Science World Journal Vol. 20(No 3) 2025 https://dx.doi.org/10.4314/swj.v20i3.54
www.scienceworldjournal.org
ISSN: 1597-6343 (Online), ISSN: 2756-391X (Print)
Published by Faculty of Science, Kaduna State University

EVALUATION OF NEW LIU-RIDGE TYPE ESTIMATORS FOR IMPROVED POISSON REGRESSION: THEORY AND SIMULATION

Kayode Olakunle, *Abiola T. Owolabi, and Timothy O. Olatayo

Department of Statistics, Ladoke Akintola University of Technology, Ogbomoso, Nigeria

*Corresponding Author Email Address: atowolabi@lautech.edu.ng

ABSTRACT
Poisson regression is a common tool for modelling count data, but its performance deteriorates under multicollinearity among predictors, leading to unstable and inefficient maximum likelihood estimates. This study proposes a Poisson New Liu-Ridge Type Estimator (PNLRTE), a hybrid approach that combines features of the Liu and ridge estimators with multiple shrinkage strategies to improve estimation accuracy. The properties of the new estimator were derived, and its performance was evaluated against existing estimators, including Poisson Maximum Likelihood Estimator (PMLE), Poisson Ridge Estimator (PRE), Poisson Liu Estimator (PLE), Poisson Kibria-Lukman (PK-L), and Poisson Modified Ridge Type (PMRT). Performance was assessed across varying sample sizes, degrees of multicollinearity, number of predictors, and intercept specifications. Results consistently show that the PNLRTE with median shrinkage (PNLRT-ME) outperforms alternatives, achieving the lowest mean squared error (MSE) in most scenarios, particularly under severe multicollinearity and larger samples. Other estimators, such as PLE and PK-L, perform competitively under moderate correlations, while PRE gains efficiency as sample size increases. In contrast, PMLE consistently yields poor results. The findings establish PNLRTE, especially PNLRT-ME, as an efficient alternative for Poisson regression with multicollinearity, providing researchers with a reliable tool for practical applications.

Keywords: Poisson Regression, Multicollinearity, Ridge Regression, Liu Estimator, Shrinkage Techniques, Mean Squared Error

INTRODUCTION

In modern statistical analysis, the choice of a regression model is largely determined by the nature of the response variable under investigation. While linear regression is well-suited for continuous outcomes and logistic regression addresses binary responses, many real-world phenomena involve data that naturally arise as counts, such as the number of hospital visits, traffic accidents, or product purchases. In such cases, the Poisson regression model provides a powerful framework for exploring relationships between explanatory variables and a count-based response. (Abdelwahab *et al.* 2024) described the Poisson Regression Model as a crucial statistical tool applied in various fields for the analysis of count data. It is valuable when examining the relationships between one or more explanatory variables and a response variable that represents rare events or non–negative integer counts (Lukman *et al.*, 2021). Poisson regression models are a fundamental component of statistical methodology for analysing count data, particularly in areas such as epidemiology, ecology, insurance, and public health (Cameron and Trivedi, 2013). These models assume that the dependent variable follows a Poisson distribution, with the logarithm of its mean related linearly to a set of predictor variables. Despite their widespread application and utility, Poisson regression models encounter significant challenges when predictor variables are highly correlated, a situation known as multicollinearity (Olakunle *et al.* 2025). The Poisson Regression Model (PRM) employs maximum likelihood estimation (MLE) to estimate its regression coefficients, making it a reliable tool for analysing count data that follows a Poisson distribution (Alrweili, 2024). However, in the presence of multicollinearity among predictors, the MLE can become unstable, leading to large variances and biased estimates of the regression coefficients (Montgomery *et al.*, 2012). Multicollinearity in the maximum likelihood estimators (MLEs), can also lead to reduced precision and unreliable inferential results (Lukman *et al.*, 2024).

The idea of ridge regression originated from the work of Hoerl and Kennard (1970), whose backgrounds in engineering problem solving, natural sciences, and mathematical statistics were instrumental in its development (Hoerl, 2020). Their primary goal was to address the persistent challenge of multicollinearity in linear regression, particularly in engineering data. They demonstrated that introducing a nonzero constant k, known as the ridge parameter, could reduce the mean square error (MSE) of the ridge regression estimator to a level smaller than the variance of the ordinary least squares (OLS) estimator (Kibria and Banik, 2016). Since then, numerous researchers have proposed various estimators for the ridge parameter k, including Hoerl and Kennard (1970), Hoerl *et al.* (1975), McDonald and Galarneau (1975), Lawless and Wang (1976), Dempster *et al.* (1977), Gibbons (1981), Kibria (2003), Khalaf and Shukur (2005), Alkhamisi and Shukur (2008), Muniz and Kibria (2009), Muniz *et al.* (2012), Ayinde *et al.* (2018), Oladapo *et al.* (2022), Owolabi *et al.* (2022a, 2022b) and Idowu *et al.* (2023) among others. In a related development, Liu (1993) introduced the Liu parameter *d* as an alternative approach to mitigating multicollinearity. Building on both concepts, subsequent studies have proposed combined ridge–Liu estimators to further enhance estimation performance under multicollinearity. While these techniques are effective in linear models, direct application to Poisson regression requires adjustments due to its specific distributional and link function characteristics.

For Poisson Regression Models, the following works have been carried out to mitigate the problem of multicollinearity in PRM; Algamal (2018), Amin *et al.* (2021), Lukman *et al.* (2021; 2022) and Olakunle *et al.* (2025). This research introduces a new hybrid estimator that combines the ridge-type and the Liu–Ridge Type estimators based on two shrinkage parameters, k and d, for Poisson regression models (PRMs). The new estimation method also utilizes some shrinkage parameters to produce efficient estimators in PRMs.

MATERIALS AND METHODS

Existing Estimators

Poisson Maximum Likelihood Estimator (PMLE)

Assume that the outcome variable under consideration follows a count distribution. In this case, the probability mass function of the Poisson distribution is given by

$$f(y_i) = \frac{\exp(-\mu_i)\mu_i^{yi}}{y_i!}, \; y_i = 0,1,2,... \quad (1)$$

with parameter μ, a key property of the Poisson model is that the mean and variance are equal; $E(y) = Var(y) = \mu$. To link the expected value of the response with a set of predictors, a transformation function, *g*, is introduced such that

$$g(\mu_i) = \eta_i = \beta_0 + \beta_1 x_1 + ... + \beta_p x_p = x_i'\beta \quad (2)$$

Here, g denotes the link function. Among several possible choices, the log link is most commonly employed in Poisson regression because it ensures that predicted values of the response remain strictly positive. The model parameters are typically estimated via the method of maximum likelihood. The likelihood function for the Poisson regression model is expressed as

$$l(\beta) = \prod_{i=1}^{n} \frac{\exp(-\mu_i)\mu_i^{yi}}{y_i!}. = \frac{\prod_{i=1}^{n} \mu_i^{yi} \exp\left(-\sum_{i=1}^{n}\mu_i\right)}{\prod_{i=1}^{n} y_i!} \quad (3)$$

$\mu_i = g^{-1}(x_i'\beta)$. Once a link function is specified, the estimation process involves maximizing the log-likelihood,

$$\ln l(\beta) = \sum_{i=1}^{n} y_i \ln(\mu_i) - \sum_{i=1}^{n}\mu_i - \sum_{i=1}^{n}\ln(y_i!) \quad (4)$$

which is nonlinear in the coefficients β. As closed-form solutions are not available, iterative numerical procedures are applied. One such procedure is the Fisher Scoring algorithm, where the update rule is given by:

$$\beta^{t+1} = \beta^t + I^{-1}(\beta^t)S(\beta^t) \quad (5)$$

where $S(\beta) = \frac{\partial l(\beta)}{\partial \beta}$ and $I^{-1}(\beta) = \left(-E(\partial^2 l(\beta)/\partial\beta\partial\beta')\right)^{-1}$. At convergence, the estimator of β is obtained as:

$$\hat{\beta}^{PMLE} = (X'\hat{W}X)^{-1} X'\hat{W}\hat{z} \quad (6)$$

where $\hat{W} = diag(\mu_i^2)$ matrix and $\hat{z} = x_i'\hat{\beta}^{MLE} + \frac{y_i - \hat{\mu}_i}{\hat{\mu}_i^2}$, is the adjusted dependent variable. Both $\hat{W}$ and $\hat{z}$ are computed iteratively through the Fisher Scoring steps (Hardin and Hilbe, 2012). The asymptotic covariance matrix of the estimated coefficients and the mean square error are given by:

$$Cov(\hat{\beta}_{MLE}) = \varphi(X'\hat{W}X)^{-1} \quad (7)$$

where $\varphi = (1/(n-p))\sum_{i=1}^{n}\left((y_i - \hat{\theta}_i)^2 / \hat{\theta}_i^2\right)$

and

$$MSE(\hat{\beta}_{MLE}) = \varphi\sum_{i=i}^{p}\frac{1}{\lambda_i}, \quad (8)$$

where λ_i is the i[th] eigenvalue of the matrix $X'\hat{W}X$.

Liu Estimator in Linear and Its Application in Poisson

The Poisson Liu Estimator (PLE) was also defined by Qasim *et al.* (2020) and Amin *et al.* (2021) as follows:

$$\hat{\beta}_{PLE} = (X'\hat{W}X + I)^{-1}(X'\hat{W}X + dI)\hat{\beta}_{MLE},$$

0 < d <1 (9)

Qasim *et al.* (2020) and Amin *et al.* (2021), Akay and Ertan (2022), Lukman *et al.* (2022), and Abonazel *et al.* (2023) going by their scholarly work determine the optimal value of d as:

$$\hat{d}_1 = \max_j\left(0, \frac{1}{p+1}\sum_{j=1}^{p+1}\left(\frac{\hat{\alpha}_j^2 - 1}{max_j\left(\frac{1}{u_j}\right) + max_j(\hat{\alpha}_j^2)}\right)\right) \quad (10)$$

$$\hat{d}_2 = \max_j\left(0, \; \min_j\left(\frac{\hat{\alpha}_j^2 - 1}{max_j\left(\frac{1}{u_j}\right) + max_j(\hat{\alpha}_j^2)}\right)\right) \quad (11)$$

Ridge Estimator in Linear and its Application in Poisson

The Poisson Ridge Estimator was proposed by Mansson and Shukur (2011) while Kibria (2003) proposed estimators by taking the Geometric Mean, Arithmetic Mean, and Median (p≥ 3) of the ridge parameter. These estimators are respectively defined as:

$$\hat{K}_{HK}^{GM} = \frac{\hat{\sigma}^2}{(\prod_{i=1}^{p}\hat{\alpha}_1^2)^{\frac{1}{p}}} \quad (12)$$

$$\hat{K}_{HK}^{AM} = \frac{\hat{\sigma}^2}{p}\sum_{i=1}^{p}\frac{1}{\hat{\alpha}_i^2} \quad (13)$$

$$\hat{K}_{HK}^{M} = Median\left(\frac{\hat{\sigma}^2}{\hat{\alpha}_1^2}\right) \quad (14)$$

The Poisson Ridge Estimator proposed by Månsson and Shukur (2011) is defined as:

$$\hat{\beta}_{PRE} = (X'\hat{W}X + kI)^{-1}X'\hat{W}X\hat{\beta}_{MLE}, \; k>0 \quad (15)$$

Kibria – Lukman Estimator in Linear Regression and Its Applications in Poisson (PKLE)

The Kibria – Lukman (KL) estimator proposed by Kibria and Lukman (2020) is defined as:

$$\hat{\beta}_{KL} = (X'X + kI)^{-1}(X'X - kI)\hat{\beta}_{ML} \quad (16)$$

Lukman *et al.* (2021) developed a Poisson Kibria-Lukman estimator as a remedy to multicollinearity in PRM, and it is defined as:

$$\hat{\beta}_{PKL} = (X'\hat{W}X + kI)^{-1}(X'\hat{W}X - kI)\hat{\beta}_{PML} \quad (17)$$

Proposed Poisson New Liu Ridge Type Estimator (PNLRTE)

Following the works of Amin *et al.* (2021) and Olakunle *et al.* (2025), this research developed the Poisson New Liu Ridge Type (PNLRT) Estimator**.** The newly proposed PNLRT Estimator is defined as:

$$\hat{\beta}_{PNLRT} = (X'WX + I)^{-1}(X'WX + dI)\hat{\beta}_{PNRT} \quad (18)$$

The properties of the newly proposed estimator $\hat{\alpha}_{PNLRT}$ in its canonical form, which are the expectation, bias, and dispersion matrix are determined in the following ways, respectively:

$$E(\hat{\alpha}_{PNLRT}) = E\left((\Lambda + I)^{-1}(\Lambda + dI)\hat{\alpha}_{PNRT}\right)$$

$$= E\left((\Lambda + I)^{-1}(\Lambda + dI)(\Lambda + (k+d)I)^{-1}\Lambda\hat{\alpha}_{PMLE}\right)$$

(19)

$$Bias(\hat{\alpha}_{PNLRT}) = E(\hat{\alpha}_{PNLRT}) - \alpha$$
$$= -(\Lambda k + \Lambda + k + d)(\Lambda + I)^{-1}(\Lambda + (k+d)I)^{-1}\alpha$$
(20)

$$Cov(\hat{\alpha}_{PNLRT}) = (\Lambda + I)^{-1}(\Lambda + dI)(\Lambda + (k+d)I)^{-1}\Lambda(\Lambda + I)^{-1}(\Lambda + dI)(\Lambda + (k+d)I)^{-1}$$
(21)

The MSEM and MSE in terms of eigenvalues are defined respectively as:

$$MSEM(\hat{\alpha}_{PNLRT}) = Cov(\hat{\alpha}_{PNLRT}) + Bias(\hat{\alpha}_{PNLRT})Bias(\hat{\alpha}_{PNLRT})'$$
$$= (\Lambda + I)^{-1}(\Lambda + dI)(\Lambda + (k+d)I)^{-1}\Lambda(\Lambda + I)^{-1}(\Lambda + dI)(\Lambda + (k+d)I)^{-1}$$
$$+ ((\Lambda + I)^{-1}(\Lambda + dI)(\Lambda + (k+d)I)^{-1}\Lambda - I)\alpha\alpha'((\Lambda + I)^{-1}(\Lambda + dI)(\Lambda + (k+d)I)^{-1}\Lambda - I)'$$
(22)

$$MSE(\hat{\alpha}_{PNLRT}) = tr(MMSE(\hat{\alpha}_{PNLRT}))$$

$$MSE(\hat{\alpha}_{PNLRT}) = \sum_{j=1}^{p}\frac{\lambda_j(\lambda_j + d)^2}{(\lambda_j + I)^2(\lambda_j + (k+d))^2} + \sum_{j=1}^{p}\frac{(\lambda_j k + \lambda_j + k + d)^2\alpha_j^2}{(\lambda_j + I)^2(\lambda_j + (k+d))^2}$$
(23)

Superiority of the PNLRT with existing estimators using the MSEM criterion

The newly proposed estimator PNLRT is compared with Poisson Maximum Likelihood (ML), Poisson ridge estimator (PRE), Poisson Liu estimator (PLE), Poisson Kibria-Lukman estimator (PKLE), Poisson Modified Ridge-Type estimator (PMRTE), and Poisson Liu Two-Parameter Estimator (PLTPE).

Comparisons of $\hat{\alpha}_{PNLRT}$ with existing estimators

1. Comparison of $\hat{\alpha}_{PNLRT}$ and $\hat{\alpha}_{PMLE}$

$$\hat{\alpha}_{PMLE} = \Lambda^{-1}Z'\hat{W}\hat{c} \quad \text{with} \quad Cov(\hat{\alpha}_{PMLE}) = \phi\Lambda^{-1}$$
(24)

Theorem 1: $\hat{\alpha}_{PNLRT}$ is better than $\hat{\alpha}_{PMLE}$ if

$$\alpha'((\Lambda + I)^{-1}(\Lambda + dI)(\Lambda + (k+d)I)^{-1}\Lambda - I)'(F)^{-1}((\Lambda + I)^{-1}(\Lambda + dI)(\Lambda + (k+d)I)^{-1}\Lambda - I)\alpha < 1$$
(25)

$$F = \Lambda^{-1} - (\Lambda + I)^{-1}(\Lambda + dI)(\Lambda + (k+d)I)^{-1}\Lambda(\Lambda + I)^{-1}(\Lambda + dI)(\Lambda + (k+d)I)^{-1}$$
(26)

Proof:
The difference of the dispersion is

$$Cov(\hat{\alpha}_{PMLE}) - Cov(\hat{\alpha}_{PNRTE}) = \Lambda^{-1} - (\Lambda + I)^{-1}(\Lambda + dI)(\Lambda + (k+d)I)^{-1}\Lambda(\Lambda + I)^{-1}(\Lambda + dI)(\Lambda + (k+d)I)^{-1}$$
(27)

$$= diag\left[\frac{1}{\lambda_j} - \frac{\lambda_j(\lambda_j + d)^2}{(\lambda_j + 1)^2(\lambda_j + (k+d))^2}\right]$$
(28)

It is observed that

$$\Lambda^{-1} - (\Lambda + I)^{-1}(\Lambda + dI)(\Lambda + (k+d)I)^{-1}\Lambda(\Lambda + I)^{-1}(\Lambda + dI)(\Lambda + (k+d)I)^{-1}$$

is positive definite for

$$(\lambda_j + 1)^2(\lambda_j + (k+d))^2 - \lambda_j^2(\lambda_j + d)^2 > 0$$

, for k > 0 and 0 < d < 1.

2. Comparison of $\hat{\alpha}_{PNLRT}$ and $\hat{\alpha}_{PRE}$

$$\hat{\alpha}_{PRE} = (\Lambda + kI)^{-1}\Lambda\hat{\alpha}_{PMLE}$$
(29)

And the matrix mean squared error is given as:

$$MMSE(\hat{\alpha}_{PRE}) = (\Lambda + kI)^{-1}\Lambda(\Lambda + kI)^{-1} + ((\Lambda + kI)^{-1} - I)\alpha\alpha'((\Lambda + kI)^{-1} - I)$$
(30)

Theorem 2: $\hat{\alpha}_{PNLRT}$ is better than $\hat{\alpha}_{PRE}$ if

$$\alpha'((\Lambda + I)^{-1}(\Lambda + dI)(\Lambda + (k+d)I)^{-1}\Lambda - I)'(G + ((\Lambda + kI)^{-1} - I)\alpha\alpha'((\Lambda + kI)^{-1} - I))^{-1}$$
$$* ((\Lambda + I)^{-1}(\Lambda + dI)(\Lambda + (k+d)I)^{-1}\Lambda - I)\alpha < 1$$
(31)

$$G = (\Lambda + kI)^{-1}\Lambda(\Lambda + kI)^{-1} - (\Lambda + I)^{-1}(\Lambda + dI)(\Lambda + (k+d)I)^{-1}\Lambda(\Lambda + I)^{-1}(\Lambda + dI)(\Lambda + (k+d)I)^{-1}$$
(32)

Proof:
The difference of the dispersion is

$$Cov(\hat{\alpha}_{PRE}) - Cov(\hat{\alpha}_{PNLRT})$$
$$= (\Lambda + kI)^{-1}\Lambda(\Lambda + kI)^{-1} - (\Lambda + I)^{-1}(\Lambda + dI)(\Lambda + (k+d)I)^{-1}\Lambda(\Lambda + I)^{-1}(\Lambda + dI)(\Lambda + (k+d)I)^{-1}$$
(33)

$$= diag\left[\frac{\lambda_j}{(\lambda_j + k)^2} - \frac{\lambda_j(\lambda_j + d)^2}{(\lambda_j + 1)^2(\lambda_j + (k+d))^2}\right]$$
(34)

It is observed that

$$(\Lambda + kI)^{-1}\Lambda(\Lambda + kI)^{-1} - (\Lambda + I)^{-1}(\Lambda + dI)(\Lambda + (k+d)I)^{-1}\Lambda(\Lambda + I)^{-1}(\Lambda + dI)(\Lambda + (k+d)I)^{-1}$$

is positive definite for

$$(\lambda_j + 1)^2(\lambda_j + (k+d))^2 - (\lambda_j + k)^2(\lambda_j + d)^2 > 0$$

, for k > 0 and 0 < d < 1.

3. Comparison of $\hat{\alpha}_{PNLRT}$ and $\hat{\alpha}_{PLE}$

$$\hat{\alpha}_{PLE} = (\Lambda + I)^{-1}(\Lambda + dI)\hat{\alpha}_{PMLE}$$
(35)

The matrix mean squared error (MMSE) of PLE is defined as:

$$MMSE(\hat{\alpha}_{PLE}) = (\Lambda + I)^{-1}(\Lambda + dI)\Lambda^{-1}(\Lambda + dI)(\Lambda + I)^{-1} + ((\Lambda + dI)(\Lambda + I)^{-1} - I)\alpha\alpha'((\Lambda + dI)(\Lambda + I)^{-1} - I)$$
(36)

Theorem 3: $\hat{\alpha}_{PNLRT}$ is better than $\hat{\alpha}_{PLE}$ if

$$\alpha'((\Lambda + I)^{-1}(\Lambda + dI)(\Lambda + (k+d)I)^{-1}\Lambda - I)'(H + ((\Lambda + dI)(\Lambda + I)^{-1} - I)\alpha\alpha'((\Lambda + dI)(\Lambda + I)^{-1} - I))^{-1}$$
$$* ((\Lambda + I)^{-1}(\Lambda + dI)(\Lambda + (k+d)I)^{-1}\Lambda - I)\alpha < 1$$
(37)

$$H = (\Lambda + I)^{-1}(\Lambda + dI)\Lambda^{-1}(\Lambda + dI)(\Lambda + I)^{-1} -$$
$$(\Lambda + I)^{-1}(\Lambda + dI)(\Lambda + (k+d)I)^{-1}\Lambda(\Lambda + I)^{-1}(\Lambda + dI)(\Lambda + (k+d)I)^{-1}$$
(38)

Proof:
The difference of the dispersion is

Science World Journal Vol. 20(No 3) 2025 https://dx.doi.org/10.4314/swj.v20i3.54
www.scienceworldjournal.org
ISSN: 1597-6343 (Online), ISSN: 2756-391X (Print)
Published by Faculty of Science, Kaduna State University

$$Cov(\hat{\alpha}_{PLE}) - Cov(\hat{\alpha}_{PNLRT}) =$$

$$(\Lambda+I)^{-1}(\Lambda+dI)\Lambda^{-1}(\Lambda+dI)(\Lambda+I)^{-1} - (\Lambda+I)^{-1}(\Lambda+dI)(\Lambda+(k+d)I)^{-1}\Lambda(\Lambda+I)^{-1}(\Lambda+dI)(\Lambda+(k+d)I)^{-1} \quad (39)$$

$$= diag\left[\frac{(\lambda_j+d)^2}{\lambda_j(\lambda_j+1)^2} - \frac{\lambda_j(\lambda_j+d)^2}{(\lambda_j+1)^2(\lambda_j+(k+d))^2}\right] \quad (40)$$

It is observed that

$(\Lambda+I)^{-1}(\Lambda+dI)\Lambda^{-1}(\Lambda+dI)(\Lambda+I)^{-1} - (\Lambda+I)^{-1}(\Lambda+dI)(\Lambda+(k+d)I)^{-1}\Lambda(\Lambda+I)^{-1}(\Lambda+dI)(\Lambda+(k+d)I)^{-1}$ is positive definite for $(\lambda_j+1)^2(\lambda_j+(k+d))^2 - \lambda_j^2(\lambda_j+1)^2 > 0$, for k > 0 and 0 < d <1. s

4. Comparison of $\hat{\alpha}_{PNLRTE}$ and $\hat{\alpha}_{PKLE}$

$$\hat{\alpha}_{PKLE} = (\Lambda + kI)^{-1}(\Lambda - kI)\hat{\alpha}_{PMLE} \quad (41)$$

The matrix mean squared error (MMSE) of PKLE is defined as:

$$MMSE\,(\hat{\alpha}_{PKLE}) = (\Lambda+kI)^{-1}(\Lambda-kI)\Lambda^{-1}(\Lambda-kI)(\Lambda+kI)^{-1} + \left((\Lambda+kI)^{-1}(\Lambda-kI)-I\right)\alpha\alpha'\left((\Lambda+kI)^{-1}(\Lambda-kI)-I\right)' \quad (42)$$

Theorem 4: $\hat{\alpha}_{PNLRT}$ is better than $\hat{\alpha}_{PKLE}$ if

$$\alpha'\left((\Lambda+I)^{-1}(\Lambda+dI)(\Lambda+(k+d)I)^{-1}\Lambda - I\right)'\left(I + \left((\Lambda+kI)^{-1}(\Lambda-kI)-I\right)\alpha\alpha'\left((\Lambda+kI)^{-1}(\Lambda-kI)-I\right)'\right)^{-1}$$
$$* \left((\Lambda+I)^{-1}(\Lambda+dI)(\Lambda+(k+d)I)^{-1}\Lambda - I\right)\alpha < 1 \quad (43)$$

$$I = (\Lambda+kI)^{-1}(\Lambda-kI)\Lambda^{-1}(\Lambda-kI)(\Lambda+kI)^{-1} - (\Lambda+I)^{-1}(\Lambda+dI)(\Lambda+(k+d)I)^{-1}\Lambda(\Lambda+I)^{-1}(\Lambda+dI)(\Lambda+(k+d)I)^{-1} \quad (44)$$

Proof:
The difference of the dispersion is

$$Cov(\hat{\alpha}_{PKLE}) - Cov(\hat{\alpha}_{PNLRT})$$
$$= (\Lambda+kI)^{-1}(\Lambda-kI)\Lambda^{-1}(\Lambda-kI)(\Lambda+kI)^{-1} - (\Lambda+I)^{-1}(\Lambda+dI)(\Lambda+(k+d)I)^{-1}\Lambda(\Lambda+I)^{-1}(\Lambda+dI)(\Lambda+(k+d)I)^{-1} \quad (45)$$

$$= diag\left[\frac{(\lambda_j-k)^2}{\lambda_j(\lambda_j+k)^2} - \frac{\lambda_j(\lambda_j+d)^2}{(\lambda_j+1)^2(\lambda_j+(k+d))^2}\right] \quad (46)$$

It is observed that

$(\Lambda+kI)^{-1}(\Lambda-kI)\Lambda^{-1}(\Lambda-kI)(\Lambda+kI)^{-1} - (\Lambda+I)^{-1}(\Lambda+dI)(\Lambda+(k+d)I)^{-1}\Lambda(\Lambda+I)^{-1}(\Lambda+dI)(\Lambda+(k+d)I)^{-1}$ is positive definite for $(\lambda_j-k)^2(\lambda_j+1)^2(\lambda_j+(k+d))^2 - \lambda_j^2(\lambda_j+d)^2(\lambda_j+k)^2 > 0$, for k > 0 and 0 < d < 1.

5. Comparison of $\hat{\alpha}_{PNLRT}$ and $\hat{\alpha}_{PMRTE}$

$$\hat{\alpha}_{PMRTE} = (\Lambda + k(1+d)I)^{-1}\Lambda\hat{\alpha}_{PMLE} \quad (47)$$

The matrix mean squared error (MMSE) of PKLE is defined as:

$$MMSE\,(\hat{\alpha}_{PMRTE}) = (\Lambda+I)^{-1}(\Lambda+dI)(\Lambda+I)^{-1}(\Lambda+dI)(\Lambda+k(1+d)I)^{-1}\Lambda(\Lambda+k(1+d)I)^{-1}$$
$$+ \left((\Lambda+k(1+d)I)^{-1}\Lambda - I\right)\alpha\alpha'\left((\Lambda+k(1+d)I)^{-1}\Lambda - I\right)' \quad (48)$$

Theorem 5: $\hat{\alpha}_{PNLRT}$ is better than $\hat{\alpha}_{PMRTE}$ if

$$\alpha'\left((\Lambda+I)^{-1}(\Lambda+dI)(\Lambda+(k+d)I)^{-1}\Lambda - I\right)'\left(J + \left((\Lambda+k(1+d)I)^{-1}\Lambda - I\right)\alpha\alpha'\left((\Lambda+k(1+d)I)^{-1}\Lambda - I\right)'\right)^{-1}$$
$$* \left((\Lambda+I)^{-1}(\Lambda+dI)(\Lambda+(k+d)I)^{-1}\Lambda - I\right)\alpha < 1 \quad (49)$$

$$J = (\Lambda+I)^{-1}(\Lambda+dI)(\Lambda+I)^{-1}(\Lambda+dI)(\Lambda+k(1+d)I)^{-1}\Lambda(\Lambda+k(1+d)I)^{-1}$$
$$-(\Lambda+I)^{-1}(\Lambda+dI)(\Lambda+(k+d)I)^{-1}\Lambda(\Lambda+I)^{-1}(\Lambda+dI)(\Lambda+(k+d)I)^{-1} \quad (50)$$

Proof:
The difference of the dispersion is

$$Cov(\hat{\alpha}_{PMRTE}) - Cov(\hat{\alpha}_{PNLRT})$$
$$= (\Lambda+I)^{-1}(\Lambda+dI)(\Lambda+I)^{-1}(\Lambda+dI)(\Lambda+k(1+d)I)^{-1}\Lambda(\Lambda+k(1+d)I)^{-1}$$
$$-(\Lambda+I)^{-1}(\Lambda+dI)(\Lambda+(k+d)I)^{-1}\Lambda(\Lambda+I)^{-1}(\Lambda+dI)(\Lambda+(k+d)I)^{-1} \quad (51)$$

$$= diag\left[\frac{\lambda_j(\lambda_j+d)^2}{(\lambda_j+1)^2(\lambda_j+k(1+d))^2} - \frac{\lambda_j(\lambda_j+d)^2}{(\lambda_j+1)^2(\lambda_j+(k+d))^2}\right] \quad (52)$$

It is observed that $(\Lambda+I)^{-1}(\Lambda+dI)(\Lambda+I)^{-1}(\Lambda+dI)(\Lambda+k(1+d)I)^{-1}\Lambda(\Lambda+k(1+d)I)^{-1}$

$$-(\Lambda+I)^{-1}(\Lambda+dI)(\Lambda+(k+d)I)^{-1}\Lambda(\Lambda+I)^{-1}(\Lambda+dI)(\Lambda+(k+d)I)^{-1} \quad (53)$$

is positive definite for $(\lambda_j+(k+d))^2-(\lambda_j+k(1+d))^2>0$, for k > 0 and 0 < d < 1.

6. Comparison of $\hat{\alpha}_{PNLRT}$ and $\hat{\alpha}_{PTPE}$

$$\hat{\alpha}_{PTPE} = (\Lambda+kI)^{-1}(\Lambda-dI)\hat{\alpha}_{PMLE} \quad (54)$$

The matrix mean squared error (MMSE) of PKLE is defined as:

$$MMSE\,(\hat{\alpha}_{PMRTE}) = (\Lambda+kI)^{-1}(\Lambda-dI)(\Lambda+kI)^{-1}(\Lambda-dI)+((\Lambda+kI)^{-1}(\Lambda-dI)-I)\alpha\alpha'((\Lambda+kI)^{-1}(\Lambda-dI)-I)' \quad (55)$$

Theorem 6: $\hat{\alpha}_{PMNLRT}$ is better than $\hat{\alpha}_{PTPE}$ if

$$\alpha'((\Lambda+I)^{-1}(\Lambda+dI)(\Lambda+(k+d)I)^{-1}\Lambda-I)'(H+((\Lambda+kI)^{-1}(\Lambda-dI)-I)\alpha\alpha'((\Lambda+kI)^{-1}(\Lambda-dI)-I)')^{-1}$$
$$*\,((\Lambda+I)^{-1}(\Lambda+dI)(\Lambda+(k+d)I)^{-1}\Lambda-I)\alpha<1$$

(56)

$$H=(\Lambda+kI)^{-1}(\Lambda-dI)(\Lambda+kI)^{-1}(\Lambda-dI)-(\Lambda+I)^{-1}(\Lambda+dI)(\Lambda+(k+d)I)^{-1}\Lambda(\Lambda+I)^{-1}(\Lambda+dI)(\Lambda+(k+d)I)^{-1} \quad (57)$$

Proof:
The difference of the dispersion is

$$Cov(\hat{\alpha}_{PTPE})-Cov(\hat{\alpha}_{PNLRT})$$
$$=(\Lambda+kI)^{-1}(\Lambda-dI)(\Lambda+kI)^{-1}(\Lambda-dI)-(\Lambda+I)^{-1}(\Lambda+dI)(\Lambda+(k+d)I)^{-1}\Lambda(\Lambda+I)^{-1}(\Lambda+dI)(\Lambda+(k+d)I)^{-1} \quad (58)$$

$$= diag\left[\frac{(\lambda_j-d)^2}{(\lambda_j+k)^2} - \frac{\lambda_j(\lambda_j+d)^2}{(\lambda_j+1)^2(\lambda_j+(k+d))^2}\right] \quad (59)$$

It is observed that
$(\Lambda+kI)^{-1}(\Lambda-dI)(\Lambda+kI)^{-1}(\Lambda-dI)-(\Lambda+I)^{-1}(\Lambda+dI)(\Lambda+(k+d)I)^{-1}\Lambda(\Lambda+I)^{-1}(\Lambda+dI)(\Lambda+(k+d)I)^{-1}$
is positive definite for $(\lambda_j-d)^2(\lambda_j+1)^2(\lambda_j+(k+d))^2-\lambda_j(\lambda_j+d)^2(\lambda_j+k)^2>0$, for k > 0 and 0 < d < 1.

Selection of parameters k and d for Poisson new Liu-ridge type estimator

$$MSE\,(\hat{\alpha}_{PNLRT}) = \sum_{j=1}^{p}\frac{\lambda_j(\lambda_j+d)^2}{(\lambda_j+I)^2(\lambda_j+(k+d))^2}+\sum_{j=1}^{p}\frac{(\lambda_jk+\lambda_j+k+d)^2\alpha_j^2}{(\lambda_j+I)^2(\lambda_j+(k+d))^2} \quad (60)$$

With *d* fixed, the optimal value of *k* is the one that minimizes $MSE(\hat{\alpha}_{PNLRT})$. Then, by differentiating g(k, d) with respect to *k* and equating to zero, we get

$$k=\sum_{j=1}^{p}\frac{\lambda_j(1-\alpha_j^2)+d(1-\alpha_j^2)}{\alpha_j^2(\lambda_j+1)} \quad (61)$$

However, *k* depends on the unknown α_j. For practical purposes, it will be replaced by its unbiased estimator $\hat{\alpha}_j$. Hence, this will be obtained as:

$$\hat{k}=\sum_{j=1}^{p}\frac{\lambda_j(1-\hat{\alpha}_j^2)+d(1-\hat{\alpha}_j^2)}{\hat{\alpha}_j^2(\lambda_j+1)}=k_1 \quad (62)$$

Three shrinkage parameters namely; median of *k* $(\hat{k}_{MED})$, Geometric Mean of *k* $(\hat{k}_{GM})$ and midrange of *k* $(\hat{k}_{MR})$ including the generalized form, are examined and studied for the Poisson New Ridge-Type (PNRT) Estimator. They are specified as follows:

$$k_{MED} = MED\left(\frac{1}{\hat{\alpha}_j^2} - d\right) \quad (63)$$

$$k_{GM} = \frac{1}{p}\sum_{j=1}^{p}\frac{1}{\alpha_j^2} - d \quad (64)$$

$$\hat{k}_{MR} = \frac{1}{2}\left[MAX\left(\frac{1}{\hat{\alpha}_j^2} - d\right) - MIN\left(\frac{1}{\hat{\alpha}_j^2} - d\right)\right] \quad (65)$$

Then, the optimal value of *d* can be considered to be the d that minimize $MSE(\hat{\alpha}_{PNLRT})$

Let

$$g(k,d) = MSE\left(\hat{\alpha}_{PNLRT}\right) = \sum_{j=1}^{p}\frac{\lambda_j(\lambda_j + d)^2}{(\lambda_j + I)^2(\lambda_j + (k+d))^2} + \sum_{j=1}^{p}\frac{(\lambda_j k + \lambda_j + k + d)^2\alpha_j^2}{(\lambda_j + I)^2(\lambda_j + (k+d))^2}$$

Then, by differentiating $MSE(\hat{\alpha}_{PNLRT})$ w.r.t. *d* and equating to 0, we have

$$d = \sum_{j=1}^{p}\left[\frac{\alpha_j^2 k(\lambda_j + 1) - \lambda_j(1 - \alpha_j^2)}{(1 - \alpha_j^2)}\right] \quad (66)$$

However, d depends on the unknown α_j. For practical purposes, it will be replaced by its unbiased estimator $\hat{\alpha}_j$. Hence, this will be obtained

$$\hat{d} = \sum_{j=1}^{p}\left[\frac{\alpha_j^2 k(\lambda_j + 1) - \lambda_j(1 - \alpha_j^2)}{(1 - \alpha_j^2)}\right] \quad (67)$$

Monte Carlo Simulation

Simulation Design

Several simulation experiments were conducted to evaluate the performance of the proposed biased estimator for Poisson regression under multicollinearity, in comparison with existing biased estimators. The study examined the influence of sample size (n), degree of multicollinearity (ρ), and number of explanatory variables (p). Following procedures used by earlier scholars (McDonald and Galarneau, 1975; Gibbons, 1981; Kibria, 2003; Lukman and Ayinde, 2017; Oladapo *et al.*, 2023), explanatory variables were generated as:

$$x_{ij} = (1-\rho^2)^{1/2} z_{ij} + \rho z_{ip+1}, i = 1,2,\ldots,p.\ j = 1,2,\ldots,p. \quad (68)$$

with z_{ij} drawn from a standard normal distribution and ρ^2 the correlation between the explanatory variable, set at 0.75, 0.85, 0.95, and 0.99. Simulations considered p=2, 4, and intercept values of -1, 0, and 1, and slope coefficients satisfying $\sum_{j=1}^{p}\beta_j^2 = 1$ and $\beta_1 = \beta_2 = \cdots = \beta_p$. Sample sizes are 25, 50, 75, 100, 150, 200. The experiments, implemented in R, involved 5000 replications, and performance was assessed using the mean squared error (MSE):

$$MSE\ (\beta^*) = \frac{1}{5000}\sum_{l=1}^{5000}(\beta^* - \beta)'\ (\alpha^* - \beta) \quad (69)$$

where α∗ denotes any of the estimators considered. The estimator with the smallest MSE was identified as the best-performing and most suitable under the simulation settings. The experiment is replicated 5000 times.

RESULTS

Table 1: Computed Mean Square Error (MSE) for the difference estimator P=2 and γ_{-1}

N	P	PMLE	PRE	PLE	PK-L	PMRT	PNLRT	PNLRT-ME	PNLRT-MR	PNLRT-GM
25	0.75	5.8368	0.5276	0.5290	0.4610	0.5290	0.5291	**0.3089**	0.6014	0.6197
	0.85	5.8029	0.6164	0.5895	0.5218	0.5895	0.5960	**0.3059**	0.6513	0.6649
	0.95	6.3423	1.0109	0.8215	0.8116	0.8215	0.8235	**0.4353**	0.9054	0.8402
	0.99	10.9002	3.2940	2.2094	2.4506	2.2094	2.2054	**1.0370**	2.5980	10.1644
50	0.75	5.9780	0.1743	0.2609	0.1733	0.2609	0.2609	**0.1616**	0.2860	0.3002
	0.	4.89	0.2	0.3	0.2	0.3	0.34	**0.18**	0.273	0.28

Science World Journal Vol. 20(No 3) 2025 https://dx.doi.org/10.4314/swj.v20i3.54
www.scienceworldjournal.org
ISSN: 1597-6343 (Online), ISSN: 2756-391X (Print)
Published by Faculty of Science, Kaduna State University

	85	35	073	421	018	421	21	**43**	1	61
	0.95	5.7837	0.3566	0.4760	0.3194	0.4764	0.4786	**0.2493**	0.4258	0.4346
	0.99	7.0140	1.0603	0.8448	0.8648	0.8451	1.3737	**0.3354**	662.3766	4.3403
75	0.75	5.8491	0.1258	0.1553	0.1242	0.1553	0.1553	**0.1202**	0.1341	0.1430
	0.85	4.8761	0.1462	0.1518	0.1420	0.1518	0.1518	**0.1327**	0.1472	0.1540
	0.95	5.4525	0.2366	0.2456	0.2168	0.2456	0.2456	**0.1856**	0.2491	0.2530
	0.99	6.2084	0.6416	0.5757	0.5336	0.5757	2.0716	**0.2142**	0.6586	0.6148
100	0.75	5.2017	0.0969	0.1000	0.0968	0.1000	0.1000	**0.0946**	0.0966	0.1016
	0.85	5.6027	0.1127	0.1093	0.1106	0.1093	0.1093	**0.1046**	0.1050	0.1080
	0.95	5.5114	0.1941	0.1853	0.1789	0.1853	0.1853	**0.1581**	0.1887	0.1900
	0.99	6.0615	0.5971	0.4760	0.4984	0.4759	0.4795	**0.4389**	0.5700	0.5718
150	0.75	5.1316	0.0496	0.0489	0.0493	0.0489	0.0489	**0.0489**	0.0495	0.0520
	0.85	4.8014	0.0559	0.0548	0.0561	0.0538	**0.0528**	0.0547	0.0539	0.0554
	0.95	5.3836	0.0980	0.0836	0.0939	0.0836	0.0836	0.0884	**0.0805**	0.0808
	0.99	5.6602	0.3164	0.2747	0.2718	0.2747	0.2750	**0.1904**	0.3406	0.3410
200	0.75	4.8607	0.0416	0.0408	0.0418	0.0398	**0.0388**	0.0414	0.0414	0.0425
	0.85	5.5702	0.0497	0.0478	0.0495	0.0468	**0.0458**	0.0481	0.0470	0.0476
	0.95	5.5497	0.0928	0.0779	0.0885	0.0779	0.0779	0.0838	0.0731	**0.0730**
	0.99	5.7262	0.3069	0.2584	0.2652	0.2584	0.2587	**0.1907**	0.3098	0.3098

NOTE: Estimators with the minimum MSE in each row are bold

Science World Journal Vol. 20(No 3) 2025 https://dx.doi.org/10.4314/swj.v20i3.54
www.scienceworldjournal.org
ISSN: 1597-6343 (Online), ISSN: 2756-391X (Print)
Published by Faculty of Science, Kaduna State University

Table 2: Computed Mean Square Error (MSE) for the difference estimator P=2 and γ_0

N	ρ	PMLE	PRE	PLE	PK-L	PMRT	PNLRT	PNLRT-ME	PNLRT-MR	PNLRT-GM
25	0.75	2.8037	0.1750	0.2832	0.1672	0.2832	0.2831	**0.1072**	0.2763	0.2762
	0.85	2.7365	0.2019	0.2746	0.1934	0.2746	0.2755	**0.0936**	0.2271	0.2263
	0.95	2.9196	0.3621	0.3332	0.3463	0.3330	0.3334	**0.1005**	0.2129	0.2139
	0.99	4.5159	1.1575	0.8329	1.0137	0.8324	0.6531	0.5145	**0.4999**	0.7329
50	0.75	2.9700	0.0618	0.2178	0.0602	0.2178	0.2178	**0.0567**	0.2717	0.2729
	0.85	2.5479	0.0729	0.2272	0.0708	0.2273	0.2273	**0.0628**	0.2737	0.2744
	0.95	2.8416	0.1315	0.2548	0.1274	0.2548	0.2548	**0.0760**	0.2576	0.2576
	0.99	3.2412	0.3950	0.4019	0.3779	0.4019	0.4083	**0.1201**	0.2886	0.2969
75	0.75	2.7013	0.0446	0.2038	0.0434	0.2038	0.2038	**0.0413**	0.2604	0.2607
	0.85	2.3338	0.0527	0.1955	0.0514	0.1956	0.1956	**0.0463**	0.2354	0.2355
	0.95	2.6584	0.0874	0.2183	0.0851	0.2183	0.2183	**0.0568**	0.2260	0.2256
	0.99	2.9445	0.2479	0.2926	0.2392	0.2926	0.2940	**0.0733**	0.2376	0.2415
100	0.75	2.6519	0.0356	0.1899	0.0350	0.1899	0.1899	**0.0337**	0.2672	0.2675
	0.85	2.8161	0.0400	0.1890	0.0393	0.1891	0.1891	**0.0343**	0.2298	0.2299
	0.95	2.7162	0.0752	0.1985	0.0734	0.1985	0.1985	**0.0446**	0.1887	0.1882
	0.99	2.8652	0.2266	0.2666	0.2187	0.2666	0.2675	**0.0557**	0.1982	0.2007
150	0.75	2.5550	0.0179	0.1745	0.0177	0.1745	0.1745	**0.0173**	0.2288	0.2290
	0.85	2.4907	0.0213	0.1677	0.0211	0.1677	0.1677	**0.0198**	0.2050	0.2051
	0.9	2.6694	0.0384	0.1767	0.0377	0.1767	0.1767	**0.0215**	0.1866	0.1867

	5									
	0.99	2.7745	0.1280	0.2227	0.1242	0.2227	0.2227	**0.0416**	0.1936	0.1940
200	0.75	2.4833	0.0151	0.1897	0.0149	0.1897	0.1897	**0.0145**	0.2526	0.2527
	0.85	2.7845	0.0183	0.1815	0.0182	0.1815	0.1815	**0.0168**	0.2286	0.2287
	0.95	2.7387	0.0338	0.1885	0.0332	0.1885	0.1885	**0.0298**	0.2113	0.2114
	0.99	2.7964	0.1165	0.2287	0.1131	0.2287	0.2287	**0.0397**	0.2084	0.2091

NOTE: Estimators with the minimum MSE in each row are bold

Table 3: Computed Mean Square Error (MSE) for the difference estimator P=2 and γ_1

N	ρ	PMLE	PRE	PLE	PK-L	PMRT	PNLRT	PNLRT-ME	PNLRT-MR	PNLRT-GM
25	0.75	5.0740	0.0687	0.0597	0.0675	0.0597	0.0597	**0.0541**	0.0610	0.0604
	0.85	5.1520	0.0835	0.0670	0.0812	0.0670	0.0670	**0.0581**	0.0671	0.0663
	0.95	5.4010	0.1665	0.1050	0.1542	0.1050	0.1052	0.1341	0.1045	**0.1032**
	0.99	5.8297	0.5296	0.2946	0.4466	0.2946	**0.2942**	0.6423	0.3196	0.3184
50	0.75	4.8877	0.0231	0.0254	0.0231	0.0244	**0.0224**	0.0231	0.0229	0.0228
	0.85	5.2975	0.0276	0.0264	0.0275	0.0264	0.0264	**0.0262**	0.0271	0.0269
	0.95	5.0004	0.0518	0.0469	0.0507	0.0469	0.0469	**0.0374**	0.0462	0.0457
	0.99	5.2264	0.1863	0.1063	0.1686	0.1061	0.1065	0.1252	0.0984	**0.0975**
75	0.75	4.9986	0.0171	0.0166	0.0171	0.0166	0.0166	**0.0165**	0.0169	0.0168
	0.85	5.1768	0.0196	0.0189	0.0196	0.0189	0.0189	**0.0184**	0.0192	0.0191
	0.95	5.0417	0.0345	0.0316	0.0339	0.0316	0.0316	**0.0272**	0.0319	0.0317
	0.99	5.0494	0.1131	0.0699	0.1047	0.0699	0.0699	**0.0604**	0.0668	0.0660
10	0.7	5.2030	0.0130	0.0129	0.0130	0.0129	0.0129	**0.0126**	0.0129	0.0129

0	5									
	0.85	5.1050	0.0152	0.0149	0.0151	0.0149	0.0149	**0.0143**	0.0149	0.0149
	0.95	5.3295	0.0283	0.0263	0.0278	0.0263	0.0263	**0.0227**	0.0264	0.0262
	0.99	5.0473	0.1062	0.0677	0.0983	0.0677	0.0677	**0.0516**	0.0636	0.0629
150	0.75	5.1827	0.0068	0.0068	0.0068	0.0068	**0.0067**	0.0068	0.0068	0.0068
	0.85	5.2953	0.0078	0.0078	0.0078	0.0078	0.0078	**0.0077**	0.0078	0.0078
	0.95	5.0466	0.0147	0.0143	0.0146	0.0143	0.0143	**0.0134**	0.0143	0.0142
	0.99	4.9570	0.0536	0.0433	0.0512	0.0433	0.0433	**0.0309**	0.0433	0.0429
200	0.75	5.2543	0.0056	0.0056	0.0056	0.0056	0.0056	**0.0056**	0.0056	0.0056
	0.85	5.0336	0.0068	0.0068	0.0068	0.0068	0.0068	**0.0067**	0.0068	0.0067
	0.95	4.9723	0.0129	0.0127	0.0129	0.0127	0.0127	**0.0119**	0.0127	0.0126
	0.99	5.1271	0.0472	0.0389	0.0452	0.0389	0.0389	**0.0280**	0.0390	0.0386

NOTE: Estimators with the minimum MSE in each row are bold

Table 4: Computed Mean Square Error (MSE) for the difference estimator P=4 and γ_{-1}

N	ρ	PMLE	PRE	PLE	PK-L	PMRT	PNLRT	PNLRT-ME	PNLRT-MR	PNLRT-GM
25	0.75	5.3239	0.6010	0.7189	0.5295	0.7189	0.7189	**0.4085**	1.0254	0.9390
	0.85	5.0204	0.6704	0.7887	0.5777	0.7888	0.7898	**0.4619**	1.0325	0.9412
	0.95	5.2124	1.1889	0.8852	0.9185	0.8853	0.8906	**0.5802**	1.1667	1.0700
	0.99	10.2456	4.1987	2.5822	2.9561	1.5823	1.6198	**1.5822**	3.9701	1.6043
50	0.75	4.1293	0.3657	0.4827	0.3298	0.4827	0.4827	**0.2658**	0.6351	0.5449
	0.85	4.1475	0.4525	0.5297	0.3938	0.5297	0.5299	**0.2966**	0.7380	0.6361
	0.	5.35	0.9	0.6	0.7	0.6	0.64	**0.44**	0.99	0.89

	95	58	040	370	048	373	42	**20**	94	80
	0.99	8.7681	3.2733	2.1178	2.3602	1.1181	1.2043	**1.1178**	1.7299	1.2075
75	0.75	4.5838	0.1780	0.1934	0.1733	0.1934	0.1934	**0.1723**	0.3007	0.2630
	0.85	3.8489	0.2178	0.2369	0.2078	0.2369	0.2369	**0.1835**	0.3557	0.3010
	0.95	4.6079	0.4493	0.4077	0.3930	0.4074	0.4077	**0.2909**	0.7124	0.6081
	0.99	6.3080	1.6461	0.9102	1.2573	0.9100	0.9507	**0.4957**	0.9863	0.9101
100	0.75	4.6717	0.1303	0.1399	**0.1286**	0.1401	0.1401	0.1308	0.2132	0.1875
	0.85	4.3599	0.1633	0.1870	0.1586	0.1870	0.1870	**0.1435**	0.2514	0.2138
	0.95	4.4277	0.3254	0.3411	0.2960	0.3412	0.3412	**0.2330**	0.5643	0.4701
	0.99	5.5181	1.1586	0.5951	0.9082	0.5949	0.5960	**0.5187**	0.9529	0.8626
150	0.75	3.8003	0.0812	0.0831	0.0804	0.0831	0.0831	**0.0802**	0.0965	0.0940
	0.85	3.5179	0.1005	0.0967	0.0985	0.0967	0.0967	**0.0908**	0.1044	0.0972
	0.95	4.3228	0.2070	0.1915	0.1937	0.1915	0.1915	**0.1582**	0.2893	0.2395
	0.99	5.0137	0.7983	0.4553	0.6518	0.4554	0.4555	**0.4217**	0.8200	0.7254
200	0.75	4.4174	0.0549	0.0614	0.0546	0.0614	0.0614	**0.0546**	0.0640	0.0652
	0.85	3.7184	0.0633	0.0686	0.0627	0.0686	0.0686	**0.0606**	0.0627	0.0619
	0.95	3.6877	0.1191	0.1121	0.1146	0.1121	0.1121	**0.0974**	0.1132	0.1013
	0.99	4.3436	0.4491	0.3099	0.3834	0.3099	0.3103	**0.2767**	0.5663	0.4828

NOTE: Estimators with the minimum MSE in each row are bold

Table 5: Computed Mean Square Error (MSE) for the difference estimator P=4 and γ_0

N	ρ	PMLE	PRE	PLE	PK-L	PMRT	PNLRT	PNLRT-ME	PNLRT-MR	PNLRT-GM
2	0.	2.8	0.1	0.2	0.1	0.2	0.25	**0.12**	0.27	0.23

Science World Journal Vol. 20(No 3) 2025 https://dx.doi.org/10.4314/swj.v20i3.54
www.scienceworldjournal.org
ISSN: 1597-6343 (Online), ISSN: 2756-391X (Print)
Published by Faculty of Science, Kaduna State University

5	75	070	951	587	723	587	87	**69**	09	77
	0.85	2.7013	0.2314	0.2643	0.2027	0.2643	0.2643	**0.1388**	0.2522	0.2225
	0.95	2.7091	0.4051	0.3157	0.3515	0.3154	0.3157	**0.1651**	0.2240	0.1940
	0.99	4.2182	1.4041	0.6033	1.1295	0.6032	0.6087	0.8522	**0.2559**	0.3114
50	0.75	2.3676	0.1269	0.1986	0.1167	0.1986	0.1986	**0.0908**	0.2019	0.1716
	0.85	2.3344	0.1666	0.2086	0.1517	0.2086	0.2086	**0.1046**	0.1736	0.1481
	0.95	2.7603	0.3377	0.2602	0.2974	0.2603	0.2602	**0.1283**	0.1572	0.1327
	0.99	3.9707	1.2133	0.4923	0.9957	0.4923	0.4909	**0.1721**	0.1981	0.1741
75	0.75	2.4222	0.0639	0.1421	0.0605	0.1421	0.1421	**0.0536**	0.1488	0.1247
	0.85	2.1987	0.0802	0.1473	0.0755	0.1473	0.1473	**0.0631**	0.1305	0.1102
	0.95	2.4214	0.1645	0.1709	0.1510	0.1709	0.1709	**0.0878**	0.1077	0.0905
	0.99	3.0406	0.6123	0.3448	0.5292	0.3448	0.3446	**0.1084**	0.1316	0.1113
100	0.75	2.4242	0.0476	0.1665	0.0458	0.1665	0.1665	**0.0406**	0.1897	0.1524
	0.85	2.3705	0.0594	0.1686	0.0568	0.1686	0.1686	**0.0445**	0.1686	0.1360
	0.95	2.3985	0.1171	0.1907	0.1094	0.1907	0.1907	**0.0820**	0.1417	0.1146
	0.99	2.7899	0.4360	0.2965	0.3852	0.2963	0.2964	**0.1180**	0.1648	0.1336
150	0.75	2.2612	0.0297	0.1193	0.0289	0.1193	0.1194	**0.0265**	0.1411	0.1131
	0.85	2.2073	0.0372	0.1169	0.0361	0.1169	0.1169	**0.0309**	0.1174	0.0929
	0.95	2.3893	0.0770	0.1372	0.0731	0.1372	0.1372	**0.0611**	0.0961	0.0771
	0.99	2.6288	0.2872	0.2182	0.2580	0.2181	0.2181	0.1025	0.0980	**0.0777**
2	0.	2.3	0.0	0.1	0.0	0.1	0.11	**0.01**	0.14	0.11

Science World Journal Vol. 20(No 3) 2025 https://dx.doi.org/10.4314/swj.v20i3.54
www.scienceworldjournal.org
ISSN: 1597-6343 (Online), ISSN: 2756-391X (Print)
Published by Faculty of Science, Kaduna State University

00	75	952	198	186	194	187	87	**79**	50	35
	0.85	2.1323	0.0235	0.1069	0.0230	0.1069	0.1069	**0.0200**	0.1148	0.0901
	0.95	2.1910	0.0447	0.1130	0.0431	0.1130	0.1130	**0.0386**	0.0887	0.0700
	0.99	2.4037	0.1645	0.1723	0.1505	0.1722	0.1722	0.0844	0.0814	**0.0637**

NOTE: Estimators with the minimum MSE in each row are bold

Table 6: Computed Mean Square Error (MSE) for the difference estimator P=4 and γ_1

N	ρ	PMLE	PRE	PLE	PK-L	PMRT	PNLRT	PNLRT-ME	PNLRT-MR	PNLRT-GM
25	0.75	6.0323	0.0771	0.0740	0.0762	0.0740	0.0740	**0.0725**	0.0811	0.0762
	0.85	6.1153	0.0879	0.0830	0.0866	0.0830	0.0830	**0.0796**	0.0829	0.0806
	0.95	5.7954	0.1677	0.1421	0.1596	0.1421	0.1421	**0.1342**	0.1465	0.1344
	0.99	6.3329	0.6123	0.3332	0.5088	0.2796	0.2795	**0.2792**	0.4347	0.3791
50	0.75	5.9994	0.0493	0.0475	0.0490	0.0475	0.0475	**0.0442**	0.0453	0.0451
	0.85	6.2026	0.0645	0.0600	0.0636	0.0600	0.0600	**0.0535**	0.0560	0.0559
	0.95	5.6654	0.1383	0.1129	0.1314	0.1128	0.1128	**0.0968**	0.1016	0.0998
	0.99	6.1372	0.5271	0.2451	0.4390	0.2451	0.2500	**0.2450**	0.3688	0.3202
75	0.75	5.4013	0.0239	0.0252	0.0239	0.0242	**0.0232**	0.0234	0.0244	0.0243
	0.85	5.9889	0.0303	0.0295	0.0303	0.0295	0.0295	**0.0292**	0.0295	0.0294
	0.95	5.5063	0.0640	0.0551	0.0629	0.0551	0.0551	**0.0471**	0.0512	0.0523
	0.99	5.7313	0.2583	0.1403	0.2328	0.1403	0.2268	**0.1403**	0.1977	0.1700
100	0.75	5.4067	0.0179	0.0177	0.0179	0.0176	0.0179	**0.0174**	0.0181	0.0181
	0.85	5.6131	0.0223	0.0218	0.0223	0.0218	0.0218	**0.0217**	0.0219	0.0218
	0.	5.6	0.0	0.0	0.0	0.0	0.04	**0.03**	0.03	0.03

	95	944	451	418	446	418	18	**65**	97	99
	0.99	5.8261	0.1811	0.1198	0.1694	0.1198	0.1198	0.1412	0.1196	**0.1100**
150	0.75	5.8950	0.0113	0.0111	0.0110	0.0111	0.0111	**0.0110**	0.0112	0.0113
	0.85	5.9358	0.0138	0.0137	0.0138	0.0137	0.0137	0.0137	0.0137	**0.0137**
	0.95	5.5525	0.0292	0.0277	0.0290	0.0277	0.0277	**0.0255**	0.0270	0.0269
	0.99	5.6357	0.1203	0.0875	0.1150	0.0875	0.0875	**0.0778**	0.0793	0.0790
200	0.75	5.3284	0.0074	0.0077	0.0074	0.0076	**0.0074**	0.0076	0.0075	0.0076
	0.85	5.6819	0.0088	0.0088	0.0088	0.0089	**0.0087**	0.0088	0.0088	0.0088
	0.95	5.8138	0.0170	0.0165	0.0170	0.0165	0.0165	**0.0158**	0.0163	0.0163
	0.99	5.5880	0.0671	0.0557	0.0655	0.0557	0.0557	**0.0487**	0.0587	0.0504

NOTE: Estimators with the minimum MSE in each row are bold

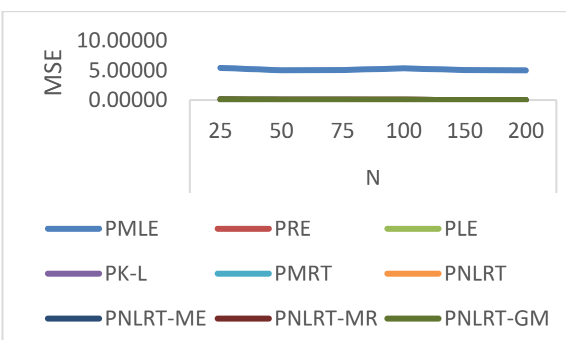

Figure 1: Effect of MSE with increasing Sample Size among Estimators when p=2 and γ_1

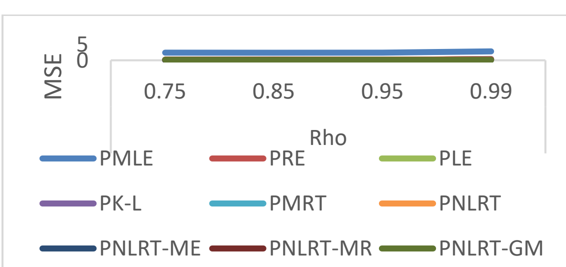

Figure 2: Effect of Correlation (ρ) on MSE among Estimators when n=100, p=4, and γ_0

DISCUSSION

Table 1 presents the Mean Squared Error (MSE) values for proposed and existing estimators under varying sample sizes and correlation levels (ρ), with number of parameters p=2 and intercept γ=−1. Across all scenarios, the Poisson Maximum Likelihood Estimator (PMLE) consistently produced the highest MSEs, confirming its poor performance in the presence of multicollinearity (see Figures 1 and 2). In contrast, PNLRT-ME, PK-L, and PMRT frequently emerged as the top three estimators with the lowest MSEs. Particularly, PNLRT-ME dominated most settings, especially at higher correlations and larger sample sizes, showcasing its robustness and adaptability. PK-L estimator showed strong performance in moderate sample sizes, often closely following PNLRT-ME, especially for lower ρ values. MLRT also ranked among the best in many instances, especially when ρ = 0.75 or 0.85. The performance of all estimators generally improved with increasing sample size (see Figure 1), and their MSEs increased with rising multicollinearity (see Figure 2), though the top three estimators maintained relative superiority across conditions. These patterns suggest that PNLRT-ME is the most efficient estimator across different scenarios, with PK-L and MLRT as reliable alternatives depending on the specific correlation level and sample size.

Table 2 evaluates the performance of proposed and existing estimators based on their Mean Squared Error (MSE) when the intercept γ=0 and number of parameters p=2. Across all combinations of sample size and correlation level (ρ), the PMLE continues to display the highest MSEs, confirming its sensitivity to multicollinearity. In contrast, the three most consistently best-performing estimators are PNLRT-ME, K-L, and PRE. The PNLRT-ME estimator clearly dominates, frequently attaining the lowest MSE across nearly all sample sizes and correlation levels. The PNLRT-ME especially excels under higher correlations and smaller samples, where other estimators tend to underperform. The PK-L estimator often comes close behind PNLRT-ME, particularly for moderate sample sizes and lower ρ values, while PRE exhibits strong performance in larger samples, typically emerging as the second or third best. As the sample size increases, MSE values generally decline across all estimators (see Figure 1), but the relative advantage of PNLRT-ME remains stable. Notably, PNLRT-ME consistently adapts well across varying conditions, reinforcing its robustness, while PK-L and PRE serve as strong alternatives depending on the multicollinearity level and size of data.

Table 3 assesses the Mean Squared Error (MSE) of proposed and existing estimators under the condition γ=1 and p=2, across different sample sizes and correlation levels (ρ). Once again, the PMLE consistently shows the highest MSEs, confirming its poor suitability in multicollinear settings. In contrast, the best-performing estimators across nearly all settings are PNLRT-ME, PLE, and MLRT, with PNLRT-ME emerging as the most dominant. Its performance advantage is especially noticeable at higher values of ρ (0.95 and 0.99), where multicollinearity is most severe. The PLE estimator performs remarkably well in low to moderate correlations, often tying with or trailing PNLRT-ME by a small margin. MLRT also stands out in several instances, particularly in smaller samples, maintaining competitiveness even under extreme correlations. As sample size increases, all estimators tend to improve (i.e. lower MSEs), but the relative superiority of PNLRT-ME remains consistent. The consistently minimal MSE values of PNLRT-ME highlight its robustness across scenarios, making it the most reliable choice when dealing with collinearity and varying sample sizes, while PLE and PMRT serve as strong secondary options.

Table 4 examines the Mean Squared Error (MSE) of proposed and existing estimators when the number of parameters p=4 and the intercept γ=−1, under different correlation levels (ρ) and sample sizes. As in Tables 1-3, the PMLE consistently yields the highest MSEs, indicating poor performance in multicollinear settings. In contrast, the standout performers across most scenarios are PNLRT-ME, PK-L, and PRE. Notably, PNLRT-ME frequently records the lowest MSE, especially at higher sample sizes and extreme correlations, highlighting its robustness under severe multicollinearity. The PK-L estimator performs particularly well in small to moderate samples and lower correlation scenarios, occasionally outperforming others. PRE consistently performed well, especially in larger samples. While the PLE and PMRT estimators occasionally show competitive MSEs, they generally lag behind the top three. As the sample size increases, the MSEs of all estimators reduce significantly, but PNLRT-ME maintains a consistent edge, followed closely by PK-L and PRE, depending on the ρ value. This pattern shows that PNLRT-ME is the most efficient estimator, especially in situations with extreme multicollinearity.

Table 5 presents the Mean Squared Error (MSE) values for proposed and existing estimators when the number of parameters p=4 and intercept γ=0, across a range of sample sizes and correlation coefficients (ρ). As expected, PMLE continues to perform poorly across all scenarios, with the highest MSEs in every case. On the other hand, the three best-performing estimators are consistently PNLRT-ME, PK-L, and PRE. Among them, PNLRT-ME stands out as the most robust, recording the lowest MSEs in a majority of settings, especially as sample size increases or correlation becomes extreme (ρ=0.99). The PK-L estimator performs well at lower correlations and smaller samples, often ranking second to PNLRT-ME. PRE maintains steady performance, particularly in moderate correlation settings, where it occasionally outperforms others in terms of MSE. As sample size increases, all estimators show decreased MSEs (as shown in Figure 1), but PNLRT-ME's dominance remains evident, especially for extreme multicollinearity situations. This confirms PNLRT-ME as the most efficient estimator overall, with PK-L and PRE serving as reliable alternatives based on the data structure and multicollinearity intensity.

Table 6 presents the Mean Squared Error (MSE) for different estimators under the condition p=4 and intercept γ=1, across varying sample sizes and correlation levels (ρ). As expected, PMLE again records the highest MSEs in all cases (as evident in Figures 1 and 2), confirming its sensitivity to multicollinearity. The three consistently best-performing estimators are PNLRT-ME, PLE, and PRE. However, PNLRT-ME shows consistency, often getting the lowest MSE or coming very close to the best result across nearly all sample sizes and ρ values. It is especially effective at high correlation levels (ρ=0.95 and ρ=0.99), where other estimators tend to degrade. The PLE also performs strongly, often producing MSEs very close to or equal to those of PNLRT-ME, particularly in moderate sample sizes and mid-range correlations. PRE, while slightly less competitive than PNLRT-ME and PLE, performs steadily and reliably, especially in smaller to moderate correlation settings. A clear pattern observed is that as the sample size increases, MSEs for all estimators reduce substantially. However, PNLRT-ME retains a consistent edge across all levels, reinforcing its efficiency for estimating in the presence of multicollinearity and large parameter settings.

This study comprehensively evaluated the performance of several existing estimators, namely, PMLE, PRE, PLE, PK-L, PMRT, and proposed estimators like PNLRT, PNLRT-ME, PNLRT-MR, and PNLRT-GM under various configurations of intercept (γ = −1,0,1), number of parameters (P=2,4), sample sizes (n = 25, 50, 75, 100, 150 and 200), and degrees of multicollinearity (ρ = 0.75, 0.85, 0.95, 0.99). The comparison was based on the Mean Squared Error (MSE) of the difference estimators across all conditions.

Across all scenarios, the Poisson Maximum Likelihood Estimator (PMLE) consistently recorded the highest MSE, clearly indicating its inefficiency in the presence of multicollinearity. This reaffirms its sensitivity to the ill-conditioning of the design matrix, especially under high correlations.

Among the proposed estimators, PNLRT-ME stood out as the most efficient and consistent performer across all six tables. It exhibited minimal MSE in the majority of cases, regardless of the number of parameters, intercept level, or sample size. It was particularly superior under high multicollinearity (ρ=0.95, 0.99) and with larger sample sizes, confirming its efficiency.

The PLE and PK-L estimators also showed strong performance, especially under moderate correlations and smaller samples. PLE was often among the top three performing estimators, particularly when the intercept was positive. PK-L, though not always the best, consistently remained competitive and showed low MSE in low to moderate correlation scenarios. PRE estimator, while not leading, offered reliable performance and was particularly effective as sample sizes increased.

In contrast, PNLRT-GM and PNLRT-MR showed inconsistent performance, sometimes coming close to the best estimators, but in many cases underperforming. This variability makes them less desirable for general-purpose estimation under multicollinearity.

CONCLUSION

In conclusion, PNLRT-ME is the most efficient estimator across all examined conditions, while PLE and PK-L are good alternatives under moderate multicollinearity. All estimators improve with increasing sample size, but relative efficiency remains largely unchanged.

Further studies will consider higher-dimensional parameter settings beyond p = 4 and incorporate real-world datasets to better assess the estimators' performance under practical conditions, including challenges such as outliers and heteroscedasticity.

REFERENCES

Abdelwahab, M. M., Abonazel, M. R., Hammad, A. T., & El-Masry, A. M. (2024). Modified two parameter Liu estimator for addressing multicollinearity in the Poisson regression model. *Axioms, 13*(1). https://doi.org/10.3390/axioms13010046

Abonazel, M. (2023). New modified two-parameter Liu estimator for the Conway–Maxwell Poisson regression model. *Journal of Statistical Computation and Simulation, 93*, 1976–1996.

Akay, K. U., & Ertan, E. (2022). A new improved Liu-type estimator for Poisson regression models. *Hacettepe Journal of Mathematics and Statistics, 51*, 1484–1503.

Algamal, Z. Y. (2018). Biased estimators in Poisson regression model in the presence of multicollinearity: A subject review. *Al-Qadisiyah Journal for Administrative and Economic Sciences, 20*(1).

Alkhamisi, M., & Shukur, G. (2008). Developing ridge parameters for SUR model. *Communications in Statistics—Theory and Methods, 37*(4), 544–564. https://doi.org/10.1080/03610920701469152

Alrweili, H. (2024). Kibria–Lukman hybrid estimator for handling multicollinearity in Poisson regression model: Method and application. *International Journal of Mathematics and Mathematical Sciences, 2024*, Article ID 1053397.

Amin, M., Akram, M. N., & Kibria, B. M. G. (2021). A new adjusted Liu estimator for the Poisson regression model. *Concurrency and Computation: Practice and Experience, 33*(20).

Ayinde, K., Lukman, A. F., Samuel, O. O., & Ajiboye, S. A. (2018). Some new adjusted ridge estimators of linear regression model. *International Journal of Civil Engineering and Technology, 9*(11), 2838–2852.

Cameron, A. C., & Trivedi, P. K. (2013). *Regression analysis of count data* (2nd ed.). Cambridge University Press.

Dempster, A. P., Schatzoff, M., & Wermuth, N. (1977). A simulation study of alternatives to ordinary least squares. *Journal of the American Statistical Association, 72*(357), 77–91. https://doi.org/10.1080/01621459.1977.10479910

Gibbons, D. G. (1981). A simulation study of some ridge estimators. *Journal of the American Statistical Association, 76*(373), 131–139. https://doi.org/10.1080/01621459.1981.10477619

Hardin, J. W., & Hilbe, J. M. (2012). *Generalized linear models and extensions* (3rd ed.). Stata Press.

Hoerl, A. E., & Kennard, R. W. (1970). Ridge regression: Biased estimation for non-orthogonal problems. *Technometrics, 12*(1), 55–67. https://doi.org/10.1080/00401706.1970.10488634

Hoerl, A. E., Kennard, R. W., & Baldwin, K. F. (1975). Ridge regression: Some simulations. *Communications in Statistics, 4*(2), 105–123. https://doi.org/10.1080/03610927508827232

Hoerl, R. (2020). Ridge regression: A historical context. *Technometrics, 62*(4), 420–425. https://doi.org/10.1080/00401706.2020.1742207

Idowu, J. I., Owolabi, A. T., Oladapo, O. J., ʹAyinde, K., Oshuoporu, O. A., & Alao, A. N. (2023). Mitigating multicollinearity in linear regression model with two parameter Kibria–Lukman estimators. *WSEAS Transactions on Systems and Control, 18*, 612–627. https://doi.org/10.37394/23203.2023.18.63

Khalaf, G., & Shukur, G. (2005). Choosing ridge parameters for regression problems. *Communications in Statistics—Theory and Methods, 34*(5), 1177–1182. https://doi.org/10.1081/STA-200056836

Kibria, B. (2003). Performance of some new ridge regression estimators. *Communications in Statistics—Simulation and Computation, 32*(2), 419–435.

Kibria, B. M. G., & Banik, S. (2016). Some ridge regression estimators and their performances. *Journal of Modern Applied Statistical Methods, 15*, 206–238. https://doi.org/10.22237/jmasm/1462075860

Kibria, B. M. G., & Lukman, A. F. (2020). A new ridge-type estimator for the linear regression model: Simulations and applications. *Scientifica*. https://doi.org/10.1155/2020/9758378

Lawless, J. F., & Wang, P. (1976). A simulation study of ridge and other regression estimators. *Communications in Statistics—Theory and Methods, 5*(4), 307–323. https://doi.org/10.1080/03610927608827353

Liu, K. (1993). A new class of biased estimate in linear regression. *Communications in Statistics, 22*(2), 393–402.

Lukman, A. F., Adewuyi, E., Månsson, K., & Kibria, B. M. G. (2021). A new estimator for the multicollinear

Poisson regression model: Simulation and application. *Scientific Reports, 11*, 3732. https://doi.org/10.1038/s41598-021-82582-w

Lukman, A. F., Aladeitan, B., Ayinde, K., & Abonazel, M. R. (2022). Modified ridge-type for the Poisson regression model: Simulation and application. *Journal of Applied Statistics, 49*(8), 2124–2136.

Lukman, A. F., Owolabi, A. T., Akanni, O. O., Kporxah, C., & Farghali, R. (2024). Robust enhanced ridge-type estimation for the Poisson regression models: Application to English League Football data. *International Journal of Uncertainty.*

Lukman, A. F., & Ayinde, K. (2017). Review and classifications of the ridge parameter estimation techniques. *Hacettepe Journal of Mathematics and Statistics, 46*, 953.

Månsson, K., & Shukur, G. (2011). A Poisson ridge regression estimator. *Economic Modelling, 28*, 1475–1481.

McDonald, G. C., & Galarneau, D. I. (1975). A Monte Carlo evaluation of ridge-type estimators. *Journal of the American Statistical Association, 70*(350), 407–416. https://doi.org/10.1080/01621459.1975.10479882

Montgomery, D. C., Peck, E. A., & Vining, G. G. (2012). *Introduction to linear regression analysis* (5th ed.). Wiley.

Muniz, G., & Kibria, B. M. G. (2009). On some ridge estimators: An empirical comparison. *Communications in Statistics—Simulation and Computation, 38*(3), 621–630. https://doi.org/10.1080/03610910802592838

Muniz, G., Kibria, B. M. G., Månsson, K., & Shukur, G. (2012). On developing ridge regression parameters: A graphical investigation. *Statistics and Operations Research Transactions, 36*(2), 115–138.

Oladapo, J. O., Idowu, J. I., Owolabi, A. T., & Ayinde, K. (2023). A new biased two-parameter estimator in linear regression model. *Equations, 3*(10), 73–81. https://doi.org/10.37394/232021.2023.3.10

Oladapo, O. J., Owolabi, A. T., Idowu, J. I., & Ayinde, K. (2022). A new modified Liu ridge-type estimator for the linear regression model: Simulation and application. *International Journal of Clinical Biostatistics and Biometrics, 8*(048).

Olakunle, K., Owolabi, A. T., & Olatayo, T. O. (2025). New ridge-type estimator to mitigate multicollinearity in the Poisson regression model: Theory and simulation. Hamburg: ZBW – Leibniz Information Centre for Economics.

Owolabi, A. T., Ayinde, K., & Alabi, O. O. (2022a). A new ridge-type estimator for the linear regression model with correlated regressors. *Concurrency and Computation: Practice and Experience, e6933.*

Owolabi, A. T., Ayinde, K., & Alabi, O. O. (2022b). A modified two-parameter estimator with different forms of biasing parameters in the linear regression model. *African Scientific Reports, 1*(3), 212–228. https://doi.org/10.46481/asr.2022.1.3.6

Qasim, M., Kibria, B., Månsson, K., & Sjölander, P. (2020). A new Poisson Liu regression estimator: Method and application. *Journal of Applied Statistics, 47*, 2258–2271.