly useful for thermal engineering, process control, and safety analysis in critical infrastructures where stratification and flow stability are of paramount importance. It also models a realistic scenario where temperature- and concentration-induced stratification influence buoyancy-driven flows around cylindrical structures. It will improve predictions, enhance thermal system design, and contribute new insights to the scientific community.

## Author Contributions

**U.C. Ugwu:** Writing – review & editing, Writing – original draft, Software, Methodology, Conceptualization. **M.O. Durojaye:** Supervision, Resources, Project administration, Formal analysis, Data curation. **A.O. Adeniji:** Supervision, Visualization, Validation, Methodology. **M. Modebei:** Visualization, Supervision, Resources, Investigation, Funding acquisition. **A.M. Ayinde:** Resources, Investigation, Funding acquisition, Formal analysis, Supervision.

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**Declarations & Limitation:** This manuscript has not been presented anywhere else for publication. It is limited to the mathematical analysis of the existence and uniqueness of the solution to the modeled equation.

**Conflict of interest:** The authors have no conflict of interest to declare that are relevant to the content of this article.

**Data availability:** Not applicable

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