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# MODELLING THE VOLATILITY OF STOCK EXCHANGE MARKET CAPITALIZATION RETURNS IN NIGERIA USING GARCH MODELS

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## ABSTRACT

Financial market volatility remains a significant concern for investors and policymakers, particularly in emerging economies, where market inefficiencies exacerbate risks. This study provided fresh insights into Nigeria's stock market volatility by comprehensively evaluating Generalized Autoregressive Conditional Heteroscedasticity (GARCH)-family models with alternative error distributions for market capitalization returns from 1990 to 2023. The analysis revealed striking findings. While standard GARCH models captured basic volatility clustering, only specifications incorporating heavy-tailed distributions adequately addressed the extreme fluctuations characteristic of this emerging market. The Threshold GARCH(1,1) model with Student-t innovations emerged as superior in modelling asymmetric volatility responses, with the EGARCH-Generalized Error Distribution (GED) specification showing infinite persistence - a remarkable finding suggesting shock impacts may never fully dissipate. Through rigorous comparison of Normal, Student-t and GED innovations, the study demonstrated that distributional assumptions significantly influenced volatility persistence estimates and forecast accuracy. The results challenged conventional modelling approaches by showing that even sophisticated GARCH variants leave some nonlinear dependencies unaccounted for, pointing to potential avenues for future methodological improvements. These findings carry important implications for risk management practices and regulatory policies in volatile emerging markets, particularly for portfolio managers seeking to mitigate downside risks in Nigeria's equity market. The study advances the empirical literature on volatility modelling while providing practical guidance for financial market participants operating in similar emerging market contexts.

**Keywords**: GARCH models, Emerging markets, Market Capitalization, Risk Management, Nigeria stock Market.

## INTRODUCTION

The Nigerian Stock Market, established on August 15, 1961, is recognized as an emerging market by the International Finance Corporation and ranks among Africa's largest in liquidity, market capitalization, and trade volume. It serves as a key platform for portfolio investments in Africa (Oloko, 2016). However, stock market volatility poses significant risks, deterring investment, destabilizing returns, and undermining investor confidence (Ndwiga & Muriu, 2016). Volatility clustering, asymmetry, and leptokurtosis further complicate forecasting and valuation (Onoh *et al*., 2017). Despite extensive research on volatility modelling using GARCH-family models, the role of error distributions in enhancing model efficiency remains underexplored.

Studies on volatility in Nigeria and other markets highlight the prevalence of ARCH and GARCH models. Emenogu and Adenomon (2023) identified EGARCH with Student-t distribution as optimal for modelling First Bank returns, while Bala and Asemota (2013) found that volatility breaks improve GARCH performance in exchange-rate modelling. Yaya *et al*. (2016) demonstrated the superiority of Beta-t-EGARCH for Nigeria's All Share Index, whereas Aako and Alabi (2019) confirmed leverage effects in Nigeria using EGARCH. Comparative studies, such as Onyele and Nwadike (2021), revealed asymmetric responses to news, with negative shocks amplifying volatility. Internationally, Caporale *et al*. (2020) observed mean-reverting volatility in Russia, while Saeed *et al.* (2021) linked COVID-19 to heightened volatility in Pakistan's stock market.

While prior studies extensively applied GARCH models, few systematically evaluated the impact of error distributions (normal, Student-t, GED) on volatility forecast and persistence in Nigeria's market capitalization returns. This gap limits the precision of risk assessments and investment strategies. This study aimed at modelling the volatility in Nigeria's stock market capitalization returns using GARCH-family models under three error distributions.

## MATERIALS AND METHODS

### Data

The study used secondary data obtained from the Central Bank of Nigeria 2023 Statistical Bulletin for the period January 1990 to December 2023.

### Techniques for Data Analysis

### Returns on Stock Market Capitalization

The returns series was derived from the monthly stock market capitalization data through this computation

$$r_t = \ln\left(\frac{p_t}{p_{t-1}}\right) \quad (1)$$

### Unit Root Test

The Augmented Dickey-Fuller (ADF) test was used to check stationarity in stock market capitalization returns. It corrects for autocorrelation by modelling the data as an AR(p) process, including p lagged differences of the dependent variable in the regression. The test equation is specified as:

$$\Delta y_t = a y_{t-1} + x_t'\delta + B_1 \Delta y_{t-1} + B_2 \Delta y_{t-2} + \cdots + B_p \Delta y_{t-p} + v_t \quad (2)$$

Where $x_t$ are optional exogenous regressors, which may consist of a constant or a constant and trend.

### ARCH (P) Model

Engle (1982) pioneered the concept of conditional heteroscedasticity, challenging the assumption of constant variance in time series. He proposed the ARCH model, where volatility varies over time based on past squared errors while

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maintaining stable unconditional variance. Essentially, the ARCH model captures dependence in uncorrelated shocks $\varepsilon_t$ through lagged squared error terms.
The ARCH(p) model is given by:

$$r_t = \mu_t + \varepsilon_t\ ,\ \varepsilon_t = \sigma_t z_t, \quad z_t \sim N(0,1) \tag{3}$$

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 \tag{4}$$

where $\omega > 0$, $\alpha_i \geq 0$, $i = 1, ..., p$, and $p > 0$ is the order of ARCH model, $\omega$ represents the average values of $\sigma_t^2$, $z_t$ is a white noise with mean zero and variance 1. The $\mu_t$ is the appropriate structure explaining the mean equation. The ARCH coefficients $\alpha_i$ must satisfy the stationarity condition to ensure that the unconditional variation exists. If $\sum_{i=1}^{p} \alpha_i < 1$ the ARCH model is weakly stationary

**Standard GARCH (p, q) Model**
Engle's ARCH model, widely used in finance and economics, has limitations; it treats positive/negative shocks equally and risks negative variance due to its reliance on squared lags and numerous parameters. Bollerslev (1986) addressed these issues with the GARCH model, linking current volatility to both past shocks (p) and prior volatility (q). The standard GARCH formulation is:

$$r_t = \mu_t + \varepsilon_t\ ,\ \varepsilon_t = \sigma_t z_t, \quad z_t \sim N(0,1) \tag{5}$$

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 \tag{6}$$

Equations (5) and (6) are the mean and variance equations, respectively, $\omega > 0$, $\alpha_i \geq 0$, $i = 1, ..., p$ and $\beta_j \geq 0$, $i = 1, ..., q$, are sufficient conditions to ensure that the conditional variance $\sigma_t^2 > 0$. Also, $\mu_t$ is the average value of $r_t$, $\omega$ represents the average values of $\sigma_t^2$, $z_t$ is a white noise with mean zero and variance 1. $r_t$ is the continuous compounding log return series. The parameters $\alpha_i$ represents the ARCH effect and $\beta j$ represents the GARCH effect.

**Power GARCH (1,1) Model**
The power GARCH model (PGARCH) model – PGARCH (1,1) is given by:

$$\sigma_t^\delta = \alpha_0 + \beta_1 \sigma_{t-1}^\delta + \alpha_1 (|\varepsilon_{t-1}| - \gamma_1 \varepsilon_{t-1})^\delta \tag{7}$$

where $\alpha_0$ is the constant, $\alpha_1$ and $\beta_1$ are the standard ARCH and GARCH parameters, $\gamma$ is the leverage parameter and $\delta$ is the parameter for the power term, and $\delta > 0, |\gamma_1| \leq 1$.

**Threshold GARCH (TGARCH) Model**
The Threshold GARCH (TGARCH) model, an asymmetric extension of GARCH, effectively captures leverage effects in volatility. Its general form is specified as:

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{p} (\alpha_1 + \varphi_i N_{t-i}) a_{t-1}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 \tag{8}$$

Where $N_{t-i}$ is an indicator for negative $a_{t-i}$, that is,

$$N_{t-i} = \begin{cases} 1 & if\ a_{t-i} < 0, \\ 0 & if\ a_{t-i} \geq 0 \end{cases} \tag{9}$$

And $\alpha_i, \varphi_i$ and $\beta_j$ are nonnegative parameters satisfying conditions similar to those of GARCH models (Tsay 2005).

**EGARCH Model**
Nelson (1991) developed the EGARCH model to address GARCH limitations in financial time series, particularly to capture asymmetric effects between positive and negative returns. The EGARCH(1,1) specification is:

$$g(\varepsilon_t) = \theta \varepsilon_t + \varphi[|\varepsilon_t| - E(|\varepsilon_t|)] \tag{10}$$

Where $\theta$ and $\varphi$ are real constants. Both $\varepsilon_t$ and $[|\varepsilon_t| - E(|\varepsilon_t|)$ are zero-mean sequences with continuous distributions. Therefore, $E[g(\varepsilon_t)] = 0$. The asymmetry of $g(\varepsilon_t)$ can easily be seen by rewriting it as

$$a_t = \sigma_t \varepsilon_t \tag{11}$$

$$\ln(\sigma_t^2) = \omega + \sum_{i=1}^{s} \alpha_i \frac{|a_{t-i}| + \theta_i a_{t-i}}{\sigma_{t-i}} + \sum_{j=1}^{m} \beta_j \ln(\sigma_{t-1}^2) \tag{12}$$

which specifically results in EGARCH (1,1) being written as

$$a_t = \sigma_t \varepsilon_t \tag{13}$$

$$\ln(\sigma_t^2) = \omega + \alpha(\left[|a_{t-1}| - \sqrt{2/n}\right] + \theta a_{t-1} + \beta \ln(\sigma_t^2) \tag{14}$$

**Error Distribution forms and Estimation of GARCH models**

**a) The Normal Distribution**

The log-likelihood from the normal distribution is

$$l_t = -\frac{1}{2}\left[N log(2\pi) + \sum_{t=1}^{N} \frac{\varepsilon_t^2}{\sigma_t^2} + \sum_{t=1}^{s} log \sigma_t^2\right] \tag{15}$$

And with $\varepsilon_t = \sigma_t z_t$ where $z_t$ is the GARCH time series innovations and N is the sample size of the time series.

**b) Students' t Distribution**

The log-likelihood of Student t-distribution is given as below

$$l_t = -\frac{1}{2}\left\{N log\left(\frac{\pi(v-2)\Gamma(v/2)^2}{\Gamma\left(\frac{v+1}{2}\right)^2}\right) + \sum_{t=1}^{N} log \sigma_t^2 + (v + 1)\sum_{t=1}^{N} log[1 + \frac{\varepsilon_t^2}{\sigma_t^2(v-1)}]\right\} \tag{16}$$

Where v is the degrees of freedom to be estimated and $\Gamma(.)$ is the gamma function.

**c) The Generalized Error Distribution**

The log-likelihood of generalized error distribution is given by:

$$l_t = -\frac{1}{2}\left\{N log\left(\frac{\Gamma(v^{-1})}{\Gamma(3v^{-1})(v/2)^2}\right) + \sum_{t=1}^{N} log \sigma_t^2 + (v + 1)\sum_{t=1}^{N} log\left(\frac{\Gamma(3v^{-1})\varepsilon_t^2}{\sigma_t^2 \Gamma(v^{-1})}\right)^{v/2}\right\} \tag{17}$$

Where v is the tail thickness parameter.

**Diagnostic Check**
A well-specified GARCH model must fully capture dynamics in both the mean and variance equations. The standardized residuals should exhibit: No serial correlation (tested via Ljung-Box Q-statistics), No remaining volatility clustering (Q-statistics) and White noise properties. Failure to meet these conditions indicates model misspecification (Enders, 2004).

**Model Selection Criterion and Forecast Performance Evaluation**
Information criteria Akaike Information Criterion (AIC), Bayesian Information Criterion (SBIC) and Hannan-Quinn Information Criterion (HQIC) evaluate GARCH model fit, with lower values indicating better performance. The preferred model among alternatives is the one that minimizes these metrics. Their formulations are (Adenomon *et al.*, 2022):

$$AIC = -2log(\hat{\sigma}^2) + 2(k) - 1 - log(2\pi) \tag{18}$$

$$SBIC = -2log(\hat{\sigma}^2) + (k) * \log(n) - 1 - log(2\pi) \tag{19}$$

$$HQIC = -2log(\hat{\sigma}^2) + 2(k) * \log(\log(n)) - 1 - log(2\pi) \tag{20}$$

Forecasting is the ultimate objective of time series modelling,

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aiming to predict future values using fitted GARCH models. In this study, the forecast performance was evaluated using the metrics: Theil's coefficient, Mean Absolute Error **(**MAE), Mean Absolute Percentage Error (MAPE) and Root Mean Square Error (RMSE) - with the best model demonstrating the lowest error values

**Half-Life Volatility**
The mean reversion pace, or average time, of returns on stock market capitalization was measured by half-life volatility. Mathematically, the half-life volatility is given as below

$$Half-life = \frac{\ln(0.5)}{\ln(\alpha_1+\beta_1)} \quad (21)$$

**Persistence**
Volatility persistence measures how long shocks affect volatility. In GARCH models, it is calculated as the sum of ARCH and GARCH coefficients (Banerjee & Sarkar 2006; Ahmed *et al.,* 2018). Persistence can be:

i) If $\alpha_1+\beta_1<1$: the model ensures positive conditional variance as well as stationary.
ii) If $\alpha_1+\beta_1=1$; we have an exponential decay model, then the half-life becomes infinite, meaning that the model is strictly stationary.
iii) If $\alpha_1+\beta_1>1$; the GARCH model is said to be non-stationary, meaning that the volatility ultimately detonates toward the infinitude (Ahmed et al., 2018).

In addition, the model shows that the conditional variance is unstable, unpredictable and the process is non-stationary (Kuhe, 2018).

## RESULTS AND DISCUSSION

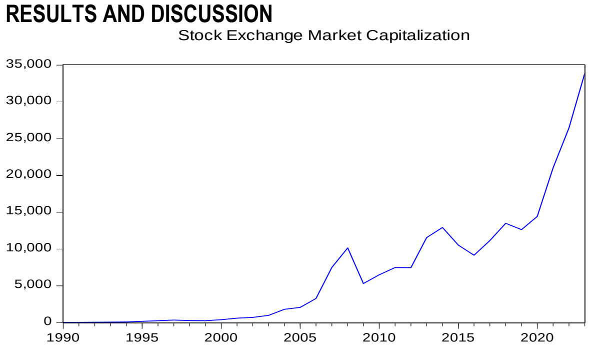

**Figure 1**: Graph of actual stock market capitalization

The graph in Figure 1 shows a dramatic increase in market capitalization from 1990 to early 2023. Starting near zero in the early 1990s, market capitalization remained relatively flat until around 2000, when it began a gradual rise. Around 2005, growth accelerated more noticeably, reaching approximately 10,000 (presumably in billions of currency units) by 2010. Between 2010 and 2020, the market experienced several fluctuations with both upward and downward movements, though maintaining an overall upward trajectory. Most striking is the explosive growth after 2020, where market capitalization more than doubled in just a few years, reaching approximately 33,000 by early 2023. This sharp upward trajectory at the end of the graph suggests an extraordinary period of market expansion, possibly influenced by economic policies, market conditions, or investor sentiment following the global events of 2020.

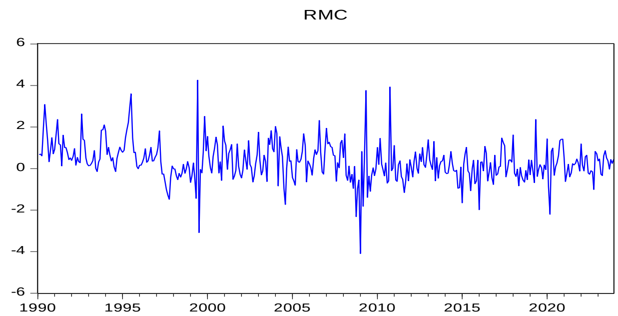

**Figure 2**: Graph of stock market capitalization returns

Figure 2 depicts a time series that exhibits mean reversion and volatility clustering which are key characteristics of financial time series data. The series fluctuates primarily within a stable range of -2 to +2, frequently returning to its central tendency after deviations, which aligns with mean-reverting behaviour. However, the occasional sharp spikes reaching extremes around -4 and +4 cluster around specific periods such as 2000, 2010, and 2020, indicating volatility clustering where large movements tend to occur consecutively before stabilizing. The absence of a long-term trend and the persistence of these bounded yet erratic swings suggest a stationary process where short-term shocks create temporary disruptions before the series reverts to its mean. This pattern is common in financial volatility measures, economic indicators, or model residuals, where external shocks induce bursts of instability that dissipate over time. The combination of mean reversion and clustered volatility implies that while the series remains range-bound in the long run, it experiences periods of heightened turbulence that are not uniformly distributed.

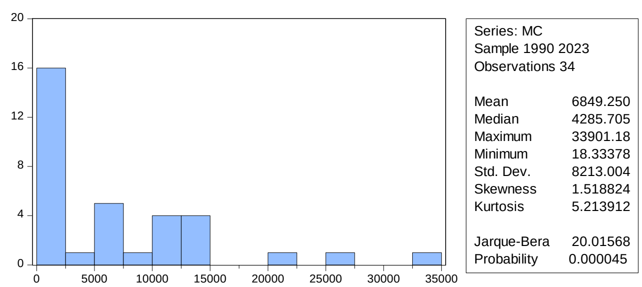

**Figure 3**: Histogram of actual Stock Market Capitalization

Figure 3 describe a right-skewed distribution of stock market capitalization (MC) from 1990 to 2023, with most observations clustered at lower values (nearly half below 5000) and a few extreme values reaching up to 33,901. The mean (6849) exceeds the median (4286), confirming right skewness, while the high kurtosis (5.21) and significant Jarque-Bera test ($p \approx 0.000045$) indicated heavy tails and non-normality. This pattern suggests that the market is dominated by smaller-cap stocks, with a few large-cap outliers pulling the average upward. The wide range (18 to 33,901) and high standard deviation (8213) reflect substantial volatility, implying that traditional models assuming normality may underestimate tail risks. For accurate analysis, transformations or fat-tailed distributions may be necessary to account for the skewness and extreme values inherent in market capitalization data.

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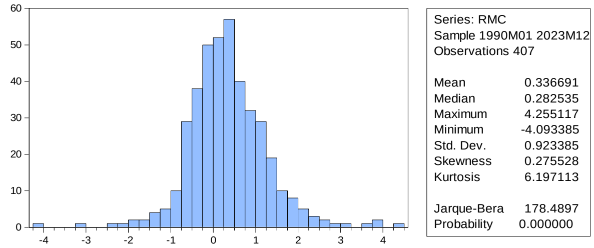


**Figure 4**: Histogram of Stock Market Capitalization Returns

Figure 4 depicts the distribution of return on market capitalization from January 1990 to December 2023 (407 observations). The histogram shows a near-symmetric, bell-shaped pattern centered close to zero (range: −4.09 to 4.26), with frequencies peaking at ~55. Despite its Gaussian-like appearance, the series exhibits key deviations: a mild positive skewness (0.28), extreme kurtosis (6.20), and a mean (0.34) slightly above the median (0.28), all hallmarks of financial returns. The moderate standard deviation (0.92) masks the heavy tails, evidenced by the Jarque-Bera test (178.49, p ≈ 0.00), which rejects normality consistent with the leptokurtic, outlier-prone behaviour typical of asset returns. This aligns with empirical finance, where returns often cluster near zero but exhibit fat tails due to volatility clustering and rare extreme events.

**Table 1:** Descriptive Statistics Results

| | MC | RMC |
|---|---|---|
| Mean | 6866.042 | 0.336691 |
| Median | 4483.500 | 0.282535 |
| Maximum | 40917.51 | 4.255117 |
| Minimum | 15.11593 | -4.093385 |
| Std. Dev. | 8183.493 | 0.923385 |
| Skewness | 1.573605 | 0.275528 |
| Kurtosis | 5.576181 | 6.197113 |
| Jarque-Bera | 280.5188 | 178.4897 |
| Probability | 0.000000 | 0.000000 |
| Sum | 2794479. | 137.0331 |
| Sum Sq. Dev. | 2.72E+10 | 346.1716 |

**Source**: EVIEWs output

The descriptive statistics in table 1 revealed key characteristics of the market capitalization (MC) and return on market capitalization (RMC) series. For MC, the mean (6,866) significantly exceeds the median (4,484), indicating a right-skewed distribution (skewness = 1.57) with a few high-value outliers, as further evidenced by the wide range (15 to 40,918) and high standard deviation (8,183). The kurtosis (5.58) confirms heavy tails, and the Jarque-Bera test (p = 0.000) rejects normality. For RMC, the near-zero mean (0.34) and median (0.28) suggested symmetric returns, yet the positive skewness (0.28) and extreme kurtosis (6.20) highlight fat tails and excess extreme values compared to a normal distribution, further supported by the Jarque-Bera test's rejection of normality (p = 0.000). These results reflected typical financial series behaviour: MC exhibits positive skewness and concentration of lower values with sporadic large outliers, while RMC, though roughly symmetric, displays leptokurtosis consistent with the frequent presence of volatility and tail risks in market returns.

**Table 2:** ADF and PP Unit Root Result of actual market capitalization

| Method | Difference Order | Test Values 1% | 5% | Critical 10% | Test Statistic | P-value | Remark |
|---|---|---|---|---|---|---|---|
| ADF | 0 | -3.4462 | -2.8684 | -2.5705 | 3.5024 | 1.0000 | Not Stationery |
| | 1 | -3.4462 | -2.8684 | -2.5705 | -15.8874 | 0.0000 | Stationery |
| PP | 0 | -3.4462 | -2.8684 | -2.5705 | 4.5889 | 1.0000 | Not Stationery |
| | 1 | -3.4462 | -2.8684 | -2.5705 | -16.0256 | 0.0000 | Stationery |

**Source**: Extracted from EVIEWs Output

Table 2 presents the results of the Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) unit root tests for actual market capitalization data. In their original level form (difference order 0), both tests fail to reject the null hypothesis of a unit root, as the test statistics (3.5024 for ADF and 4.5889 for PP) are greater than all critical values and have p-values of 1.0000, indicating non-stationarity. However, after taking the first difference (difference order 1), the test statistics (-15.8874 for ADF and -16.0256 for PP) became significantly negative than all critical values with p-values of 0.0000, strongly rejecting the null hypothesis and confirming stationarity. This demonstrated that while the raw market capitalization series is non-stationary, its first-differenced form becomes stationary, suggesting that the series is integrated of order one (I(1)), a common characteristic of financial time series data that often requires differencing for meaningful analysis.

**Table 3:** ADF and PP Unit Root Result of Market Capitalization Returns

| Method | Difference Order | Test Critical Values 1% | 5% | 10% | Test Statistic | P-value | Remark |
|---|---|---|---|---|---|---|---|
| ADF | 0 | -3.4463 | -2.8685 | -2.5705 | -10.2996 | 0.0000 | Stationery |
| PP | 0 | -3.4462 | -2.8684 | -2.5705 | -16.4502 | 0.0000 | Stationery |

**Source:** Extracted from EVIEWs Output

Table 3 presents the results of the Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) unit root tests for market capitalization returns. Both tests in their level form (difference order 0) strongly reject the null hypothesis of a unit root, as the test statistics (-

10.2996 for ADF and -16.4502 for PP) are significantly negative than all critical values at the 1%, 5%, and 10% significance levels, with p-values of 0.0000. This indicated that the market capitalization returns series is stationary in its original form, without requiring any differencing. The results suggested that the returns series does not exhibit a stochastic trend and maintains stable statistical properties over time, which is a desirable characteristic for time series analysis and modelling in financial applications. The consistency between both test methods further reinforces the reliability of this conclusion.

**Table 4:** ARCH LM Test of Stock Exchange Market Capitalization Returns

| Lags | Test Statistics | P-value |
|---|---|---|
| 10 | 5.9420 | 0.0000 |
| 20 | 3.2110 | 0.0000 |
| 30 | 2.7987 | 0.0000 |

**Source:** Extracted from EVIEWs Output

The ARCH-LM test results in Table 4 for stock exchange market capitalization returns show statistically significant evidence of autoregressive conditional heteroskedasticity (ARCH effects) across all lag lengths (10, 20, and 30). With all p-values at 0.0000, the test strongly rejects the null hypothesis of no ARCH effects, indicating that volatility clustering is present in the returns series. This finding suggested that large returns tend to be followed by large returns (of either sign) and small returns by small returns, a characteristic feature of financial time series that warrants the use of ARCH/GARCH-type models for proper volatility modelling and risk assessment. The consistent significance across increasing lag lengths further confirms the persistence of volatility clustering in the data.

**Table 5:** Information Criteria of GARCH models on the stock exchange market capitalization returns

| Models | Information Criteria | Normal Innovation | Student t Innovation | GED Innovation |
|---|---|---|---|---|
| SGARCH (1,1) | AIC | 2.4569 | *2.3476* | 2.3652 |
| | SIC | 2.5062 | *2.4068* | 2.4244 |
| | HQ | 2.4764 | *2.3710* | 2.3886 |
| EGARCH (1,1) | AIC | 2.4512 | *2.3509* | 2.3677 |
| | SIC | 2.5104 | *2.4199* | 2.4368 |
| | HQ | 2.4747 | *2.3782* | 2.3950 |
| TGARCH (1,1) | AIC | 2.4439 | *2.3475* | 2.3687 |
| | SIC | 2.5031 | *2.4166* | 2.4377 |
| | HQ | 2.4673 | *2.3748* | 2.3960 |
| PGARCH (1,1) | AIC | 2.4444 | *2.3533* | 2.3721 |
| | SIC | 2.5135 | *2.4323* | 2.4510 |
| | HQ | 2.4718 | *2.3846* | 2.4033 |

**Source**: Extracted from EVIEWs Output

Table 5 compared four GARCH models (SGARCH, EGARCH, TGARCH, and PGARCH) with different error distributions (Normal, Student-t, GED) for modelling stock exchange market capitalization returns. Across all models, the Student-t innovation consistently show the lowest AIC values (ranging 2.3476-2.3533), suggesting it best captured the returns' fat-tailed characteristics. The TGARCH(1,1) model with Student-t errors emerged as the optimal model with the lowest AIC (2.3475), followed closely by PGARCH (2.3533) and EGARCH (2.3509), indicating that accounting for asymmetric volatility effects (leverage) improves model fit. The consistent superiority of Student-t innovations across all specifications confirms the presence of excess kurtosis in the data, while the similar performance of asymmetric models (TGARCH/EGARCH/PGARCH) suggests significant leverage effects in market capitalization returns volatility.

**Table 6:** Parameter Estimates of GARCH models with Student t Distribution Innovation

| Models | $\alpha$ | $\beta$ | $\gamma$ | $\delta$ |
|---|---|---|---|---|
| SGARCH (1,1) | 0.4000 | -0.0026 | - | - |
| | (0.0000) | (0.0000) | - | - |
| EGARCH (1,1) | 0.6088 | 0.3058 | 0.0966 | - |
| | (0.0000) | (0.1271) | (0.3262) | - |
| TGARCH (1,1) | 0.4764 | -0.1560 | - | - |
| | (0.0123) | (0.0421) | - | - |
| PGARCH (1,1) | 0.3917 | -0.1323 | -0.2219 | 1.2902 |
| | (0.0003) | (0.3645) | (0.1695) | (0.0464) |

**Source**: Extracted from EVIEWs Output *Note:* The P-values are presented in parentheses

Table 6 present parameter estimates of GARCH models with Student-t distribution for market returns. The SGARCH (1,1) show significant ARCH effects (α=0.4000) but a negative GARCH term (β=-0.0026), indicating volatility clustering without persistence. The EGARCH (1,1) revealed strong ARCH effects (α=0.6088) but insignificant GARCH (β=0.3058) and leverage terms (γ=0.0966). The TGARCH (1,1) displayed significant ARCH effects (α=0.4764) with a negative GARCH coefficient (β=-0.1560). The PGARCH (1,1) show significant ARCH (α=0.3917) and power term (δ=1.2902), but other parameters are insignificant. These mixed results suggested the Student-t GARCH models show volatility clustering but fail to properly capture persistence and asymmetric effects.

**Table 7**: Parameter Estimates of GARCH models with Generalized Error Distribution

| Models | $\alpha$ | $\beta$ | $\gamma$ | $\delta$ |
|---|---|---|---|---|
| SGARCH (1,1) | 0.3055 | 0.3025 | - | - |
| | (0.0000) | (0.0000) | - | - |
| EGARCH (1,1) | 0.5546 | 0.5782 | -0.0125 | - |
| | (0.0000) | (0.0001) | (0.8863) | - |
| TGARCH (1,1) | 0.2296 | 0.3772 | - | - |
| | (0.0507) | (0.0028) | - | - |
| PGARCH (1,1) | 0.2202 | 0.2564) | 0.1356 | 3.7707 |
| | (0.1455) | (0.2898) | (0.3015) | (0.1299) |

**Source**: Extracted from EVIEWs Output *Note:* The P-values are presented in parentheses

Table 7 show parameter estimates of GARCH models using Generalized Error Distribution (GED) innovations. The SGARCH (1,1) model demonstrated significant volatility clustering ($\alpha = 0.3055$) and persistence ($\beta = 0.3025$), with both parameters highly significant ($p = 0.0000$). The EGARCH(1,1) revealed strong ARCH ($\alpha = 0.5546$) and GARCH ($\beta = 0.5782$) effects, but the leverage term ($\gamma = -0.0125$) is insignificant (p=0.8863), suggesting no asymmetric volatility response. The TGARCH(1,1) show significant ARCH ($\alpha = 0.2296$) and GARCH ($\beta = 0.3772$) effects at 5% significance level. However, the PGARCH(1,1) model show all parameters ($\alpha = 0.2202, \beta = 0.2564, \gamma = 0.1356, \delta = 3.7707$) as statistically insignificant (p>0.10). These results indicated that while basic GARCH models (SGARCH, EGARCH, TGARCH) with GED innovations effectively captured volatility clustering and persistence, more complex specifications (PGARCH) may not provide additional explanatory power for this dataset. The absence of significant leverage effects in EGARCH suggests symmetric volatility responses to market shocks.

**Table 8**: Forecast performance measures of GARCH models on the stock exchange market capitalization returns

| Models | Forecast Performance Measures | Student t Innovation | GED Innovation |
|---|---|---|---|
| SGARCH (1,1) | RMSE | 0.9011 | 0.8987 |
| | MAE | 0.6160 | 0.6157 |
| | MAPE | 192.2714 | 194.0538 |
| EGARCH (1,1) | RMSE | 0.9000 | 0.8981 |
| | MAE | 0.6158 | 0.6156 |
| | MAPE | 194.2331 | 194.2986 |
| TGARCH (1,1) | RMSE | 0.9001 | 0.8978 |
| | MAE | 0.6158 | 0.6156 |
| | MAPE | 192.0132 | 192.2184 |
| PGARCH (1,1) | RMSE | 0.9003 | 0.8978 |
| | MAE | 0.6159 | 0.6156 |
| | MAPE | 194.3343 | 191.7160 |

**Source**: Extracted from EVIEWs Output

The forecast performance of various GARCH models with Student-t and GED innovations was evaluated using RMSE, MAE, and MAPE metrics and the results were presented in Table 8. Results show minimal differences between model specifications, with all RMSE values ranging between 0.8978-0.9011 and MAE values between 0.6156-0.6160, indicating similar point forecast accuracy across models. The GED innovation models consistently show slightly better RMSE performance compared to Student-t, particularly for TGARCH and PGARCH specifications. However, MAPE values are substantially higher (191.7160-194.3343) and more variable across models, suggesting greater relative error in percentage terms. The narrow range of RMSE and MAE values across different GARCH specifications implies that model choice has limited impact on forecast accuracy for this dataset, with simpler models (SGARCH) performing nearly as well as more complex variants (EGARCH, TGARCH, PGARCH). The GED distribution appears marginally superior to Student-t for volatility forecasting in this context.

**Table 9:** Persistence and half-life volatility of stock exchange market capitalization returns

| Models | Student *t* Innovation | | GED Innovation | |
|---|---|---|---|---|
| | Persistence | Half-Life | Persistence | Half-Life |
| SGARCH (1,1) | 0.3974 | 1.7511 | 0.6080 | 2.3929 |
| EGARCH (1,1) | 0.9146 | 8.7666 | 1.1328 | *Infinity* |
| TGARCH (1,1) | 0.3204 | 1.6090 | 0.6068 | 2.3876 |
| PGARCH (1,1) | 0.2594 | 1.5137 | 0.4766 | 1.9354 |

**Source:** Extracted from EVIEWs Output

Table 9 show volatility persistence and half-life estimates across GARCH models with Student-t and GED innovations. For Student-t models, persistence ranges from 0.2594 (PGARCH) to 0.9146 (EGARCH), with corresponding half-lives of 1.51 to 8.77 days, indicating moderate volatility clustering. The EGARCH model shows particularly strong persistence (0.9146) with an 8.77-day half-life. GED innovations yield higher persistence values (0.4766-1.1328) and longer half-lives (1.94 days to infinity), with EGARCH-GED showing infinite half-life (1.1328 persistence), suggesting non-mean-reverting volatility. Across both distributions, EGARCH displays the strongest persistence, while PGARCH shows the quickest volatility decay. The results suggested that GED innovations generally produce more persistent volatility effects than Student-t, and model choice significantly impacts volatility duration estimates, with asymmetric models (EGARCH) capturing longer-lasting volatility shocks compared to symmetric specifications.

**Table 10:** Residuals diagnostic testing for serial correlation and remaining ARCH effect.

| | Parameters | Student t Innovation | GED Innovation |
|---|---|---|---|
| SGARCH (1,1) | Q(24) | 32.349 (0.119) | 29.868 (0.189) |
| | $Q^2$ (24) | 62.363** (0.000) | 45.915** (0.005) |
| | ARCH-LM | 0.0023 (0.8798) | 0.0512 (0.8211) |
| EGARCH (1,1) | Q(24) | 31.001 (0.154) | 29.601 (0.198) |
| | $Q^2$ (24) | 50.962** (0.001) | 41.230** (0.016) |
| | ARCH-LM | 0.0160 (0.8995) | 0.0189 (0.8908) |
| TGARCH (1,1) | Q(24) | 30.189 (0.179) | 29.521 (0.201) |
| | $Q^2$ (24) | 47.018** (0.003) | 41.180** (0.016) |
| | ARCH-LM | 0.1430 (0.7055) | 0.1620 (0.6875) |
| PGARCH (1,1) | Q(24) | 32.132 (0.124) | 33.834 (0.088) |

| | | |
|---|---|---|
| $Q^2$ (24) | 62.186** | 64.788** |
| | (0.000) | (0.000) |
| ARCH-LM | 0.0253 | 0.0011 |
| | (0.8737) | (0.9735) |

**Source**: Extracted from E-VIEWS output

Table 10 assess whether the GARCH models adequately captured serial correlation and ARCH effects in market capitalization returns. For both Student-t and GED innovations, the Ljung-Box Q(24) tests for serial correlation show insignificant p-values (ranging 0.088-0.201), indicating no remaining linear dependence in the standardized residuals. However, the squared residual $Q^2(24)$ tests revealed significant p-values (0.000-0.016) across all models, suggesting some remaining nonlinear dependence. The ARCH-LM tests show insignificant results (p-values 0.6875-0.9735), indicating no remaining ARCH effects in the residuals. While all models successfully eliminated linear autocorrelation and ARCH effects, the presence of significant $Q^2$ statistics suggests they may not fully capture higher-order volatility dynamics. The results are consistent across both distributional assumptions, with GED innovations showing marginally better performance in some specifications. The PGARCH model show the strongest remaining nonlinear dependencies, particularly with GED innovations. Overall, the models adequately address basic volatility clustering but may require more sophisticated specifications to capture all nonlinear features of the data.

The findings revealed that market capitalization returns exhibit significant volatility clustering, leptokurtosis, and asymmetry—consistent with stylized facts in financial time series. Among the competing models, the TGARCH(1,1) with Student-t innovations emerged as the optimal specification based on information criteria, effectively capturing volatility persistence and leverage effects. However, residual diagnostics indicate some remaining nonlinear dependencies, suggesting that higher-order volatility dynamics may require more sophisticated modelling approaches.

The EGARCH model with GED innovations demonstrated the strongest volatility persistence, including infinite half-life, implying prolonged market reactions to shocks. Forecast evaluations showed minimal differences across models, with GED distributions marginally outperforming Student-t in predictive accuracy. Despite this, simpler models (e.g., SGARCH) performed comparably to more complex variants, highlighting a trade-off between model sophistication and incremental forecasting gains.

## CONCLUSION

This study examined the volatility dynamics of Nigeria's stock market capitalization returns using various GARCH-family models under different error distributions (Normal, Student-t, and GED).

These results have critical implications for investors and policymakers. The presence of asymmetric volatility responses suggests that negative shocks have a more pronounced impact on market stability, necessitating risk management strategies that account for tail risks. Additionally, the persistence of volatility underscores the need for regulatory frameworks that mitigate prolonged market turbulence. Future research could explore hybrid or regime-switching GARCH models to better capture nonlinearities and structural breaks in Nigeria's evolving equity market.

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