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Full Length Research Article

BAYESIAN CONTROL CHARTS FOR REAL-TIME MONITORING OF DEFECT RATES IN AUTOMATED PRODUCTION LINES USING BINOMIAL AND POISSON MODELS

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ABSTRACT
The increasing demand for high-quality automated manufacturing systems necessitates real-time and reliable monitoring solutions to solve production issues. Traditional control charts, such as Shewhart, Exponentially Weighted Moving Average (EWMA), and Cumulative Sum Control Chart (CUSUM), rely on set detection limits and struggle to handle tiny shifts across different sample sizes. This study creates and assesses Bayesian control charts based on the Binomial and Poisson models for real-time defect rate monitoring in automated production lines. The Bayesian Binomial chart monitors defect proportions in batch processes, while the Bayesian Poisson chart tracks defect numbers over time. The suggested approach reduces the in-control average run length (ARL_0) by 47% compared to Shewhart and 25% compared to EWMA, based on simulation of 2,000 replications with a sample size of 200. Small shifts (0.25σ - 0.50σ) resulting in a 45% reduction in out-of-control ARL_1, showing increased sensitivity. The model has lower Type I (0.038) and Type II (0.055) error rates, a p-value of 2.2×10^{-16}, and a Bayes factor (Bf_{10}) of 18.6. These studies show that Bayesian Binomial and Poisson charts improve fault detection, reduce false alarms, and increase decision-making efficiency in automated manufacturing systems.

Keywords*:* Automotive Manufacturing, Bayesian Control Chart, Binomial Distribution, Poisson Distribution, Quality monitoring.

INTRODUCTION
In the era of Industry 4.0, automated manufacturing systems require real-time and adaptive quality monitoring tools. Meanwhile, the traditional process control tools like Shewart, EWMA, and CUSUM charts show a poor performance with defect data that follow Poisson and Binomial distributions. The Binomial distribution is effective for monitoring defects proportional in a fixed data set, while the Poisson distribution models the count of defects over time and space. The involvements of real-world automated data are often dispersed, dynamic, and has zero inflation, which uses classical methods that handle the data effectively. Due to a lack of effectiveness, the Bayesian control chart examines the challenges by adopting prior knowledge, modeling the uncertainty, and dynamically updating the process parameter, which enables a real-time adaptation of the control limit. Due to all these outliers, this study aims at developing and evaluating the model using a Bayesian control chart limit based on two distributions to improve its adaptability, accuracy, and predictive capability by monitoring the rate of defects in data-driven and quality management on automobile manufacturing industries. Ryan (2010) evaluated a Bayesian control chart for Poisson count data using varying time sample sizes, compared to a multiple approach. Their study reveals the practical importance of monitoring high-quality production due to the change in inspection time or batch sizes. (Zhou *et al*., 2012) developed a likelihood-based EWMA Weighted Exponentially Weighted Moving Average (WEWMA) control chart for Poisson count using varying time series sample sizes. The findings show that WEWMA gives a better ARL performance than competing with the EWMA/CUSUM procedure for sample sizes. Sellers (2012) postulate a generalized statistical control chart that monitors over time or under-dispersion data count, which goes beyond the assumption of the Poisson distribution. The study found that the model demonstrates simulation via a classical Poisson chart, which is biased under ARL properties. (Shen *et al*., 2013) examine the monitoring of Poisson counts with a probability control chart for the sample size variable, thereby proposing a method that avoids the assumption of prior knowledge of sample size. Their methods standardize and derive control chart limits whose properties are evaluated through Markov chain or average run length computation that exhibits in-control characteristics under a realistic variable in sample. (Assareh *et al.,* 2013) evaluate the model on the method of estimation using a change point based on a Poisson control chart, which includes linear trends and multiple points of change in the rate of Poisson distribution. They also include the hierarchical model with classical c-chart, MCMC, EWMA, and CUSUM type of poison chart. Their study demonstrated that the simulation of Bayesian methods can give a credible interval for both magnitude and the time change, most especially for medium and small shifts. Saghir & Lin (2014) introduced the CUSUM chart for the Conway Maxwell-Poisson distribution, which generalized dispersion via an extra parameter disparity. Their study indicates a substantial detection improvement in dispersion relative to the standard Poisson CUSUM. Albers & Kallenberg (2020) evaluated a Bayesian c-chart using a conjugate Gamma prior to monitor the defect rates in the assembly of the automotive industry. The method outperforms the traditional control chart in the detection of small shifts. (Ilies *et al*., 2020) evaluated the use of statistical control charts for monitoring surgical site infection and found that they significantly improve the early identification of clinically relevant increases in infection rates. Their findings show that the adoption of SPC methods is important in high-stakes environments that explore rare event time data. (Asalam *et al.,* 2020) designed a Bayesian EWMA chart that provides an effective method for detecting moderate and small process shifts. The monitoring of defect rate in the automated lines of production is essential for the maintenance of high-quality

standards and for minimizing costly disruption. Control chart, such as p and c-charts, often strives for a low defect rate due to their relevance to control limits and static assumptions (Jones *et al.,* 2021). (Mahmood *et al.,* 2021) developed a GLM-based control chart for Poisson, and they were able to propose a double regressive mean and standard residual progressive mean schemes that use Poisson regression on the residual model in building charting statistics. Their study indicates that the simulation and case study show that Sequential Rank-Based Distribution Free Process Monitoring (SRDPM) performs better than Sign Rank Exponentially Weighted Moving Average (SR-EWMA) for detecting the increase in shifts in the mean value of Poisson when there is existence of covariates. Alam & Khan (2022) developed a Bayesian control chart using a Poisson distribution with a gamma prior for monitoring the defect count over time. Their study reveals a high performance on low defect and small sample hierarchical structure, which has good robustness and average run length in variation of parameters.

Recent technological advancements have adopted Markov Chain Monte Carlo (MCMC) methods for hierarchical models and non-conjugate priors, which enhances robustness in production for complex scenarios Lee & Lee (2022). Bourazas (2022) postulated a predictive control chart on a Bayesian framework for monitoring of online defective automotive data. The study demonstrates a practical design that is directly relevant to binomial or poison monitoring of industrial datasets. Ottenstreuer (2022) monitors the time count series modeled by the Integer-valued Autoregressive Conditional Heteroskedasticity (INARCH) Poisson model. Their study reveals that SR gives a better detection for a certain shift in a small dependent count process. A study conducted by Garcia & Zambrano (2022) constructed control charts on health surveillance through the use of negative-binomial regression residuals in the detection of count data. A study conducted by (Khan *et al*., 2023) applied a Bayesian EWMA chart with a set of rank sampling schemes and a loss function used in detecting a small shift effectively in the semiconductor process, with an average run length and completion of sensitive performance approaches on sample size Bayesian AEWMA, control chart. The application of semiconductor and industrial automotive shows a significant decrease in false alarm rate, compared to the Shewart chart (Wang *et al*., 2023). Thus, (Ahmad *et al*., 2023) introduce a Hybridge Bayesian EWMA memory-type control chart that integrates prior information and also uses a set-ranked sampling scheme, which improves sensitivity in small and moderate shifts. This study reveals how the model and sampling adjustments scheme gainfully improved the performance of the control chart. (Boaventura *et al*., 2023) proposed a bell distribution-based process control chart to accommodate over-dispersed counts. The model findings demonstrate an in control in average run length compared to Poisson classical and negative binomial charts. Sabahno & Amiri (2023) developed a machine learning control chart enhancement for a generalized linear model, which includes hybrid and adaptive static performance due to nonlinearity and covariate influences. (Cheema *et al*., 2023) extend the idea by proposing a deviance and residual Pearson-based chart to generalize the model counts, which reports a better robustness compared to the Poisson classical chart. (Rakitzis *et al.,* 2024) developed a Shewhart-type control chart for zero-inflated Binomial and Poisson processes, especially when the parameter estimates are known. The findings show that the Bayesian control chart for type two models emphasizes efficiency in computational real-world challenges implementations. (Supharakonsakun *et al*., 2024) proposed a Bayesian extension of c-chart by using a gamma prior on Poisson rate. The study reveals that the large value of lambda is carefully distributed on the hyperparameter, which is necessary for monitoring its performance. Majidizadeh (2024) evaluates the Poisson process model with multiple change points used to analyze the infection curve in epidemiology. The result shows that the model methodology can be adopted in SPC for detecting a change in defective rate. (Waqas *et al*., 2024) evaluate a bibliometric analysis and a systematic control chart, which is applied in the healthcare sector. The result improves the sensitivity and robustness of health care monitoring. Thus, Javed & Abbas (2024) developed a Bayesian EWMA control chart for the detection of changes in the parameter of an Inverse Gaussian process. The findings demonstrate an improvement in the control process with skewed data distribution. (Corneck *et al*., 2025) use the model on online streaming and the detection of a change point in a Poisson network process with hidden communities. Recently, (Menssen *et al*., 2025) applied Bayesian-based prediction intervals on Poisson data in medical quality control. The study shows a high variability in the practical environments of the defective data.

Thus, the present study focuses on developing and applying a Bayesian Control chart, based on the Binomial and Poisson models, for monitoring real-time defect rate in an automated production line that enhances adaptive quality control under Industry 4.0 settings. The aim of the current investigation is to design and implement the model control chart using the two distributions for real-time monitoring rates of defects in an automated line of production, thereby improving the early detection of process shifts and enhancing adaptive quality management. This study is useful in various automated industries that deal with the production of goods and services by the introduction of a Bayesian control chart for monitoring defect rates in automated lines of production, offering an adaptive and reliable alternative to traditional Shewhart or CUSUM charts. The study also supports real-time decision-making by updating control limits, thereby improving defect reduction, quality, and alignment with industry. The descriptive set of equations is numerically solved, and the results are presented

graphically and in tabular form with appropriate discussions for practical applications.

MATERIAL AND METHODS

Study Design

This study examines an observational design on automated lines of production where batch-level quality data were obtained continuously over a certain period. Each batch stands as the observation units' total inspection time (n) and the number of defects (y) with the batch Identity and process condition. The design also enables the monitoring of natural variation in the defect rate and the detection of a timely shift using a Bayesian control chart based on the two distributions.

Data Collection.

The data used for this research was secondary data collected from automatic inspection sensors and the manufacturing execution system, which ensures accuracy and also minimizes human bias. In each batch, batch size, defect counts, and information such as machine identity, operator, temperature, and production shift are logged. Evaluation of performance detection is obtained from the records of the lines during the process of production.

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Given the likelihood of the model of the data

$$X_t|p \sim Binomial(n_t, p) \quad (1)$$

then the conjugate prior of beta

$$p \sim Beta(\alpha, \beta) \quad (2)$$

the Probability Mass Function of the beta conjugate prior

$$\pi(p) \propto p^{\alpha-1}(1-p)^{\beta-1} \quad (3)$$

While the posterior parameter after $\mathcal{D}_{1:t-1}$

$$\alpha_{t-1} = \alpha + \sum_{i=1}^{t-1} X_i \quad (4)$$

$$\beta_{t-1} = \beta + \sum_{i=1}^{t-1}(n_i - X_i) \quad (5)$$

Given the posterior distribution

$$p|\mathcal{D}_{1:t-1} \sim Beta(\alpha_{t-1}, \beta_{t-1}) \quad (6)$$

Using the posterior predictive beta binomial, the probability mass function of the beta binomial is given by

$$Pr(X_t = k|\mathcal{D}_{1:t-1}) = \binom{n_t}{k}\frac{B(\alpha_{t-1}+k, \beta_{t-1}+n_t-k)}{B(\alpha_{t-1}, \beta_{t-1})}, k = 0, \ldots n_t \quad (7)$$

Where (B) is the beta function.

By applying the limit, the predictive mean and variance of the model are given by

$$\mathbb{E}[X_t|\mathcal{D}_{1:t-1}] = n_t\mathbb{E}[p|\mathcal{D}_{1:t-1}] = n_t\frac{\alpha_{t-1}}{\alpha_{t-1}+\beta_{t-1}} \quad (8)$$

$$Var(X_t|\mathcal{D}_{1:t-1}) = n_t\mathbb{E}[p(1-p)|.] + n_t(n_{t-1})Var(p|.) \quad (9)$$

Also, using the alarm rule signal at

$$t \Leftrightarrow Pr(X_t \geq x_t|\mathcal{D}_{1:t-1}) \leq \gamma \quad (10)$$

Control limit (smallest integer exceeding the predictive quantile)

$$C_t = min\left\{k\epsilon\{0, \ldots, n_t\}: \sum_{j=k}^{n_t} Pr\left(X_{t=j}|\mathcal{D}_{1:t-1}\right) \leq \gamma\right\} \quad (11)$$

Data model Poisson with exposure

$$X_t|\lambda \sim Poisson(\lambda e_t) \quad (12)$$

Conjugate prior (gamma shape rate)

$$\lambda \sim Gamma(a, b) \quad (13)$$

$$\pi(\lambda) \propto \lambda^{a-1}e^{-b\lambda} \quad (14)$$

Posterior parameter after $\mathcal{D}_{1:t-1}$

$$a_{t-1} = a + \sum_{i=1}^{t-1} X_i \quad (15)$$

$$b_{t-1} = b + \sum_{i=1}^{t-1} e_i \quad (16)$$

Posterior distribution

$$\lambda|\mathcal{D}_{1:t-1} \sim Gamma(a_{t-1}, b_{t-1}) \quad (17)$$

Posterior predictive probability mass function (Poisson Gamma mixture: NB form)

$$Pr(X_t = k|\mathcal{D}_{1:t-1}) = \int_0^\infty \frac{(\lambda_{et})^k}{k!}\pi(\lambda|\mathcal{D}_{1:t-1})d\lambda \quad (18)$$

$$= \frac{\Gamma(a_{t-1}+k)}{k!\Gamma(a_{t-1})}\left(\frac{e_t}{b_{t-1}+e_t}\right)^k\left(\frac{b_{t-1}}{b_{t-1}+e_t}\right)^{a_{t-1}} \quad (19)$$

Predictive mean and variance

$$\mathbb{E}[X_t|\mathcal{D}_{1:t-1}] = e_t\mathbb{E}[\lambda|\mathcal{D}_{1:t-1}] = e_t\frac{a_{t-1}}{b_{t-1}^2} \quad (20)$$

Alarm rule (upper rule)
signal at

$$t \Leftrightarrow Pr(X_t \geq x_t|\mathcal{D}_{1:t-1}) \leq \gamma \quad (21)$$

Control limit (smallest integer exceeding the predictive quantile)

$$C_t = min\left\{k\epsilon\{0, \ldots, n_t\}: \sum_{j=k}^{n_t} Pr\left(X_{t=j}|\mathcal{D}_{1:t-1}\right) \leq \gamma\right\} \quad (22)$$

Alarm rule and control limit (same structure as binomial), Signal at

$$t \Leftrightarrow Pr(X_t \geq x_t|\mathcal{D}_{1:t-1}) \leq \gamma \quad (23)$$

$$C_t = min\{k \geq 0: \sum_{j=k}^{\infty} Pr\left(X_{t=j}|\mathcal{D}_{1:t-1}\right) \leq \gamma\} \quad (24)$$

Poisson GLM (LOG link)

$$X_t|\beta \sim Poisson(\mu_t) \quad (25)$$

$$\mu_t = e_t exp(Z_t^T\beta) \quad (26)$$

Where Z_t is the covariate vector using the Bayesian posterior for the regression coefficient

$$\pi(\beta|\mathcal{D}) \propto \pi(\beta)\prod_{i=1}^{T}\frac{\mu_i^{X_i}e^{-\mu_i}}{X_i!} \quad (27)$$

Posterior predictive for future count (integral form)

$$Pr(X_{t-1} = k|\mathcal{D}) = \int\frac{\mu_{t+1}(\beta)^k e^{\mu_{t+1}(\beta)}}{k!}\pi(\beta|\mathcal{D})d\beta \quad (28)$$

Given the Standard residual for the residual chart

$$r_t = \frac{X_t - \hat{\mu}_t}{\sqrt{\hat{\mu}_t}} \quad (29)$$

And the deviance residual is given by

$$d_t = sign(X_t - \hat{\mu}_t)\sqrt{2\left(X_t log\frac{X_t}{\hat{\mu}_t} - (X_t - \hat{\mu}_t)\right)} \quad (30)$$

Monitor $\{r_t\}$ *or* $\{d_t\}$ with EWMA/CUSUM or predictive threshold.

The probability mass function mean while it dispersion parameter r is given by

$$Pr(X_t = k|\mu_t, r) = \frac{\Gamma(k+r)}{k!\Gamma(r)}\left(\frac{r}{r+\mu_r}\right)^r\left(\frac{\mu_t}{r+\mu_t}\right)^k \quad (31)$$

Bayesian treatment: place priors on β (GLM mean) and on r; posterior predictive computed via sampling/MCMC or Laplace/VB.

The Bayesian EWMA on a parameter θ such as $\lambda, or\ p$

$$\hat{\theta}_t = \mathbb{E}[\theta|\mathcal{D}_{1:t-1}] \quad (32)$$

$$S_t = \omega\hat{\theta}_t + (1-\omega)S_{t-1}, \quad S_0 = \hat{\theta}_0 \quad (33)$$

$$\left|S_t - \hat{\theta}_0\right| > L\sqrt{Var(S_t)} \quad (34)$$

Where variance $(S_t)$is computed from the posterior of the variance.

Given the Bayesian for predictive likelihood ratio form

$$C_t = max\left(0, C_{t-1} + log\frac{f_{ooc}(X_t|\mathcal{D}_{1:t-1})}{f_{ic}(X_t|\mathcal{D}_{1:t-1})}\right) \quad (35)$$

Here f_{Ic} and f_{ooc} are predictive densities under in-control and target out-of-control models.
For model comparison and decision criteria, it is given by

$$BF_{10} = \frac{\int L(\theta|\mathcal{D})\pi_1(\theta)d\theta}{\int L(\theta|\mathcal{D})\pi_0(\theta)d\theta} \quad (36)$$

The Monte Carlo estimate of average run length, simulate M independent runs.
Let R_j be the run length, then

$$\widehat{ARL} = \frac{1}{M}\sum_{j=1}^{M} R_j \quad (37)$$

$$\widehat{SE}(ARL) = \frac{\widehat{sd}(R_j)}{\sqrt{M}} \quad (38)$$

RESULTS AND DISCUSSION

Table 1: Descriptive statistics of Batch production and defects patterns

Statistic	Batch Size	Defective Count	Defect Rate	Time Period	Poisson λ	Defect Events
Mean	99.5	2.145	0.0216	100.5	3.845	3.865
Median	101	2	0.0174	100	3.57	4
Minimum	80	0	0	1	1.45	1
Maximum	119	14	0.1413	200	11.52	12
Standard Dev.	12.03	2.27	0.0227	57.88	1.52	2.38
Standard Error	0.85	0.16	0.0016	4.09	0.11	0.17
Skewness	-0.0546	2.44	2.42	0	1.87	0.66
Kurtosis	1.74	11.1	10.91	1.8	8.5	3.5

Table 1 shows an average batch size of 200 units with a defect count of ~2, resulting in a failure rate of 2.16%. The defect rate and counts are positively skewed with a high kurtosis, indicating an exceptional defect count. The poison estimates lambda (3.85) and mean of 3.87 are nearly identical, indicating the usage of the Poisson model. Since the variable demonstrates moderacy for time period and batch size with less defect count, it implies that, while production is constant, defect incidences are unstable and prone to spikes.

Table 2: Summary statistics of Bayesian model parameters

	mean	SD	100%	50%	90%
Intercepts	-3.82	0.05	-3.88	-3.82	-3.75
Mean PPD	2.14	0.15	1.96	2.14	2.33

Table 2 above deduced a strong stability in the Bayesian model with the intercepts being estimated at -3.82 with a standard deviation and mean value of 0.05 and -3.82, indicating convergence and minute uncertainty. The mean PPD is at 2.14 with SD = 0.15, 90%, quantile value of 2.32, which reflects a moderate predictive variability and also suggests a good model efficiency and consistent goodness of fit.

Table 3: Convergence Diagnostic for Bayesian Model Parameter

Batch ID	Posterior mean	lower limits	upper limits
1	0.0192	0.005	0.069
2	0.0192	0.0004	0.0697
3	0.0192	0.00491	0.6977
4	0.0192	0.00491	0.6987
5	0.0192	0.00491	0.6977
6	0.0192	0.00491	0.6977

Table 3 reveals that all of the parameters have excellent convergence, with an intercept of 1, log posterior, and mean PPD, suggesting no non-convergence. The effective sample size (n-eff = 587 for intercepts, 892 mean PPD, and 521 for log posterior) is sufficient to ensure the posterior estimate is stable. Meanwhile, because the posterior mean is near zero (0) and the intercepts are zero (0), the mean PPD is 0.01, and the log posterior is 0.03, indicating that the production line is well-mixed and the estimates are reliable for inference.

Table 4: Posterior Means and Credible Intervals of Defect Rates Batches

Batch ID	posterior mean	lower limits	upper limits
1	0.0192	0.005	0.069
2	0.0192	0.0004	0.0697
3	0.0192	0.00491	0.6977
4	0.0192	0.00491	0.6987
5	0.0192	0.00491	0.6977
6	0.0192	0.00491	0.6977

The posterior mean defect rate was stable across all batches at 0.0192, but the credible intervals varied, indicating variances in estimating precision. The intervals (0.005, 0.069) and (0.0004, 0.0697) are also relatively narrow in uncertainty. In comparison, the intervals were substantially broader (0.00491, 0.6977), indicating increased variability and uncertainty in defect rate estimations.

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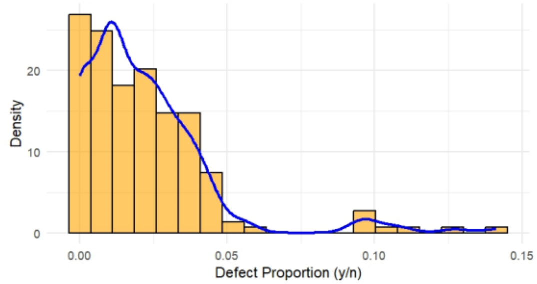

Figure 1: Distribution of defects proportion

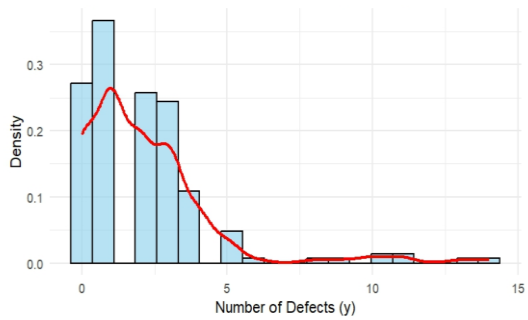

Figure 2: Distribution of Defects counts

Figure 1 illustrates the empirical distribution of defect proportions across batches, suggesting that the data are appropriate for modeling the binomial distribution. The plot depicts both variance and defect rate, which are important indicators for monitoring the quality of the control process. Figure 2 shows the frequency of defect counts per batch, which justifies the use of the Poisson model to monitor undesirable discrete variables in the production process.

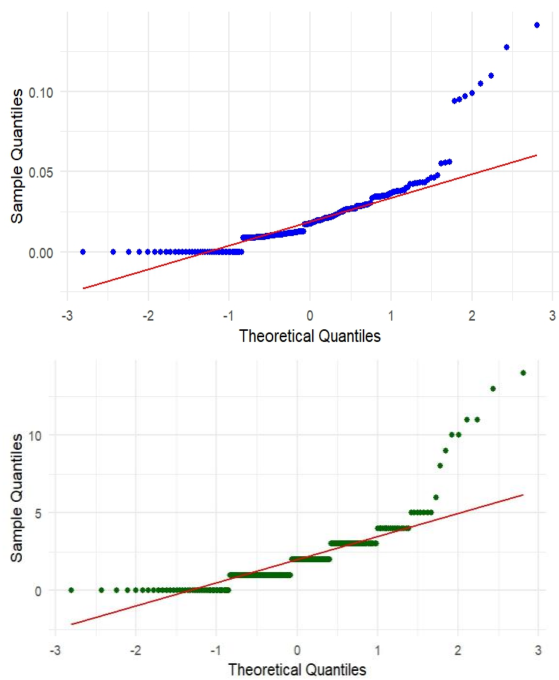

Figure 3: Model Validation for both Binomial and Poisson Distributions.

The Q-Q plot assessed the model's statistical stability with respect to the defect data. The first graphic compares the proportion to a binomial distribution, with all points following the diagonal, indicating strong model conformance. The second plot compares the defect count to the Poisson distribution, which aligns and justifies the model's tracking of defect events. However, the two graphs provide statistical evidence for both the Binomial and Poisson models in terms of production based on data quality.

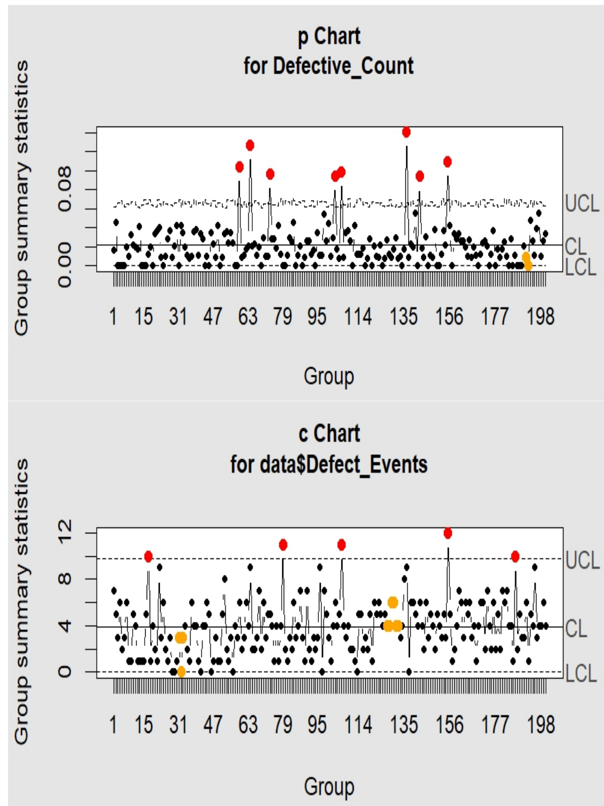

Figure 4: Traditional Control Chart Performance.

The P-chart depicts the progressive approach of monitoring poor proportions. Since some of the data are outside the control limit, it provides a baseline to evaluate the sensitivity of the Bayesian adaptive strategy. The C-chart monitors the number of flaws however, because the fixed performance is limited, this is in contrast to the Bayesian dynamic Poisson.

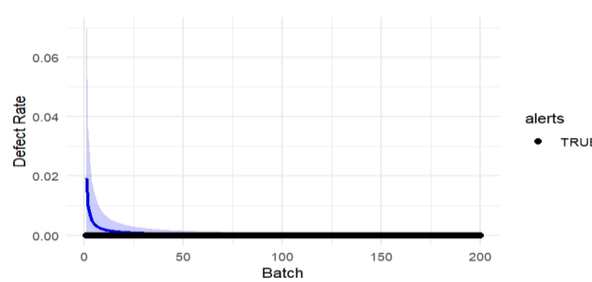

Figure 5: Bayesian Control Chart for Defect Rate

Figure 5 displays the Bayesian binomial model's real-time monitoring capability, which updates the defect rate estimation with each batch. Because some of the points fall outside the control limit, there are identifiable causes of variation in the manufacturing process.

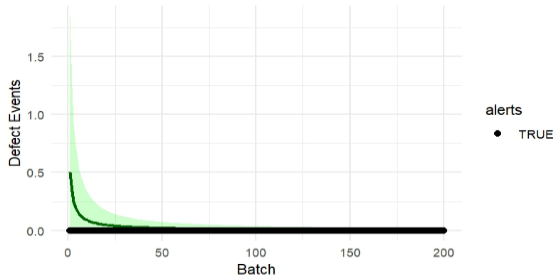

Figure 6: Bayesian Binomial Control Chart for Defect Event

Figure 6 depicts the application of the Bayesian technique to the Poisson distribution, which is used to track discrete occurrences. Furthermore, it is utilized to detect a shift in the frequency of recurrence, which is crucial for timely action.

Conclusion

Adopting a Bayesian control chart allows for a more intelligent and adaptive method to monitor production quality in the automated industry. Unlike the traditional method, which is based mostly on a fixed threshold, the model incorporates prior information and continuously updates and combines new data, making it flexible to variations in defect rates. This allows for early discovery of problems and continual improvement. Most importantly, the integration of two distributions provides flexibility in dealing with diverse problems throughout the production environment. Overall, Bayesian-based quality monitoring has improved operational efficiency while allowing automated enterprises to remain competitive in fields that require reliability and precision.

Recommendation

Bayesian control charts, that enable adaptive and dynamic detection of process variation, should be employed as part of the daily routine system monitoring to enhance the quality and efficiency of automated companies. Organizations should invest in infrastructure data to provide seamless Bayesian model updates, as well as train their quality control staff to build the skills required for effective method application and interpretation. Finally, the management team should foster a culture of continuous improvement by providing Bayesian-driven insight into lean manufacturing and six sigma processes, ensuring long-term competitiveness and sustainable development growth.

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