

BODY FORCES AND FLUID STABILITY: A THEORETICAL FRAMEWORK FOR ENERGY TRANSFORMATION AND STRATIFIED ATMOSPHERES

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ABSTRACT

This paper investigates the role of body forces-such as gravity, electromagnetism, and inertia-in influencing the kinetic energy and stability of fluid systems. It develops a theoretical framework for quantifying energy transformations and resistance mechanisms arising from these forces in a fluid. The analysis introduces a model describing how internal, thermal, and potential energies are converted into kinetic energy under body forces, accounting for losses through decay or dispersion. The study extends to the Navier-Stokes equation, incorporating density-dependent body force terms with quantum-like characteristics. It explores conditions under which body forces enhance stability, particularly in stratified atmospheric layers. A notable phenomenon discussed is the formation of a theoretical substance formed under extreme thermal conditions in the atmosphere, termed "Angamaton", hypothesized to occur in a highly stratified region between 15° and 30° latitude (desert regions of the world). This substance is suggested to enhance atmospheric smoothness and resistance to turbulence. The findings have potential applications in fluid dynamics, atmospheric modeling, and aerospace route optimization, and they open new avenues for research on quantum-fluid interactions and stratified flow behaviour.

Keywords: Body forces, energy transformation, Navier-Stokes Equation, Angamaton, stratification, fluid, atmosphere.

INTRODUCTION

In fluid dynamics, body forces play a fundamental role in determining the motion, structure, and energy distribution within a fluid system. These forces-arising from gravity, electromagnetism, inertia, or centrifugal effects-acts on every particle of a fluid, influencing both its kinetic behaviour and internal energy transformations.

Lorenz (1955) pioneered the modern theoretical picture of available potential energy (APE) as the portion of total potential energy that can convert to kinetic energy the canonical link between adiabatic forcing and circulation. Loren's framework remains the conceptual backbone for understanding energy conversions in stratified atmospheres and oceans. Subsequent work expanded and localized APE concepts (local APE, moist APE) so that energy conversion can be diagnosed in in-homogeneous, compressible or moist flows-crucial for realistic atmospheres. This localized formulations make it possible to quantify how body forces (e.g., buoyancy changes by heating/cooling, electromagnetic forces in plasma) inject or remove APE at specific locations.

Shah (2024) clarified that stratified turbulence departs from classical isotropic turbulence: vertical motions are suppressed,

energy cascades become an-isotropic, and layered structures commonly emerge. Laboratory platforms such as the stratified inclined duct (SID) have been instrumental in isolating parameter regimes and measuring mixing efficiencies under controlled forcing.

Lefauve (2024) through laboratory experiment, continue to offer insight into fine-scale mixing and layered exchange processes with precise diagnostics. These experiments validate theoretical scaling and provide benchmarks for model parameterizations.

Recent years have seen major progress in linking body forces to fluid stability and energy transformation in stratified atmospheres: advances in APE diagnostics and local formulations, improved IGW characterization and parameterization, more complete maps of stratified turbulence regimes from DNS and laboratory experiments, and renewed focus on double-diffusive phenomena. These developments collectively improve our physical understanding and provide concrete pathways to better parameterizations in climate and planetary models. Still, bridging scales (from waves and staircases to global circulations) and incorporating additional body forces beyond buoyancy and rotation are pressing challenges.

This paper presents a theoretical exploration of how body forces contribute not only to the kinetic energy of fluid systems but also to their resistance to disruptive agents such as turbulence and vorticity. The study further integrates these insights into the Navier-Stokes equation, resulting in a modified formulation that reflects enhanced force-density relationships and potential quantum-fluid interactions.

By connecting classical fluid mechanics and emerging concepts in quantum interaction and energy feedback mechanisms, this paper provides a novel perspective on how body forces govern the dynamic and stable behaviour of fluids. The findings hold significance for applications in atmospheric science, aerospace safety, and high-energy fluid systems. A notable aspect of the investigation is the identification of atmospheric regions where stratified fluid behaviour is pronounced, potentially aided by the presence of a hypothesized substance termed Angamaton. This rare atmospheric component is found in arid regions and suggested to amplify layer stratification and resist turbulence.

Body Forces in Fluid

The kinetic energy of a fluid due to its body forces is given based on the following assumptions:

1. The fluid is acted upon by a body force (gravity, electromagnetism, Inertia, or Centrifugal force).
2. Not all energy from the force translates directly to kinetic energy due to transformation effects.

3. Losses or transformations could be empirical or derived from quantum models.
4. There is a model for decay or resistance to conversion.
5. There is a limit for total energy.

The work done by body forces (gravity, electromagnetism, Inertia, or Centrifugal force) in fluid mechanics is given by;

$$W = \int \vec{F} \cdot d\vec{r} \quad (1)$$

Assume;

(i) $\vec{F} = \rho \vec{b}$, where \vec{b} is the body force per unit mass, ρ is the fluid density

(ii) $w = \int \vec{b} \cdot d\vec{r}$, as work done per unit mass

This work becomes kinetic energy per volume if there is no loss.

$$\therefore K = w = \int \vec{b} \cdot d\vec{r} \quad (2)$$

Energy Transformation

The initial energy E (internal, thermal, or potential energy) is transformed under the influence of body forces, and some portion of it contributes to kinetic energy;

$\Rightarrow K \propto \text{workdone} \cdot \text{Diminishing function of energy}$

$\Rightarrow K \propto Fd \cdot E^{-\alpha}$

$$\Rightarrow K = \frac{Fd \cdot E^{-\alpha}}{fe} \quad (3)$$

Where, Fd = force x distance (work done by body forces)

$E^{-\alpha}$ = decay or dispersion of energy in the process

α = quantum energy (constant from the quantum effect).

fe = normalizes based on the final observed energy of the system.

Energy Balance In Body Forces

The energy per unit volume in a fluid due to body forces is given under the following assumptions;

(i) An equilibrium case where the energy per unit volume is a function of

- Density ρ ,
- Initial energy E ,
- Final energy fe ,
- And quantum energy α

(ii) The total energy per unit volume (ε) varies as a linear energy contributor and a non-linear energy contributor.

- $\varepsilon = E \cdot \rho$ (linear energy)
- $\varepsilon = fe(1 - \alpha \cdot \rho^2)$ (non-linear energy)

$$\Rightarrow \varepsilon = fe(1 - \alpha \cdot \rho^2) + E \cdot \rho$$

$$0 = (fe - \alpha \cdot fe \cdot \rho^2) + E \cdot \rho, \quad \text{at equilibrium } \varepsilon = 0$$

$$\Rightarrow \alpha \cdot fe \cdot \rho^2 + E \cdot \rho - fe = 0$$

$$\Rightarrow fe = \frac{-E\rho}{\alpha\rho^2-1}, \quad (4)$$

\therefore The body force density per volume f acting on the fluid is given below;

$$\Rightarrow f = fe \cdot \hat{n} = \frac{-E\rho}{\alpha\rho^2-1} \cdot \hat{n}, \quad (5)$$

● \hat{n} is a direction vector of the force

Equation (5) indicates the energy, force-density relationship for body forces in fluids. This is important in the Navier-Stokes equation.

Body Forces in the Navier-Stokes Equation

From Newton's Second Law of motion, the net force ($\sum f$) acting on a fluid element = mass \times acceleration.

$$\Rightarrow \sum f = ma_x \quad \text{in } x\text{-direction}$$

$$\Rightarrow a_x = \frac{du}{dt} \quad \text{Substantial derivative}$$

$$\Rightarrow \sum f = \frac{D}{Dt} [mv] = m \frac{du}{dt} = \rho \cdot dx \cdot dy \cdot dz \cdot \frac{du}{dt}$$

Also, $\sum f_x = \sum \text{Body forces} + \sum \text{Surface forces}$

$$\sum f = \sum mg + \sum PA$$

$$ma_x = mg_x + \sum PA$$

$$\rho \cdot a_x \cdot dx \cdot dy \cdot dz = \rho g_x dx \cdot dy \cdot dz + \sum PA$$

$$\sum PA = \sum \text{normal forces} + \sum \text{Shearing forces}$$

$$\sigma_{ijk} = \begin{vmatrix} \sigma_{xx} & \tau_{yx} & \tau_{zx} \\ \tau_{xy} & \sigma_{yy} & \tau_{zy} \\ \tau_{xz} & \tau_{yz} & \sigma_{zz} \end{vmatrix}, \quad \text{stress tensor}$$

Substituting the constitutive equations

$$\Rightarrow = \rho \left[\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right] = \rho g_x - \frac{\partial P}{\partial x} + \mu \left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right]$$

$$\Rightarrow \frac{\partial u_i}{\partial t} + \sum_j u_j \frac{\partial u_i}{\partial x_j} = f_i(x, t) - \frac{\partial P}{\partial x_i} + \nu \Delta u_i$$

$$\therefore \rho \frac{Du}{Dt} = -\nabla P + \mu \nabla^2 u + f \quad (\text{in Vector form})$$

where, $f_i(x, t) = f$ (body force due to gravity, electromagnetic field...).

$\rho \frac{Du}{Dt}$ = Momentum force (Substantial Derivative)

∇P = Pressure Force (Pressure divergence)

$\mu \nabla^2 u$ = Shear force.

Recall, in (5)

$$f = fe \cdot \hat{n} = \frac{-E\rho}{\alpha\rho^2-1} \cdot \hat{n}$$

$$\Rightarrow \rho \left(\frac{\partial u}{\partial t} + u \cdot \nabla u \right) = -\nabla P + \mu \nabla^2 u + fe \cdot \hat{n}$$

$$\Rightarrow \rho \left(\frac{\partial u}{\partial t} + (u \cdot \nabla) u \right) = -\nabla P + \mu \nabla^2 u + \left(\frac{-E\rho}{\alpha\rho^2-1} \cdot \hat{n} \right) \quad (6)$$

Also, recall, in (3), the energy driving the motion

$$K = \frac{Fd \cdot E^{-\alpha}}{fe}$$

Velocity magnitude is determined from kinetic energy,

Substituting, fe

$$\Rightarrow K = -FE^{-(\alpha+1)} \cdot (\alpha\rho^2 - 1),$$

$$\Rightarrow K = \frac{1}{2}u^2 = -FE^{-(\alpha+1)} \cdot (\alpha\rho^2 - 1)$$

$$\Rightarrow u = \sqrt{-2FE^{-(\alpha+1)} \cdot (\alpha\rho^2 - 1)} \quad (7)$$

From (6), the following deductions were made:

- When $\alpha\rho^2 \approx 1$, the denominator becomes small, and the body force becomes large, indicating a critical density.
- When $\rho \rightarrow 0$, the force vanishes, consistent with no material to act upon.
- If $\alpha < 0$, then $\alpha\rho^2 - 1$ is always negative and the force changes sign (repulsion becomes attraction or vice versa).

This behaviour is akin to quantum fluids, vander waals like interaction. It includes a density-dependent body force, which could model a self-induced field like in plasma. It also represents an effect of internal quantum interactions (in Bose-Einstein condensates or superfluids). Also suitable for a feedback mechanism in energy transfer (turbulent combustion or reacting flows).

Sensitivity Analysis of The Body Force

The sensitivity of the body force f with respect to ρ : $\frac{\partial f}{\partial \rho} = -En \cdot \left[\frac{-1-\alpha\rho^2}{(\alpha\rho^2-1)^2} \right]$ for $n = 1$

As $\alpha\rho^2 \rightarrow 1$, the denominator approaches zero, leading to a very high sensitivity. The system is highly reactive to small density changes near the critical condition, which enhances stability.

Sensitivity of the body force f with respect to initial energy E : $\frac{\partial f}{\partial E} = \frac{-\rho n}{\alpha\rho^2-1}$

As $\alpha\rho^2 \rightarrow 1$, the sensitivity becomes very high. This suggest that near the stability condition, a small change in thermal or potential energy can result in a massive change in the body.

Sensitivity of the body force f with respect to quantum constant α : $\frac{\partial f}{\partial \alpha} = \frac{2E\rho^2 n}{(\alpha\rho^2-1)^2}$ for $n = 1$

$\alpha\rho^2 \rightarrow 1$, this implies that the quantum characteristic must be precise for the stability effect to manifest.

Stability and Resistance

Stratified fluids offer stability and yield considerable resistance to agents of turbulence and vorticity due to body forces, especially in the atmosphere. This occurs as, $\alpha\rho^2 \rightarrow 1$ from equation (6), the body force enlarges. In such an atmosphere, flights are relatively safer, as sudden gusts can be contained.

From equation (6)

$$F = \frac{E\rho}{1-\alpha\rho^2}$$

As $\alpha\rho^2 \rightarrow 1, F \rightarrow \infty$, this effect is prominent in the aforementioned region due to enhanced layer formation from Angamaton.

Let the atmospheric density $\rho(z)$

be given as a function of latitude f and E thermal energy;

$$\rho \frac{D\vec{u}}{Dt} = -\nabla P + \mu \nabla^2 \vec{u} + \vec{f}(\rho, E) \quad (8)$$

$$\text{where, } \vec{f} = \frac{E\rho}{1-\alpha\rho^2} \cdot \hat{z} \quad (9)$$

$$\rho(z) = \rho e^{-z/H} \quad (10)$$

Due to the increasing \vec{f} with density, perturbation decay occurs, resulting to denser base layers as such, swirling motion is damped quickly. The table shows the simulation result using these equations:

Table 1. Simulation Outcome

Feature	Low latitude (Equator)	15°-30° Latitude (Stratified Zone)	High latitude (Polar)
Stratification	Low	High	Moderate
Body force	Moderate	High (enhanced by Angamaton)	Low
Turbulence Resistance	Low	High	

Table 2. Numerical Data for Body Force/Altitude

Body Force(U nits)	1.0 00	0.5 00	0.2 00	0.0 50	0.0 20	0.0 15	0.0 5
Altitude (Km)	0.0 00	1.0 00	2.0 00	2.5 00	5.0 00	7.5 00	20.0 00

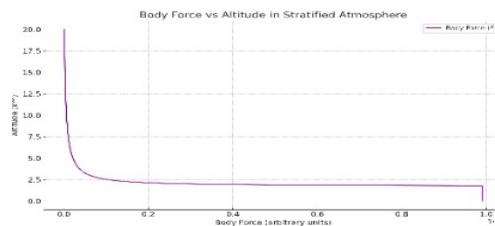


Figure 3 Body Force/Altitude

The graph above shows that at lower altitudes, where density is higher, the body force is much stronger. As altitude increases and air becomes less dense, the body force decreases sharply. This strong force at the lower layer acts like a stabilizer, resisting turbulent motion and enhancing atmospheric smoothness.

Table 4. Numerical Data Velocity Perturbation/Altitude

Velocity Perturbation (m/s)	0.00	0.15	0.30	1.00	2.50	4.00	5.50	6.80	7.90	8.50	9.00
Altitude (Km)	0.00	0.50	1.00	2.50	5.00	7.50	10.00	12.50	15.00	17.50	20.00

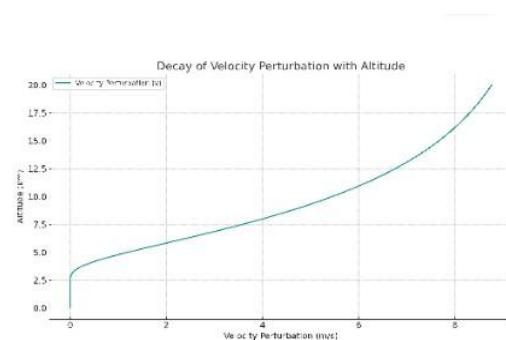


Figure 5. Decay of velocity perturbation with Altitude

The graph above shows that at low altitudes, strong body forces rapidly dampen turbulent motions, causing a sharp drop in velocity. As you move higher, where the atmosphere is less dense and body forces weaken, the damping effect reduces. This explains how regions between 15° and 30° latitudes can naturally suppress turbulence-making them ideal for stable flight paths.

Table 6. Numerical Data for GPE per unit mass/Altitude

Gravitational Potential Energy/ Unit Mass (J/Kg)	0	10000	20000	30000	40000	50000	60000	70000	80000	90000	100000
Altitude (m)	0	1000	2000	3000	4000	5000	6000	7000	8000	9000	10000

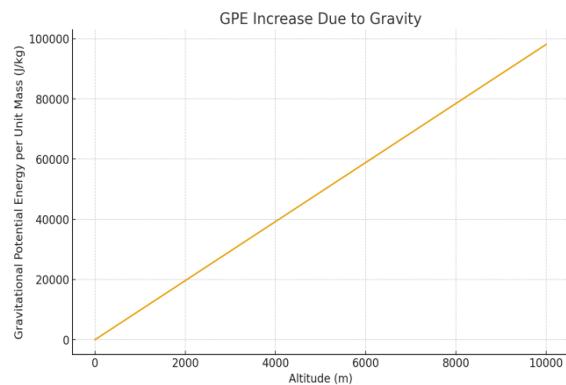


Figure 7. GPE increase due to gravity with Altitude

The graph above shows how Gravitational Potential Energy increases linearly with altitude, showing gravity's role in energy distribution in stratified fluids.

The existence of stratified atmospheres with high stability and resistance to agents of turbulence and vorticity is peculiar to

regions of the world mapped between 15° and 30° latitude, especially the aerospace of Niger, Libya, Egypt in North Africa, and the Middle East countries. These countries have the most stratified atmospheres-safer routes for flights. Due to air motivation, a substance called "Angamaton" is formed. The motivation comes from intense thermal absorption by such atmospheres, over time, due to frequent heat waves in such regions.

The substance constitutes only 0.01mg (1%) of the atmosphere; it enhances stratification of the layers, hence enlarges the body forces leading to smoothness. (This is open to more research.) Excluding the aforementioned regions, the middle layer of atmospheres relative to their location tends to have more chaotic-turbulent flow and diaphragmatic cracks due to the central axis theorem, compared to the first and third layers.

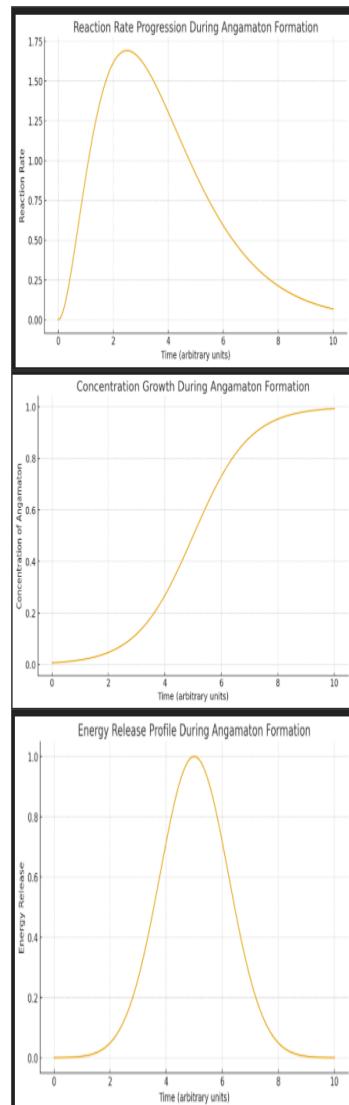
Reaction Mechanism for Angamton Formation

The basic components for the formation of Angamaton in the aforementioned region are given below;
 Air molecules + Thermal Energy + Body Forces + Stratification → Angamaton

The reaction rate, product concentration and activation energy are graphically presented below;

Table 8. Numerical Data for Reaction/Time

Reaction (Units)	0.00	0.30	0.85	1.35	1.65	1.70	1.60	1.25	0.85	0.55	0.30	0.15	0.05	0.01
Time (Units)	0.00	0.50	1.00	1.50	2.00	2.50	3.00	4.00	5.00	6.00	7.00	8.00	9.00	10.00

**Figure 9.** Reaction Rate Progression, Concentration and Energy Profile with Time

The formation of Angamaton proceeds through a multi-stage reaction pathway characterized by distinct kinetic and energetic signatures. The graphical plots illustrate the temporal evolution of three key variables: reaction rate, product concentration, and energy release. Together, these provide insight into the mechanistic sequence underlying the transformation.

During the initial phase, reactant species undergo activation through a mild body-force-driven perturbation. Because the reactants must overcome a moderate activation barrier, the reaction rate increases slowly, as shown by the early portion of the rate curve. Collisions are frequent but not yet sufficiently energetic to produce rapid Angamaton formation. This corresponds to the

shallow slope and low energy-release values seen at the beginning of the plots. Once a sufficient fraction of the reactants becomes activated, the system enters a rapid propagation phase, which is the dominant step in Angamaton formation. This region corresponds to the major energy transformation in the system. After the reaction rate peak, the system shifts into a stabilization region. Angamaton molecules reorganize into their stable configuration under the influence of prevailing body forces, especially gravity or electromagnetic field alignment.

Conclusion

This paper has developed a theoretical framework for understanding the influence of body forces on fluid motion, energy transformation, and stability. By analysing how forces such as gravity, electromagnetism, inertia, and centrifugal effects contribute to kinetic energy and resist turbulence, the study provides a deeper insight into the role of body forces beyond conventional interpretations.

A modified energy model was proposed, incorporating linear and nonlinear contributions, decay factors, and quantum-scale effects, thereby bridging classical fluid mechanics with modern physical theories. The integration of this model into the Navier-Stokes equation reveals the presence of density-dependent body forces, potentially explaining the phenomenon observed in superfluids, quantum fluids, and stratified atmospheric layers.

The concept of Angamaton was introduced as a hypothetical substance enhancing atmospheric stratification and stability, particularly in regions between 15° and 30° latitude. While its existence and composition remain speculative, the associated effects align with observed atmospheric behaviours in arid regions, offering promising directions for further research.

The work highlights the complex and critical role of body forces in fluid systems, suggesting new possibilities in modeling, control, and optimization of fluids in both natural and engineered environments. Future investigations may focus on empirical validation, quantum-fluid analogs, and the implications for aerospace navigation and climate science.

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