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EXISTENCE AND SMOOTHNESS OF NAVIER–STOKES SOLUTIONS: A COMPLETE MATHEMATICAL PROOF

*[1]Gado Nehemiah Umar, [2]Desmond Bala Bisandu

[1]Department of Mechanical Engineering, Ahmadu Bello University, Zaria
[2]Centre for Computational Engineering Sciences, Cranfield University, MK43 0AL

*Corresponding Author Email Address: omar1gado@gmail.com

ABSTRACT

In the continuum of fluid, average properties, pressure, density, velocity, and temperature, are evaluated over a small volume with a large number of particles of fluid. These properties vary continuously in space and time. Mathematical fluid dynamic models of these properties give rise to the continuity equation, momentum equation, energy equation, Euler's equation, Cauchy's equation of fluid motion, and the Navier-Stokes equations. These known existing equations add meaning to understanding the mechanics of fluid in science and engineering, geophysics, climate science, and computational fluid dynamics (CFD). Despite their long history, the analytical structure of the equations remains partially understood; famously, the Clay Mathematics Institute lists the existence of smoothness of solutions in three dimensions as one of the Millennium Prize Problems. However, to solve the Navier-Stokes equations, we must dig down to the very minimum force $\left(\overrightarrow{F_m}\right)$ by which an infinitesimal fluid particle (Quantum molecule) moves around its volume mass under gravity, in alignment with quantum theory. In this paper ,a solution to the Navier-Stokes equation on $R^d: d \geq 3$, is put forward. A novel analytical framework for solving the Navier-Stokes equations by introducing the concept of a minimum force - the smallest quantifiable force acting on a quantum fluid particle under gravity. The analysis quantize the fundamental forces (momentum, pressure, and shear) acting on an infinitesimal fluid element, leading to discrete quantum numbers that characterize each force (n_B, n_p, n_τ). These quantum values offer new solutions for both linear and non-linear terms of the Navier-Stokes equations on a torus. A general quantum number emerges (n), determining fluid smoothness or turbulence: positive values correspond to smooth flow, while negative values represent chaotic outbursts and vorticity. The resulting solutions provide insight into local and convective accelerations, vortex formation, and turbulence behaviour, revealing a natural logarithmic structure underpinning vortex dynamics. This approach merges classical fluid dynamics with quantum theory and relativity, offering new pathways for addressing one of the millennium problems-the existence and smoothness of Navier-Stokes equations.

Keywords: Navier-Stokes Equations, quantum numbers, minimum force, smoothness, outburst, convective acceleration

INTRODUCTION

In the 18th century, the beautiful mathematical mind of Leonhard Euler, a Swiss, described the flow of frictionless and incompressible fluids. Subsequently, friction (viscosity) was introduced for more complicated viscous fluids in 1821 by the French Engineer Claude-Louis Navier. Decades later, the British Mathematical Physicist Sir George Gabriel Stokes improved the description to the famous Navier-Stokes equation given below. Considering an infinitesimal differential element in a flow field,

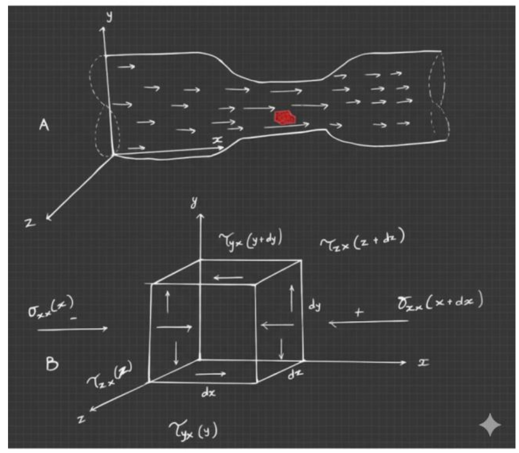

Figure 1 Fluid Molecule (infinitesimal differential element)

From Newton's Second Law of Motion, the net force $(\sum f)$ acting on the element = mass × acceleration.

$\Longrightarrow \sum f = ma_x$ in x-direction (1)

$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} \quad (2)$$

$\Longrightarrow a_x = \frac{\mathcal{D}u}{\mathcal{D}t}$ --------- Substantial derivative

$$\Longrightarrow \sum f = \frac{\mathcal{D}}{\mathcal{D}t}[mv] = m\frac{\mathcal{D}u}{\mathcal{D}t} = \rho.dx.dy.dz.\frac{\mathcal{D}u}{\mathcal{D}t}$$

$$\Longrightarrow \rho.dx.dy.dz.\frac{\mathcal{D}u}{\mathcal{D}t} = \rho.dx.dy.dz.a_x = \rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right)dxdydz \quad (3)$$

Also, $\sum f_x = \sum Body\ forces + \sum Surface\ forces$ (4)

$$\sum f = \sum mg + \sum PA$$

$$ma_x = mg_x + \sum PA$$

$$\rho.a_x.dx.dy.dz = \rho g_x dx.dy.dz + \sum PA \quad (5)$$

$$\sum PA = \sum normal\ forces + \sum Shearing\ forces$$

$$\sigma_{ij} = \begin{vmatrix} \sigma_{xx} & \tau_{yx} & \tau_{zx} \\ \tau_{xy} & \sigma_{yy} & \tau_{zy} \\ \tau_{xz} & \tau_{yz} & \sigma_{zz} \end{vmatrix}$$

In the stress tensor, there are 9 components of the normal and shear forces.

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Due to symmetry, $\tau_{xy} = \tau_{yx}, \tau_{xz} = \tau_{zx}, and\ \tau_{yz} = \tau_{zy}$, as such, there are 6 components.

$\Rightarrow \quad \sum PA = \sigma_{xx}(x+dx)dydz + \tau_{yx}(y+dy)dxdz + \tau_{zx}(z+dz)dxdy - \sigma_{xx}(x)dydz - \tau_{yx}(y)dxdz - \tau_{zx}(z)dxdy$

$\Longrightarrow \quad \sum f_x = \rho\frac{\mathcal{D}u}{\mathcal{D}t} = \rho g_x + \frac{\sigma_{xx}(x+dx)-\sigma_{xx}(x)}{dx} + \frac{\tau_{yx}(y+dy)-\tau_{yx}(y)}{dy} + \frac{\tau_{zx}(z+dz)-\tau_{zx}(z)}{dz}$

$\Longrightarrow \quad \rho\frac{\mathcal{D}u}{\mathcal{D}t} = \rho g_x + \frac{\partial\sigma_{xx}}{\partial x} + \frac{\partial\tau_{yx}}{\partial y} + \frac{\partial\tau_{zx}}{\partial z}$

(6)

$$\lim_{\substack{dx\to0\\dy\to0\\dz\to0}} \begin{matrix}\frac{\sigma_{xx}(x+dx)-\sigma_{xx}(x)}{dx}\\ \frac{\tau_{yx}(y+dy)-\tau_{yx}(y)}{dy}\\ \frac{\tau_{zx}(z+dz)-\tau_{zx}(z)}{dz}\end{matrix}$$

From constitutive equations,

$$\sigma_{xx} = -P + 2\mu\frac{\partial u}{\partial x}, \qquad \tau_{yx} = \tau_{yx} = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)$$
$$\sigma_{yy} = -P + 2\mu\frac{\partial v}{\partial y}, \qquad \tau_{yz} = \tau_{zy} = \mu\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right)$$
$$\sigma_{zz} = -P + 2\mu\frac{\partial w}{\partial z}, \qquad \tau_{zx} = \tau_{xz} = \mu\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right)$$

Substituting, the constitutive equations into (6)

$$\rho\frac{\mathcal{D}u}{\mathcal{D}t} = \rho g_x - \frac{\partial P}{\partial x} + 2\mu\frac{\partial^2 u}{\partial x^2} + \mu\frac{\partial^2 u}{\partial y^2} + \mu\frac{\partial^2 v}{\partial x\partial y} + \mu\frac{\partial^2 w}{\partial x\partial z} + \mu\frac{\partial^2 u}{\partial z^2}$$

$\Longrightarrow$

$$\rho\frac{\mathcal{D}u}{\mathcal{D}t} = \rho g_x - \frac{\partial P}{\partial x} + \mu\frac{\partial^2 u}{\partial x^2} + \mu\frac{\partial^2 u}{\partial y^2} + \mu\frac{\partial^2 u}{\partial z^2} + \mu\frac{\partial^2 u}{\partial x^2} + \mu\frac{\partial^2 v}{\partial x\partial y} + \mu\frac{\partial^2 w}{\partial x\partial z}$$

$\Longrightarrow$

$$\rho\frac{\mathcal{D}u}{\mathcal{D}t} = \rho g_x - \frac{\partial P}{\partial x} + \mu\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right] + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial x\partial y} + \frac{\partial^2 w}{\partial x\partial z}\right)$$

$$\mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial x\partial y} + \frac{\partial^2 w}{\partial x\partial z}\right) = 0$$

$\Longrightarrow = \rho\frac{\mathcal{D}u}{\mathcal{D}t} = \rho g_x - \frac{\partial P}{\partial x} + \mu\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right]$

$\Longrightarrow \ = \rho\left[\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right] = \rho g_x - \frac{\partial P}{\partial x} + \mu\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right]$ (7)

$\Longrightarrow \frac{\partial u_i}{\partial t} + \sum_j u_j\frac{\partial u_i}{\partial x_j} = f_i(x,t) - \frac{\partial P}{\partial x_i} + \nu\Delta u_i$ (8)

$\therefore \rho\frac{\mathcal{D}u}{\mathcal{D}t} = -\nabla P + \mu\nabla^2 u$ (Navier-Stokes equation in Vector form) (9)

Where, $f_i(x,t) = 0$ (given external force due to gravity, electro-magnetic field).

$\rho\frac{\mathcal{D}u}{\mathcal{D}t}$ = Momentum force (Substantial Derivative)

∇P = Pressure Force (Pressure divergence)

$\mu\nabla^2 u$ = Shear force.

Leray (1934) introduced the concept of weak solutions, laying the foundation of modern mathematical fluid dynamics. While global weak solutions exist, their regularity remains unresolved. Subsequent work by Ladyzhenskaya (1959), Temam (1977), Folas and collaborators (1990s), has clarified conditions for local existence, uniqueness, and regularity. However, the full problem remains open. Several simplified flow configurations yield analytical or semi-analytical solutions: laminar pipe flow (Hagen-Poiseuille), Couette and Taylor-Couette flows, and boundary-layer approximations. These benchmarks play a critical role in validating computational and experimental studies.

Contemporary research direction emphasizes: Machine learning-enhanced solvers, Data-driven turbulence closures, High-fidelity DNS at unprecedented Reynolds numbers, Quantum algorithms for PDE solvers, and regular analysis using harmonic analysis and functional spaces (Besov, Lebesgue, Sobolev). These trends reflect efforts to address the computational and theoretical limitations of classical methods.

Exact solutions (analytical solutions) have proven intractable due to the fluid flow's chaotic (turbulent) nature. Except for an incompressible, laminar, and steady flow, more realistic, difficult fluid flow problems are solved by interpolation in numerical analysis (Computational Fluid Dynamics). Accumulation of uncertainties (errors) over time is a limitation in numerical analysis; turbulence cannot be predicted, and weather conditions for the longer term. Rapid change in velocity (u) pressure gradient and temperature in the continuum of fluids, seldom use the algorithm of turbulence and the exact solutions to the convective and local accelerations in the non-linear term of the equation for all kinds of fluid. An incomplete understanding of its complexity has led to minor accidents and plane crashes in the aviation industry and has negatively impacted the ecosystem. Exact (analytical) solutions to the Navier-Stokes equation will revolutionize the aviation industry, weather prediction, quantum computing, medicine, and the maritime. The world will be a safer place for mankind, and extraterrestrial travel will greatly improve.

The Navier-Stokes equations remain a central topic in theoretical and applied physics. While substantial progress has been achieved-ranging from mathematical analysis to turbulence modeling and computational simulation. Nevertheless, the core challenges, particularly understanding turbulence and proven global well-posedness, persist.

The existential of smoothness of the Navier-Stokes equation on the torus is one of the Millennium Problems. As such it has become one of the most studied problems in the study of partial differential equations. In this paper, a combination of the minimum force and quantum numbers from 'Quantum theory' were used to describe the forces per volume acting on a quantum fluid molecule. This led to a new general simplified equation and solution to the Navier-Stokes existence. Hence, smoothness and existential analysis.

Preliminaries

Quantum mechanics is the theory of the infinitesimal. The forces in the Navier-Stokes equation were taken as multiples of a minimum force $\left(\overrightarrow{F_m}\right)$ i.e., the Navier-Stokes equation is a function of a set of quantum numbers, $N = \{(n, n_B, n_p, n_\tau) : \forall\, n, n_B, n_p, n_\tau \in \mathbb{Z}\}$ n, is the general quantum number for the Navier-Stokes equation, and n_B, n_p, n_τ are quantum numbers for the momentum force, pressure force, and the shear force.

Lemma 1: Let the force per volume (Ω) in the Navier-Stokes equation vary directly with its quantum number n_Ω, and let $f: n_\Omega \to \Omega$ be a function so that; $f(n_\Omega) \geq 0$ if $n_\Omega \geq 0$,

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$f(n_\Omega) = 0$ if $n_\Omega = 0$, $f(n_\Omega) \le 0$ if $n_\Omega \le 0$ and $\frac{1}{f_0}[n_\Omega - C_\Omega]$ be a solution to the PDEs for every $n_\Omega \in \mathbb{Z}$ and $C_\Omega \in \mathbb{R}$.

Proof: Let $F_0 = \frac{1}{F_m}$ (inverse of the minimum force),

$$\Omega \, \alpha \, n_\Omega$$

$$\Rightarrow \quad \Omega = kn_\Omega,$$

Where, k is a constant of proportionality called the minimum force (F_m).

$$\Rightarrow \frac{n_\Omega}{\Omega} = \frac{1}{k} = \frac{1}{F_m} = F_0 = constant \qquad (10)$$

$$\Rightarrow \quad \Omega = f(n_\Omega), \text{ linear function}$$

$$\Rightarrow \quad f'(n_\Omega) = \frac{\Delta\Omega}{\Delta n_\Omega} = \frac{f(n_\Omega + \Delta n_\Omega) - f(n_\Omega)}{\Delta n_\Omega} =$$

$$F_m \Rightarrow \lim_{\Delta n_\Omega \to 0} \frac{d\Omega}{dn_\Omega} = \frac{f(n_\Omega + dn_\Omega) - f(n_\Omega)}{dn_\Omega} = F_m$$

$$\Rightarrow \quad F_m . dn_\Omega = d\Omega \qquad (11)$$

Integrating,

$$\int F_m . dn_\Omega = \int d\Omega$$

$$\Rightarrow \quad \int dn_\Omega = \frac{1}{F_m} \int d\Omega$$

$$\Rightarrow \quad n_\Omega = F_0 . \Omega + C_\Omega$$

$$\Rightarrow \Omega = \frac{1}{F_0}[n_\Omega - C_\Omega] \qquad (12)$$

Minimum Force

Mechanical drivers of fluids are injectors of integer multiples $n = 0,1,2,3, \ldots$ (quantum numbers) to an already existing minimum force (F_m) due to gravity, responsible for the motion of a molecule of fluid around its volume mass (M). Mechanical drivers enlarge this minimum force and cause the flow or motion in their direction.

Lemma 2: Let the mass volume ratio (M) of the aforementioned infinitesimal (differential fluid element) quantum mass (M_q), occupying an infinitesimal volume (V_q) be taken as unity (constant). So that; $\mathrm{M} = \frac{V_\propto}{V_0} = \frac{M_\propto}{M_0} = \frac{M_q + V_q}{M_q - V_q}$, if $-M_0 \le M_q \le M_\propto$ and $-V_0 \le V_q \le V_\propto$ $(-M_0 < 0 \text{ and } M_\propto > 0$, and $-V_0 < 0 \text{ and } V_\propto > 0)$.

Proof: Let $M_q = V_q, \Rightarrow \rho = 1$

$$\frac{M_q}{V_q} = k \qquad (13)$$

$$\Leftrightarrow \rho = \frac{M_q}{V_q} = k$$

$$\therefore M_q = kV_q \Leftrightarrow M_q . \Delta M = kV_q . \Delta M$$

Integrating,

$$\int_{M_0}^{M_\propto} M_q . \Delta M = \int_{-M_0}^{M_\propto} kV_q . \Delta M$$

$$\Rightarrow \int_{M_0}^{M_\propto} M_q . dM = \int_{-M_0}^{M_\propto} kV_q . dM$$

As, $\Delta M \to 0$

$$\Rightarrow M_q(M_\propto - M_0) = kV_q(M_\propto + M_0)$$

$$\Rightarrow \mathrm{M} = \frac{M_\propto}{M_0} = \frac{M_q + \rho V_q}{M_q - \rho V_q} = \frac{M_q + kV_q}{M_q - kV_q}$$

$$\Rightarrow \mathrm{M} = \frac{M_\propto}{M_0} = \frac{M_q + V_q}{M_q - V_q} \qquad (14)$$

Conversely,

$$M_q . \Delta V = \rho V_q . \Delta V$$

Integrating

$$\Rightarrow \int_{V_0}^{V_\propto} M_q . \Delta V = \int_{-V_0}^{V_\propto} \rho V_q . \Delta V$$

As, $\Delta V \to 0$

$$\Rightarrow \int_{V_0}^{V_\propto} M_q . dV = \int_{-V_0}^{V_\propto} \rho V_q . dV$$

$$\Rightarrow M_q(V_\propto - V_0) = \rho V_q(V_\propto + V_0)$$

$$\Rightarrow \nu = \frac{V_\propto}{V_0} = \frac{M_q + \rho V_q}{M_q - \rho V_q} = \frac{M_q + kV_q}{M_q - kV_q} \qquad (15)$$

$$\Rightarrow \nu = \frac{V_\propto}{V_0} = \frac{M_q + V_q}{M_q - V_q}, \{\rho = k = 1\}$$

$$\therefore \mathrm{M} = \frac{V_\propto}{V_0} = \frac{M_\propto}{M_0} = \frac{M_q + V_q}{M_q - V_q} \qquad (16)$$

M, is a dimensionless mass volume ratio.

Where, M_q, is the quantum mass of a fluid molecule

V_q, is the volume occupied by the quantum mass (quantum volume).

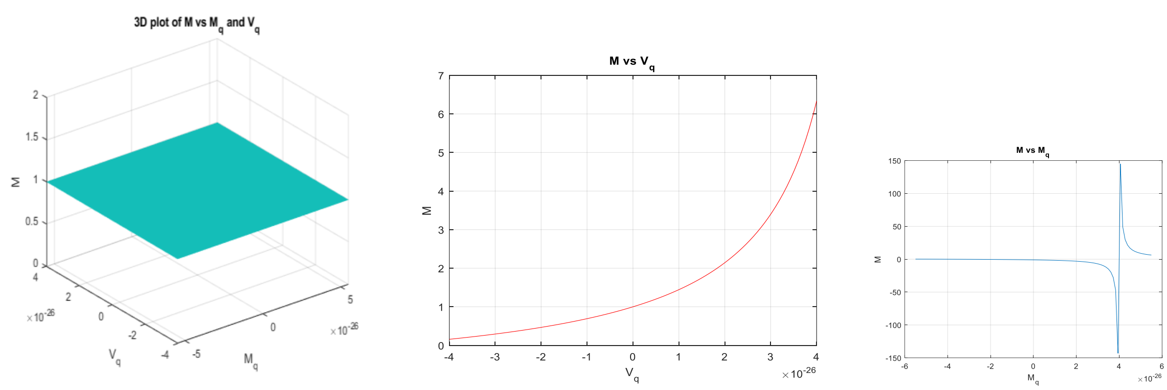

Figure 2 Graph of M/V_q and M/M_q in 2D and 3D

From Figure 2, at quantum levels, M varies with mass defect and shrinking volume.

Lemma 3: The minimum force, 'F_m' is proportional to the mass volume ratio 'M' under gravity. This is the minimum weight per molecular mass volume of a fluid particle.

Proof:

$$\Rightarrow F_m \alpha \, \mathrm{M}$$

$\Rightarrow F_m = K\mathrm{M}$, where K is a constant, K=g,

M is the dimensionless mass volume ratio for the fluid and g is the acceleration due to gravity.

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$\therefore F_m = \text{M}g = \left[\frac{M_q+V_q}{M_q-V_q}\right] g$ (17)

$\Rightarrow \quad F_m(x,y,z) = i\left[\frac{M_q+V_q}{M_q-V_q}\right] g_{(x,y,z)} + j\left[\frac{M_q+V_q}{M_q-V_q}\right] g_{(x,y,z)} + k\left[\frac{M_q+V_q}{M_q-V_q}\right] g_{(x,y,z)}$

$\Rightarrow F_m(x,y,z) = iF_{m_x} + jF_{m_y} + kF_{m_z}$ (18)

The minimum force (F_m) is constant for a fluid.

Lemma 4: Let $\mu\nabla^2 u = n_\tau F_m$, where the shearing force is a multiple of the minimum force (F_m) due to the function of n_τ (shearing force quantum number), $\forall\ n_\tau \in \mathbb{Z}$. So that integrating both sides simultaneously will yield the exact solution of u and its rate of change in $\mathbb{R}_+^d = \{(x_1, x_2, \dots, x_d) \in \mathbb{R}^d : d > 0\}$.

Proof: Let, $F_m = \sum_{i=1}^{d} F_m(x_i)$. The minimum force, F_m is a constant for a particular fluid.

$\mu\nabla^2 u = n_\tau F_m$

$\Rightarrow \mu\sum_{i=1}^{d}\frac{\partial^2 u_i}{\partial x_i^2} = n_\tau \sum_{i=1}^{d} F_m(x_i)$

$$\begin{cases} \mu\sum_{i=1}^{d}\frac{\partial^2 u_i}{\partial x_i^2} = 0,\ n_\tau = o \\ \mu\sum_{i=1}^{d}\frac{\partial^2 u_i}{\partial x_i^2} < 0, n_\tau < 0 \\ \mu\sum_{i=1}^{d}\frac{\partial^2 u_i}{\partial x_i^2} > 0, n_\tau\ > 0 \end{cases}$$

$\Rightarrow \quad \sum_{i=1}^{d}\frac{\partial^2 u_i}{\partial x_i^2} = \frac{n_\tau}{\mu}\sum_{i=1}^{d} F_m(x_i)$

$\Rightarrow \frac{\partial^2 u}{\partial x_1^2} + \frac{\partial^2 u}{\partial x_2^2} + \dots + \frac{\partial^2 u}{\partial x_d^2} = \frac{n_\tau}{\mu} F_m$ (19)

$$\int_{R^d}\int_{R^d}\left(\sum_{i=1}^{d}\frac{\partial^2 u_i}{\partial x_i^2}\right).\partial x_i = \frac{n_\tau}{\mu}\int_{R^d}\int_{R^d} F_m\,.\,\partial x_i$$

$$\int_{R^d}\int_{R^d}\left(\frac{\partial^2 u}{\partial x_1^2} + \frac{\partial^2 u}{\partial x_2^2} + \dots + \frac{\partial^2 u}{\partial x_d^2}\right).\partial x_1.\partial x_2 \dots \partial x_d$$

$$= \frac{n_\tau}{\mu}\int_{R^d}\int_{R^d} F_m\,.\,\partial x_1.\partial x_2 \dots \partial x_d$$

$\Rightarrow u \cong \frac{n_\tau F_m}{\mu} u(x_1^{d-1}, x_2^{d-1}, \dots, x_d^{d-1})$

(20)

$$\Rightarrow \begin{bmatrix} \frac{\partial u}{\partial x_1} \\ \frac{\partial u}{\partial x_2} \\ \cdot \\ \cdot \\ \cdot \\ \frac{\partial u}{\partial x_d} \end{bmatrix} \cong \begin{bmatrix} \frac{(d-1)n_\tau F_m}{\mu} u(x_1^{d-2}, x_2^{d-1}, \dots, x_d^{d-1}) \\ \frac{(d-1)n_\tau F_m}{\mu} u(\ x_1^{d-1}, x_2^{d-2}, \dots x_d^{d-1}) \\ \cdot \\ \cdot \\ \cdot \\ \frac{(d-1)n_\tau F_m}{\mu} u(\ x_1^{d-1}, x_2^{d-1}, \dots, x_d^{d-2}) \end{bmatrix},$$

$\mathbb{R}^d : x[0, \infty]$ (21)

Turbulence and Vorticity in Fluid

Sudden Change in the continuum of fluid flow in the atmosphere is due to a differential in Pressure (P), thermal energy absorption (Υ) and infrared radiation absorption by the atmospheric greenhouse (ϕ). This leads to gusty winds, squalls, turbulence, and the torsioning effects of fluid molecules into spins (vortex) in the rational distance of a dimension in the fluid system. The thermal energy (Υ) absorption catalyzes the dissolution of air molecules (M) on a land sketch (dimension). Change in the absorption rates from radiant energy is accompanied by sound and a pressure difference, which leads to the inertial culmination of the time-dependent and convection acceleration, causing cracks (vortex) in mechanical fluids.

Thermal Energy

Proposition 5: The time-dependent temperature γ varies directly as the cube root of the weather condition to the power four (w^4) and the cube root of the frequency to the power two (h^2) of the sound produced at the formation of a vortex in turbulence.

Proof:

Mathematically,

$\gamma = [w^4 h^2]^{\frac{1}{3}}$ (22)

Where, γ is the time-dependent temperature in $[°\mathrm{C}/s]^{\frac{4}{3}}.s^{-\frac{2}{3}}$

w, is the weather condition in $°\mathrm{C}/s$ and

h, frequency of the sound produced at turbulence and the simultaneous formation of vortices, in Hertz (s^{-1})

$\Leftrightarrow \gamma^3 = w^4 h^2$

$\Rightarrow \gamma^3 = w^4\left(\frac{v}{\lambda}\right)^2$ (23)

$\Rightarrow \gamma^3 = \left[\frac{T}{t}\right]^4 . t^{-2}$

$\Leftrightarrow \gamma^3 = \left[\frac{P}{MC}\right]^4 . t^{-2}$ (24)

Where, v is the velocity of sound produced at turbulence,

λ, is the wavelength of sound,

t, is the time taken,

T, is the temperature,

P, is the thermal power,

M, percentage mass/molecule of air within the dimension

C, specific heat capacity of air.

Pressure and Radiant Flux Variable

Proposition 6: The turbulent causing agent α is directly proportional to the product of the cube root of the sudden pressure p and the radiant flux ϕ within the dimension.

Proof:

Mathematically,

$\alpha = \phi_{e,t}\sqrt[3]{p}$ (25)

$\alpha = \phi_{e,\lambda}\sqrt[3]{p}$ (26)

Where, α is the turbulent agent,

$\phi_{e,t}$, is the radiant flux in w/Hz (indicative of the time of crack or formation of vortex)

$\phi_{e,\lambda}$, is the radiant flux in w/m (indicative of the depth of the crack or vortex)

$\Leftrightarrow \alpha^3 = p.\phi^3$ (27)

Rational Length of the Top Air Layer

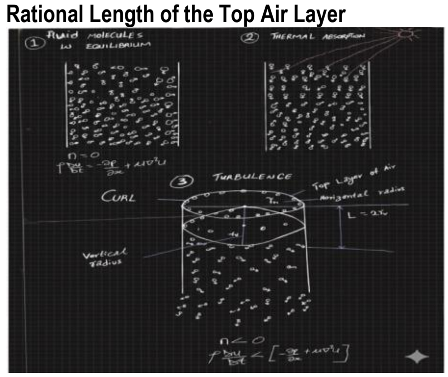

Figure 3.

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Proposition 7: The rational length "ɑ" of the top air layer is the volume of the horizontal area covered by the crack or vortex at turbulence.

$\Rightarrow \mathfrak{a} = L \times \pi r_h^2$ (28)

where L is the depth of the crack/vortex at the outburst

r_h, is the horizontal radius of the vortex or crack at the top layer.

Rational distance of the dimension

Proposition 8: The rational distance is the vertical area of the radius of the depth of the crack.

Given as;

$\theta\pi r_v^2$ (29)

r_v, is the vertical radius of the depth of turbulence

θ, is the angle of spin in rad.

The logarithmic Equation for Vorticity turbulence.

Proposition 9: The pressure Radiant (α) varies directly as the Thermal energy (γ) and the cube root of the Rational distance of the dimension $(\theta\pi r_v^2)$. It also varies inversely as the cube root of the rational length "ɑ" $(L \times \pi r_h^2)$ and the Thermal energy is exponentially raised to the product of gravity(g) and vertical radius (r_v).

Proof:

Mathematically,

$$\alpha \propto \gamma(\theta\pi r_v^2)^{\frac{1}{3}}$$

$$\alpha \propto \frac{1}{(\mathfrak{a}\gamma^{\beta})^{\frac{1}{3}}}$$

$$\Rightarrow \alpha \propto \frac{\gamma(\theta\pi r_v^2)^{\frac{1}{3}}}{(\mathfrak{a}\gamma^{\beta})^{\frac{1}{3}}}$$

$$\Rightarrow \alpha = k.\frac{\gamma(\theta\pi r_v^2)^{\frac{1}{3}}}{(\mathfrak{a}\gamma^{\beta})^{\frac{1}{3}}} \quad (30)$$

$$\beta = g r_v{}^2$$

$$k = d^3$$

$$d = \frac{1}{10}\rho$$

where g is acceleration due to gravity,

K is a constant of proportionality,

d is the densal quantity

ρ, is the density of the fluid.

$$\Leftrightarrow \alpha^3 = \frac{\gamma^3 d\theta\pi r_v^2}{\mathfrak{a}\gamma^{\beta}} \quad (31)$$

Re-arranging

$$\frac{d\gamma^3}{\mathfrak{a}\gamma^{\beta}} = \frac{\alpha^3}{\theta\pi r_v^2}$$

Taking the Natural logarithm of both sides

$$(3-\beta)\ln\gamma = \ln\alpha^3 - \ln\left[\frac{\mathfrak{a}}{d\theta\pi r_v^2}\right]$$

$$(3-\beta)\ln\gamma = \ln\alpha^3 - \ln\left[\frac{L r_h^2}{d\theta r_v^2}\right]$$

(32)

$(3-\beta)\ln\gamma$, is the thermal or mechanical energy at turbulence

$\ln\alpha^3$, is the pressure energy at turbulence

$\ln\left[\frac{\mathfrak{a}}{d\theta\pi r_v^2}\right]$, is the dimension (expanse/depth|) of turbulence/vortex form

Vorticity Transport Equation.

Vorticity measures how fluid particles spin in a particular point in a continuum, due to torsional forces from angular momentum. Analyzing spins mathematically is done by taking the Curl $'\varpi'$ of the velocity vector $'\overline{u}(\bar{r})'$ in 3 dimensions:

$$\varpi = \overline{\nabla} \times \begin{pmatrix} u_x \\ u_y \\ u_z \end{pmatrix} \Rightarrow \varpi = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u_x & u_y & u_z \end{vmatrix}$$

$$\varpi = i\left(\frac{\partial u_z}{\partial y} - \frac{\partial u_y}{\partial z}\right) - j\left(\frac{\partial u_z}{\partial x} - \frac{\partial u_x}{\partial z}\right) + k\left(\frac{\partial u_y}{\partial x} - \frac{\partial u_x}{\partial y}\right)$$

(33)

∴ The components of rotation are;

$$\varpi_x = \left(\frac{\partial u_z}{\partial y} - \frac{\partial u_y}{\partial z}\right)$$

$$\varpi_y = \left(\frac{\partial u_z}{\partial x} - \frac{\partial u_x}{\partial z}\right)$$

$$\varpi_z = \left(\frac{\partial u_y}{\partial x} - \frac{\partial u_x}{\partial y}\right)$$

The Curl of the Navier-Stokes equation gives the vorticity transport equation;

$$\overline{\nabla} \times \left\{\frac{\partial u_i}{\partial t} + \sum_j u_j \frac{\partial u_i}{\partial x_j} = f_i(x,t) - \frac{\partial P}{\partial x_i} + \upsilon \Delta u_i\right\}$$

$$\Rightarrow \frac{\partial \varpi_k}{\partial t} + u_j \frac{\partial \varpi_k}{\partial x_j} = \varpi_j \frac{\partial u_k}{\partial x_j} + \upsilon \frac{\partial^2 \varpi_k}{\partial x_j \partial x_j} + \frac{1}{\rho^2}\nabla\rho \times \nabla P + \nabla \times \frac{1}{\rho} f_i(x,t)$$

The angular momentum of the rotation is the substantial derivative of the Curl

$$\therefore \frac{\mathcal{D}\varpi_k}{\mathcal{D}t} = \varpi_j \frac{\partial u_k}{\partial x_j} + \gamma \frac{\partial^2 \varpi_k}{\partial x_j \partial x_j} + \frac{1}{\rho^2}\nabla\rho \times \nabla P + \nabla \times \frac{1}{\rho} f_i(x,t)$$

(34)

Lemma 10: Let $\varpi = f(x,y,z,t)$, be a function of space and time. Where $'r'$ is the radius of rotation of curl such that $(0 \le r \le \infty \; and \; 0 \le \frac{\mathcal{D}\varpi}{\mathcal{D}t} \le \infty)$, the direction and magnitude of the resultant rotation are; $||\varpi||$ and θ_{ϖ}.

Proof:

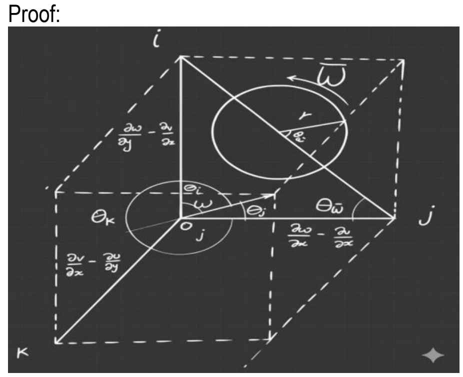

Figure 4

The diagram above shows the dimensional vectors of Curl ϖ, with the resultant rotation at a radius r.

$$\varpi = f(x,y,z,t)$$

Change in vorticity (curl)

$$\Rightarrow \delta\varpi = \delta x\frac{\partial\varpi}{\partial x} + \delta y\frac{\partial\varpi}{\partial y} + \delta z\frac{\partial\varpi}{\partial z} + \delta t\frac{\partial\varpi}{\partial t}$$

Time rate of change in vorticity (curl)

$$\Rightarrow \frac{\delta\varpi}{\delta t} = \frac{\delta x}{\delta t}.\frac{\partial\varpi}{\partial x} + \frac{\delta y}{\delta t}.\frac{\partial\varpi}{\partial y} + \frac{\delta z}{\delta t}.\frac{\partial\varpi}{\partial z} + \frac{\delta t}{\delta t}.\frac{\partial\varpi}{\partial t}$$

$$\Rightarrow \frac{d\varpi}{dt} = \frac{\partial x}{\partial t}.\frac{\partial\varpi}{\partial x} + \frac{\partial y}{\partial t}.\frac{\partial\varpi}{\partial y} + \frac{\partial z}{\partial t}.\frac{\partial\varpi}{\partial z} + \frac{\partial\varpi}{\partial t}$$

As $\delta t \to 0$

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$$\Rightarrow \frac{d\varpi}{dt} = u\frac{\partial\varpi}{\partial x} + v\frac{\partial\varpi}{\partial y} + w\frac{\partial\varpi}{\partial z} + \frac{\partial\varpi}{\partial t}$$
$$\Rightarrow \frac{\partial\varpi}{\partial t} = \frac{d\varpi}{dt} - \left\{u\frac{\partial\varpi}{\partial x} + v\frac{\partial\varpi}{\partial y} + w\frac{\partial\varpi}{\partial z}\right\}$$
(35)

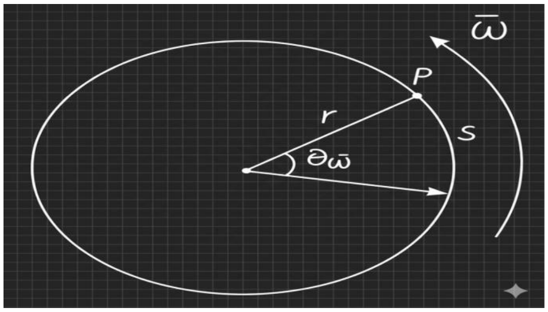

Figure 5

The angular acceleration $'\alpha'$ for a fluid particle spinning,
$$\alpha = \frac{d\varpi}{dt} = \frac{\varpi_1 - \varpi_0}{t_1 - t_0}$$
Linear velocity = radius of rotation 'r' $\times$ ϖ
$$\Rightarrow \varpi = \frac{v}{r}$$
$$\therefore \alpha = \frac{d\varpi}{dt} = \frac{\varpi_1 - \varpi_0}{t_1 - t_0} = \frac{1}{r}.\frac{v_1 + v_2}{t_1 - t_0}$$
$$\Rightarrow \alpha = \frac{1}{r}.a$$
Where a is linear acceleration
$$\Rightarrow \frac{\partial\varpi}{\partial t} = \frac{a}{r} - \left\{u\frac{\partial\varpi}{\partial x} + v\frac{\partial\varpi}{\partial y} + w\frac{\partial\varpi}{\partial z}\right\}$$
Recall from (2)
$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}$$
$$\Rightarrow \frac{\partial\varpi}{\partial t} = \frac{1}{r}\left\{\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right\} - \left\{u\frac{\partial\varpi}{\partial x} + v\frac{\partial\varpi}{\partial y} + w\frac{\partial\varpi}{\partial z}\right\}$$
$$\Rightarrow \frac{\partial\varpi}{\partial t} = \frac{1}{r}\frac{\partial u}{\partial t} + u\left(\frac{1}{r}\frac{\partial u}{\partial x} - \frac{\partial\varpi}{\partial x}\right) + v\left(\frac{1}{r}\frac{\partial u}{\partial y} - \frac{\partial\varpi}{\partial y}\right) + w\left(\frac{1}{r}\frac{\partial u}{\partial z} - \frac{\partial\varpi}{\partial z}\right) \quad (36)$$
Substituting equation (37) into (34) the substantial derivative of curl, $'\frac{\mathcal{D}\varpi}{\mathcal{D}t}'$, gives the vorticity transport equation as;
$$\frac{\mathcal{D}\varpi}{\mathcal{D}t} = \frac{1}{r}\left\{\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right\} = (\varpi.\bar{\nabla})u + \ v\nabla^2\varpi$$
$$\Rightarrow \frac{1}{r}\left\{\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right\} = \varpi_x\frac{\partial u}{\partial x} + \varpi_y\frac{\partial u}{\partial y} + \varpi_z\frac{\partial u}{\partial z} + v\left\{\frac{\partial^2\varpi}{\partial x^2} + \frac{\partial^2\varpi}{\partial y^2} + \frac{\partial^2\varpi}{\partial z^2}\right\}$$
In vector form;
$$\frac{\rho}{r}\left\{\frac{\partial u}{\partial t} + \bar{u}\nabla u\right\} = (\varpi.\bar{\nabla})u + \ u\nabla^2\varpi$$
(37)
Where r is the radius covered by the vortex.

The magnitude and direction of curl on the resultant direction of rotation, from the diagram above is given by:

$$\|\varpi\|^2 = \left\|\frac{\partial u_z}{\partial y} - \frac{\partial u_y}{\partial z}\right\|^2 - \left\|\frac{\partial u_z}{\partial x} - \frac{\partial u_x}{\partial z}\right\|^2 - \left\|\frac{\partial u_y}{\partial x} - \frac{\partial u_x}{\partial y}\right\|^2$$
(38)
$$\theta_\varpi = \tan^{-1}\left\{\frac{\frac{\partial u_z}{\partial y} - \frac{\partial u_y}{\partial z}}{\frac{\partial u_z}{\partial x} - \frac{\partial u_x}{\partial z}}\right\}$$
(39)
The direction of Curl in 3-dimensions from the diagram above;
$$\theta_i = \cos^{-1}\left\{\frac{\left(\frac{\partial u_z}{\partial y} - \frac{\partial u_y}{\partial z}\right)}{\|\varpi\|}\right\}$$
$$\theta_j = \cos^{-1}\left\{\frac{\left(\frac{\partial u_z}{\partial x} - \frac{\partial u_x}{\partial z}\right)}{\|\varpi\|}\right\}$$
$$\theta_k = \cos^{-1}\left\{\frac{\left(\frac{\partial u_y}{\partial x} - \frac{\partial u_x}{\partial y}\right)}{\|\varpi\|}\right\}$$

The Natural Logarithm of the Vorticity Transport Equation.
Formation of vortices (spins) in a turbulent flow of fluid follows a mathematical rhythm as shown below;
From (38)
$$\frac{\rho}{r}\left\{\frac{\partial u}{\partial t} + \bar{u}\nabla u\right\} = (\varpi.\bar{\nabla})u + \ u\nabla^2\varpi$$
Where r is the radius covered by the vortex.
$$\Rightarrow \frac{1}{r} = \frac{1}{\rho\left\{\frac{\partial u}{\partial t} + \bar{u}\nabla u\right\}}\{(\varpi.\bar{\nabla})u + \ u\nabla^2\varpi\}$$
$$\Rightarrow \qquad r = \frac{\rho}{\{(\varpi.\bar{\nabla})u + \ u\nabla^2\varpi\}}\left\{\frac{\partial u}{\partial t} + \bar{u}\nabla u\right\}$$
(40)
$r = r_h$, radius of vortex, from (28)
Substituting (40) into (32) to give the Natural logarithmic function of vorticity transport equation
$$\Rightarrow \qquad (3-\beta)\ln\gamma = \ln\alpha^3 - \ln\left\{\frac{\left\{\frac{\rho}{\{(\varpi.\bar{\nabla})u + u\nabla^2\varpi\}}\left\{\frac{\partial u}{\partial t} + \bar{u}\nabla u\right\}\right\}^2 L}{d\theta r_v^2}\right\}$$
(41)
Where, $(3-\beta)\ln\gamma$, is the thermal or mechanical energy at turbulence
$\ln\alpha^3$, is the pressure energy at turbulence
$\ln\left\{\frac{\left\{\frac{\rho}{\{(\varpi.\bar{\nabla})u + u\nabla^2\varpi\}}\left\{\frac{\partial u}{\partial t} + \bar{u}\nabla u\right\}\right\}^2 L}{d\theta r_v^2}\right\}$, contains the Angular momentum, vortex stretching term, rotational effect, and dimension (expanse/depth|) of the vortex formed

$$\Rightarrow \gamma^{(3-\beta)} = \frac{\alpha^3 d\theta r_v^2}{\left\{\frac{\rho}{\{(\varpi.\bar{\nabla})u + u\nabla^2\varpi\}}\left\{\frac{\partial u}{\partial t} + \bar{u}\nabla u\right\}\right\}^2 L}$$
$$\Rightarrow \gamma^{(3-\beta)} = \alpha^3 d\theta r_v^2\left\{\left\{\frac{(\varpi.\bar{\nabla})u + u\nabla^2\varpi}{\rho\left\{\frac{\partial u}{\partial t} + \bar{u}\nabla u\right\}L}\right\}\right\}^2$$
$$\Rightarrow \quad \gamma^{(3-\beta)}L\rho\left\{\frac{\partial u}{\partial t} + \bar{u}\nabla u\right\}^2 = \alpha^3 d\theta r_v^2\{(\varpi.\bar{\nabla})u + \ u\nabla^2\varpi\}^2$$
(42)

Wavelength and Frequency of Vortex Energy.
The pressure energy stimulating the molecules of a fluid into turbulent spins during the formation of vortices and cracks simultaneously travels as a waveform, using the fluid molecules as

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a material medium.

Proposition 11: The wavelength (w) varies directly as the product of pressure (P) and time (T), and inversely as the square of the volume (v) occupied by the fluid molecules.
Proof:

$$w\alpha \frac{PT}{V^2}$$

$w = k.\frac{PT}{V^2}, = \frac{1}{m}$, m is the mass per units $kgm^{-8}s^{-1}$

$$w = \frac{PT}{mV^2}$$

(43)

Proposition 12: The momentum of the force causing the vortex/crack is equal to the imparted momentum on the fluid molecules.
Proof:
The momentum of the causative force $= pressure \times Time \times Area = PT\pi r^2$
The imparted momentum $= mass\ per\ units \times volume\ cube = mV^3$

$$\Rightarrow PT\pi r^2 = mV^3$$
$$\Rightarrow PT\pi r^2 - mV^3 = 0$$
$$\therefore PT\pi r^2 - mV^3 = \boldsymbol{\varphi}$$

(44)

Proposition 13: The minimum volume (v) occupied by the air molecules is directly proportional to the wavelength (w) of the vibration of the molecules.
Proof:

$\Rightarrow v\ \alpha\ w$

$\Rightarrow v = kw$, where k is the unit area (A)

$$\Rightarrow v = Aw$$
$$\Rightarrow v = w\frac{\Delta v}{\Delta v}.1m^2$$
$$\Rightarrow v\Delta v = w\Delta v$$

Integrating

$$\int_{-v_0}^{v_\alpha} v\Delta v = \int_{-v_0}^{v_\alpha} w\Delta v$$
$$\Rightarrow \frac{v_\alpha}{v_0} = \frac{w+v}{w-v}$$

$\Rightarrow V = \frac{v_\alpha}{v_0} = \frac{w+v}{w-v}$, is the dimensionless wavelength-volume ratio.

(45)

The minimum pressure required to initiate the motion causing the cracks;

$P = \frac{\gamma V}{A}$, γ = Specific weight, V = wavelength-volume ratio

$$\Rightarrow P = \frac{\gamma}{A}\left[\frac{w+v}{w-v}\right]$$

Substituting P from (46)

$$\Rightarrow mV^3 = T\gamma\left[\frac{w+v}{w-v}\right]$$
$$\Rightarrow mV^3[w-v] = T\gamma[w+v]$$

Substituting w from (45)

$$\frac{KPT}{V} - Kmv^2 = \frac{K\gamma T}{V^2}[w+v]$$

$\frac{1}{V}\ \alpha\ P$, (Boyle's law)

$\sigma = \frac{F}{L}K = 1$ (surface tension of fluid)

$$\Rightarrow P^2T - \sigma mv^2 = \frac{\sigma\gamma T}{V^2}[w+v]$$
$$\Rightarrow P^2T - \sigma mv^2 = P^2Tf[w+v]$$
$$\Rightarrow P^2 = \frac{mv^2\sigma}{[T - fT^2(w+v)]}$$
$$\Rightarrow P = \sqrt{\frac{mv^2\sigma}{[T-fT^2(w+v)]}}$$

(46)

f, is the frequency of vortex energy.
Substituting (46) into (42) to give the Natural logarithmic function of the vorticity transport equation
Recall, from (27)

$\alpha^3 = p.\phi^3$,

$$\Rightarrow (3-\beta)\ln\gamma = \ln[p.\phi^3] - \ln\left\{\frac{\left\{\frac{\rho}{\{(\varpi.\bar{\nabla})u + u\nabla^2\varpi\}}\left\{\frac{\partial u}{\partial t} + \bar{u}\nabla u\right\}\right\}^2 L}{d\theta r_v^2}\right\}$$

$$\Rightarrow (3-\beta)\ln\gamma = \ln\left[\phi^3\sqrt{\frac{mv^2\sigma}{[T-fT^2(w+v)]}}\right] - \ln\left\{\frac{\left\{\frac{\rho}{\{(\varpi.\bar{\nabla})u + u\nabla^2\varpi\}}\left\{\frac{\partial u}{\partial t} + \bar{u}\nabla u\right\}\right\}^2 L}{d\theta r_v^2}\right\} \quad (47)$$

Where, $(3-\beta)\ln\gamma$, is the thermal or mechanical energy of the vortex formed at turbulence

$\ln\ln\left[\phi^3\sqrt{\frac{mv^2\sigma}{[T-fT^2(w+v)]}}\right]$, is the pressure/vibrational energy of the vortex formed at turbulence

$\ln\left\{\frac{\left\{\frac{\rho}{\{(\varpi.\bar{\nabla})u + u\nabla^2\varpi\}}\left\{\frac{\partial u}{\partial t} + \bar{u}\nabla u\right\}\right\}^2 L}{d\theta r_v^2}\right\}$, contains the Angular momentum, vortex stretching term, rotational effect, and dimension (expanse/depth|) of the vortex formed

$$\Rightarrow \gamma^{(3-\beta)} = \frac{\left[\phi^3\sqrt{\frac{mv^2\sigma}{[T-fT^2(w+v)]}}\right]d\theta r_v^2}{\left\{\frac{\rho}{\{(\varpi.\bar{\nabla})u + u\nabla^2\varpi\}}\left\{\frac{\partial u}{\partial t} + \bar{u}\nabla u\right\}\right\}^2 L}$$

$$\Rightarrow \gamma^{(3-\beta)} = \left[\phi^3\sqrt{\frac{mv^2\sigma}{[T-fT^2(w+v)]}}\right]d\theta r_v^2\left\{\left\{\frac{(\varpi.\bar{\nabla})u + u\nabla^2\varpi}{\rho\left\{\frac{\partial u}{\partial t} + \bar{u}\nabla u\right\}L}\right\}\right\}^2$$

$$\Rightarrow \gamma^{(3-\beta)}L\rho\left\{\frac{\partial u}{\partial t} + \bar{u}\nabla u\right\}^2 = \left[\phi^3\sqrt{\frac{mv^2\sigma}{[T-fT^2(w+v)]}}\right]d\theta r_v^2\{(\varpi.\bar{\nabla})u + u\nabla^2\varpi\}^2$$

(48)

Navier-Stokes Equation as a function of quantum integrals.
The Navier-Stokes equation can be expressed as functions of quantum numbers, as stated earlier from "Lemma 1". As a result, the following assumption is taken;
The momentum force, pressure force, and shear forces are multiples of a minimum force acting on a quantum fluid molecule (differential element) moving around its volume mass under gravity. The multiples are integral values (quantum numbers); n_B (for momentum force), n_P (pressure force) and n_τ (Shear force). Where, $n_B, n_P, n_\tau = 0,1,2,3, ...$

$$\Rightarrow \frac{1}{F_m}\left\{\frac{\partial u_i}{\partial t} + \sum_j u_j\frac{\partial u_i}{\partial x_j} = f_i(x,t) - \frac{\partial P}{\partial x_i} + \nu\Delta u_i\right\}$$

(49)

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$\Leftrightarrow \frac{1}{F_m}\left\{\rho \frac{\mathcal{D}u}{\mathcal{D}t} = -\nabla P + \mu\nabla^2 u\right\}$

$\Leftrightarrow F_0 \left\{\rho \frac{\mathcal{D}u}{\mathcal{D}t} = -\nabla P + \mu\nabla^2 u\right\}$

(50)

$\Leftrightarrow$ $$n_B = -n_P + n_\tau$$

(51)

Where, F_0 is the reciprocal of the quantum force: $F_0 = \frac{1}{F_m}$

n_B, is the momentum force quantum number
n_P, is the pressure force quantum number, and
n_τ, is the shear force quantum number.

The momentum force is a multiple of the minimum force, given as the product of the minimum force F_m and the momentum quantum number n_B.

$\Rightarrow$ $$\rho \frac{\mathcal{D}u}{\mathcal{D}t} = F_m \times n_B$$

(52)

$\Rightarrow \frac{1}{F_m} \times \rho \frac{\mathcal{D}u}{\mathcal{D}t} = n_B$

$\Rightarrow F_0 \times \rho \frac{\mathcal{D}u}{\mathcal{D}t} = n_B$

The graph of $n_B / \rho \frac{\mathcal{D}u}{\mathcal{D}t}$ is linear.

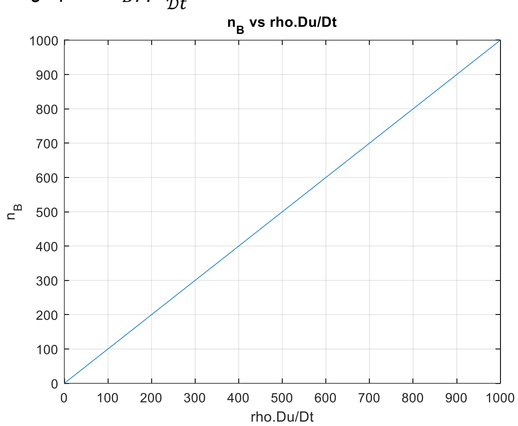

Figure 6 Graph of $n_B / \rho \frac{\mathcal{D}u}{\mathcal{D}t}$

$\Rightarrow \frac{\Delta n_B}{\Delta \rho \frac{\mathcal{D}u}{\mathcal{D}t}} = F_0$, the slope of the graph is the quantum force (F_0).

$\Rightarrow \Delta n_B = F_0 \Delta\rho \frac{\mathcal{D}u}{\mathcal{D}t}$

Integrating,

$$\int \Delta n_B = \int F_0 \Delta\rho \frac{\mathcal{D}u}{\mathcal{D}t}$$

$\Leftrightarrow$

$$\int dn_B = \int F_0 d\rho \frac{\mathcal{D}u}{\mathcal{D}t}$$

As, $\Delta\rho \frac{\mathcal{D}u}{\mathcal{D}t} \to 0$ and $\Delta n_B \to 0$

$\Rightarrow n_B = F_0 . \rho \frac{\mathcal{D}u}{\mathcal{D}t} + C_B,$

$\Rightarrow$ $$\rho \frac{\mathcal{D}u}{\mathcal{D}t} = \frac{1}{F_0}[n_B - C_B]$$

(53)

Similarly;
For pressure force,
The graph of $n_P / \nabla P$ is linear.

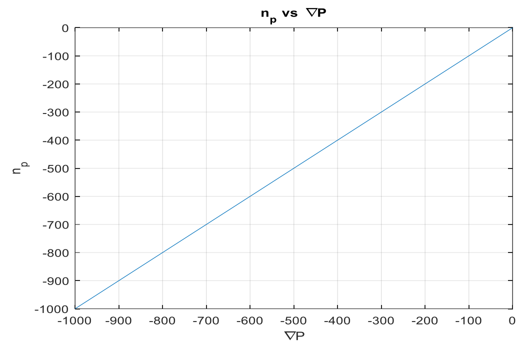

Figure 7 Graph of $n_p / \nabla P$

$\Rightarrow -n_p = -F_0 . \nabla P + C_P$

$\Rightarrow$ $$-\nabla P = -\frac{1}{F_0}[n_P + C_P]$$

(54)

Also,
Shearing force,
The graph of $n_\tau / \mu\nabla^2 u$ is linear.

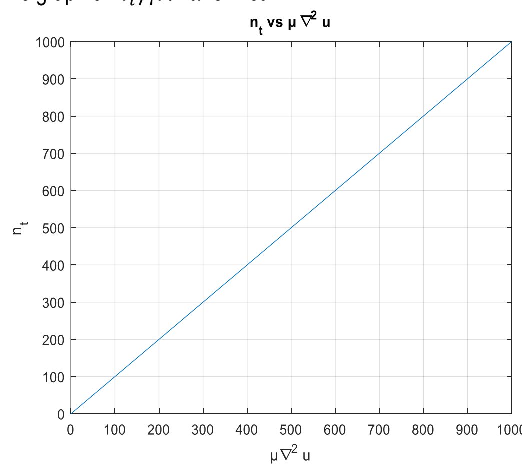

Figure 8 Graph of $n_\tau / \mu\nabla^2 u$

$\Rightarrow n_\tau = F_0 . \mu\nabla^2 u + C_\tau$

$\Rightarrow \mu\nabla^2 u = \frac{1}{F_0}[n_\tau - C_\tau]$

$\Rightarrow$ $$\nabla^2 u = \frac{1}{\mu F_0}[n_\tau - C_\tau]$$

(55)

Substituting these values from (53), (54) and (55) into equation (9)

$\frac{1}{F_0}[n_B - C_B] = -\frac{1}{F_0}[n_P + C_P] + \frac{1}{F_0}[n_\tau - C_\tau]$

(56)

Also, from (9):

$\rho \frac{\mathcal{D}u}{\mathcal{D}t} - [-\nabla P + \mu\nabla^2 u] = 0$

$\Rightarrow \frac{1}{F_q}\left[\rho \frac{\mathcal{D}u}{\mathcal{D}t} - [-\nabla P + \mu\nabla^2 u] = 0\right]$

$\Rightarrow n_B - [-n_P + n_\tau] = 0$

$\Leftrightarrow n_B - [-n_P + n_\tau] = n$

(57)

where "**n**" is the general quantum number for the N-S equation.

$\Rightarrow \frac{\Delta n}{\Delta\rho \frac{\mathcal{D}u}{\mathcal{D}t} - [-\nabla P + \mu\nabla^2 u]} = F_0$, where F_0 is the slope of the graph n/ $\rho \frac{\mathcal{D}u}{\mathcal{D}t} - [-\nabla P + \mu\nabla^2 u]$

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Integrating,

$\Rightarrow \int \Delta n = \int F_0 \Delta \rho \frac{\mathcal{D}u}{\mathcal{D}t} - [-\nabla P + \mu \nabla^2 u]$, let $\rho \frac{\mathcal{D}u}{\mathcal{D}t} - [-\nabla P + \mu \nabla^2 u] = \varphi$

$$\Rightarrow \int \Delta n = \int F_0 \Delta \varphi$$

$$\Rightarrow n = F_0\, \varphi + c$$

$$\Rightarrow \varphi = \frac{1}{F_0}(n - c)$$

(58)

$$\Rightarrow \rho \frac{\mathcal{D}u}{\mathcal{D}t} - [-\nabla P + \mu \nabla^2 u] = \frac{1}{F_0}(n - c)$$

∴ Equation (56)

$$\Leftrightarrow \frac{1}{F_0}[n_B - C_B] - \left\{-\frac{1}{F_0}[n_P + C_P] + \frac{1}{F_0}[n_\tau - C_\tau]\right\} = \frac{1}{F_0}(n - c) \quad (59)$$

New form of the N-S equation.

For a fluid system at equilibrium;

$\rho \frac{\mathcal{D}u}{\mathcal{D}t} - [-\nabla P + \mu \nabla^2 u] = 0$, at equilibrium, where $\rho \frac{\mathcal{D}u}{\mathcal{D}t} = [-\nabla P + \mu \nabla^2 u]$

$$\Rightarrow \frac{1}{F_0}[n_B - C_B] - \left\{-\frac{1}{F_0}[n_P + C_P] + \frac{1}{F_0}[n_\tau - C_\tau]\right\} = 0$$

(60)

There will be positive values for "**n**" and "$\boldsymbol{\rho \frac{\mathcal{D}u}{\mathcal{D}t} - [-\nabla P + \mu \nabla^2 u]}$".

The graph of **n** against $\boldsymbol{\rho \frac{\mathcal{D}u}{\mathcal{D}t} - [-\nabla P + \mu \nabla^2 u]}$, is shown below.

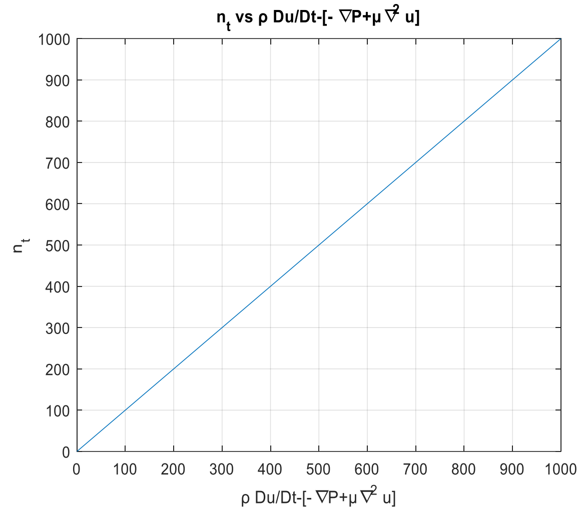

Figure 9 Graph of $\boldsymbol{n/\rho \frac{\mathcal{D}u}{\mathcal{D}t} - [-\nabla P + \mu \nabla^2 u]}$

This shift, from $n = 0$ to $n > 0$ leads from equilibrium to harmonization or smoothness of the fluid molecules and the parameters in the flow field. The greater the value of n the smoother the fluid.

Conversely:

For a non-equilibrium fluid system;

When, $\rho \frac{\mathcal{D}u}{\mathcal{D}t} < [-\nabla P + \mu \nabla^2 u]$, we have a non-equilibrium fluid system.

$$\Rightarrow \frac{1}{F_0}[n_B - C_B] - \left\{-\frac{1}{F_0}[n_P + C_P] + \frac{1}{F_0}[n_\tau - C_\tau]\right\} < 0$$

There will be negative values for "**n**" and "$\boldsymbol{\rho \frac{\mathcal{D}u}{\mathcal{D}t} - [-\nabla P + \mu \nabla^2 u]}$".

The graph of **n** against $\boldsymbol{\rho \frac{\mathcal{D}u}{\mathcal{D}t} - [-\nabla P + \mu \nabla^2 u]}$, is shown below.

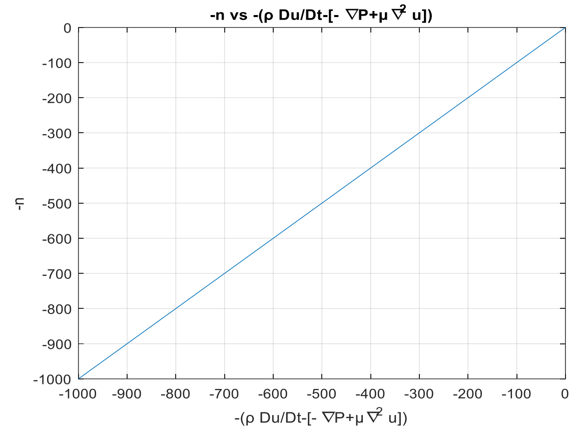

Figure 10 Graph of $\boldsymbol{-n/\rho \frac{\mathcal{D}u}{\mathcal{D}t} - [-\nabla P + \mu \nabla^2 u]}$

This shift from the equilibrium of the fluid from $n = 0$ to $n < 0$ leads to an outburst whose intensity (chaos) increases as you go down the negative axis.

Solutions to the parameters in the linear and non-linear terms of the Navier-Stokes Equation on Torus $\mathbb{R}^2/\mathbb{Z}^2$.

The shear force divergence as a linear term with its solution given in Lemma 4, due to internal friction (viscosity) in a fluid is integrated on **Torus** $\mathbb{R}^2/\mathbb{Z}^2$, yielding the solution to the divergence of velocity u in 2-dimensions. Substituting the solution of the divergence of u, into the non-linear term yield the solutions of; v, local acceleration, convective acceleration and pressure gradient. The solutions yield the subsequent graphs, topology and vorticity transport equation.

Considering the linear term from "Lemma 4":

$$\mu \nabla^2 u = \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) = \frac{1}{F_0}[n_\tau - C_\tau]$$

(61)

$$\Rightarrow \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = \frac{1}{\mu F_0}[n_\tau - C_\tau]$$

$\Rightarrow \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{1}{\mu F_0}[n_\tau - C_\tau]$ in 2-dimension.

As $\partial z \to \infty, \frac{\partial^2 u}{\partial z^2} \to 0$

Integrating,

$$\iint \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \partial x \partial y = \iint \frac{1}{\mu F_0}[n_\tau - C_\tau]\, \partial x \partial y$$

As, $\partial x \longrightarrow 0, \partial y \longrightarrow 0$

$$\Rightarrow y\frac{\partial u}{\partial x} + x\frac{\partial u}{\partial y} = \frac{xy}{\mu F_0}[n_\tau - C_\tau] + c_x y + c_y,$$

$$\left.\begin{matrix} at\ x = 0, y = 0 \\ u = o, c_y = 0 \end{matrix}\right\}$$

$$\Rightarrow c_x = \frac{\partial u}{\partial x} + \frac{l}{h}\frac{\partial u}{\partial y} - \frac{xy}{\mu F_0}[n_\tau - C_\tau],$$

$at\ x = l, and\ y = h\}$

$$\Rightarrow y\frac{\partial u}{\partial x} + x\frac{\partial u}{\partial y} = \frac{xy}{\mu F_0}[n_\tau - C_\tau] + \left[\frac{\partial u}{\partial x} + \frac{l}{h}\frac{\partial u}{\partial y} - \frac{xy}{\mu F_0}[n_\tau - C_\tau]\right] y$$

$$\Rightarrow \frac{\partial u}{\partial y} = \frac{y\left[x - \frac{l}{y}\right]}{\mu F_0\left[x - \frac{l}{h}\right]}[n_\tau - C_\tau]$$

(62)

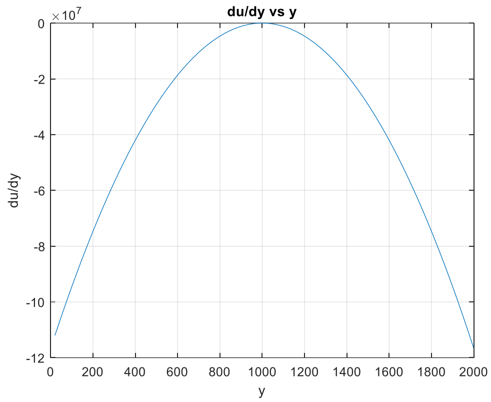

Figure 11 Graph of $\frac{\partial u}{\partial y}/y$

Similarly,

$$\frac{\partial u}{\partial x} = \frac{x\left[y - \frac{h}{x}\right]}{\mu F_0\left[y - \frac{h}{l}\right]}[n_\tau - C_\tau]$$

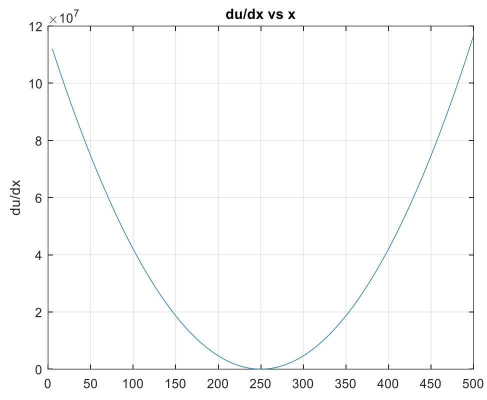

Figure 12 Graph of $\frac{\partial u}{\partial y}/x$

Let, $\varepsilon_y = \frac{y\left[x-\frac{l}{y}\right]}{\mu F_0\left[x-\frac{l}{h}\right]}, \Rightarrow \frac{\partial u}{\partial y} = \varepsilon_y[n_\tau - C_\tau]$
(63)

$\varepsilon_x = \frac{x\left[y-\frac{h}{x}\right]}{\mu F_0\left[y-\frac{h}{l}\right]}, \Rightarrow \frac{\partial u}{\partial x} = \varepsilon_x[n_\tau - C_\tau]$
(64)

Integrating (64),

$$\int \partial u = \int_0^x \varepsilon_x[n_\tau - C_\tau]\partial x$$

$u = x\varepsilon_x[n_\tau - C_\tau]$
(65)

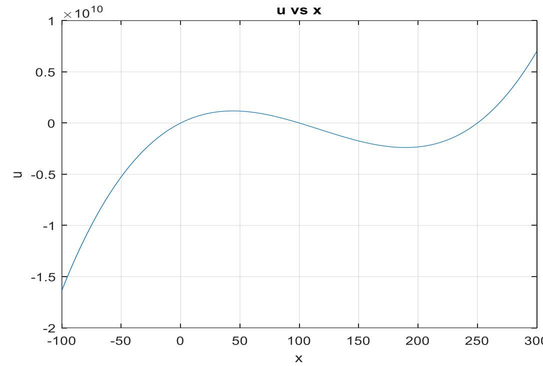

Figure 13 Graph of u/x

Similarly,

$u = y\varepsilon_y[n_\tau - C_\tau]$
(66)

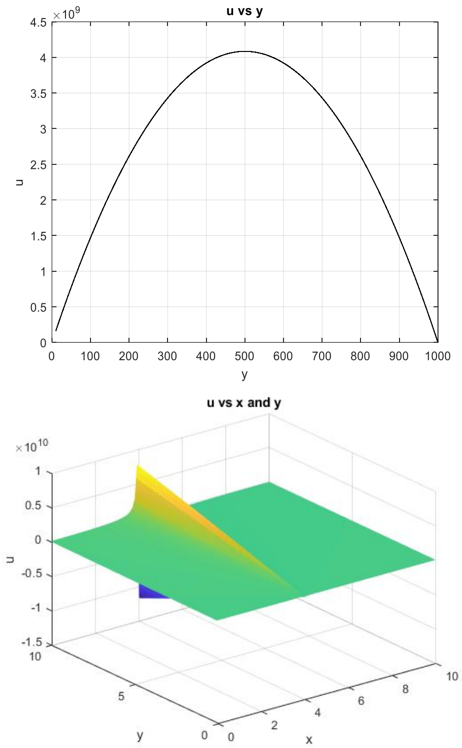

Figure 14 Graph of u/y in $2D$ & $u/x, y$ in $3D$

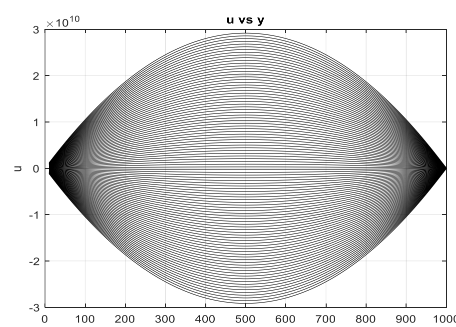

Figure 15 Graph of u/y showing the topology of the flow

https://dx.doi.org/10.4314/swj.v20i4.46

Velocities in the non-linear term and the divergence of Pressure on Torus $\mathbb{R}^2/\mathbb{Z}^2$.

$\rho\frac{\mathcal{D}u}{\mathcal{D}t}=\frac{1}{F_0}(n_B-C_B)$, from equation (53)

$$\Rightarrow \rho\left[\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right]=\frac{1}{F_0}(n_B-C_B)$$

$$\Rightarrow \left[\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right]=\frac{1}{\rho F_0}(n_B-C_B)$$

As $\partial z\longrightarrow\infty,\frac{\partial u}{\partial z}\longrightarrow 0$

$$\Rightarrow \left[\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right]=\frac{1}{\rho F_0}(n_B-C_B)$$

Substituting for $\frac{\partial u}{\partial x}$ and $\frac{\partial u}{\partial y}$

$$\frac{\partial u}{\partial t}+u\varepsilon_x[n_\tau-C_\tau]+v\varepsilon_y[n_\tau-C_\tau]=\frac{1}{\rho F_0}(n_B-C_B)$$

$$\Rightarrow \frac{\partial u}{\partial t}+[u\varepsilon_x+v\varepsilon_y][n_\tau-C_\tau]=\frac{1}{\rho F_0}(n_B-C_B)$$

$$\Rightarrow [u\varepsilon_x+v\varepsilon_y][n_\tau-C_\tau]=\frac{1}{\rho F_0}(n_B-C_B)-\frac{\partial u}{\partial t}$$

$$\Rightarrow u\varepsilon_x+v\varepsilon_y=\frac{(n_B-C_B)}{\rho F_0[n_\tau-C_\tau]}-\frac{1}{[n_\tau-C_\tau]}\cdot\frac{\partial u}{\partial t}$$

$$\Rightarrow v=\frac{(n_B-C_B)}{\varepsilon_y\,\rho F_0[n_\tau-C_\tau]}-\frac{1}{\varepsilon_y[n_\tau-C_\tau]}\cdot\frac{\partial u}{\partial t}-u\frac{\varepsilon_x}{\varepsilon_y}$$

$$\Rightarrow v=\frac{1}{2}\left\{\frac{1}{\varepsilon_y\,\rho F_0}\left[\frac{n_B-C_B}{n_\tau-C_\tau}\right]-u\frac{\varepsilon_x}{\varepsilon_y}\right\}$$

$$\Rightarrow \qquad v=\frac{1}{2}\left\{\frac{1}{\varepsilon_y\,\rho F_0}\left[\frac{n_B-C_B}{n_\tau-C_\tau}\right]-y\varepsilon_x[n_\tau-C_\tau]\right\}$$

(67)

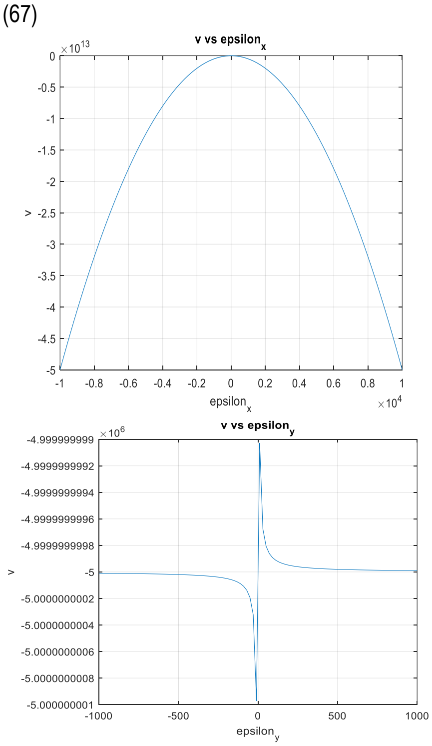

Figure 16 Graph of v/ε_x & v/ε_y

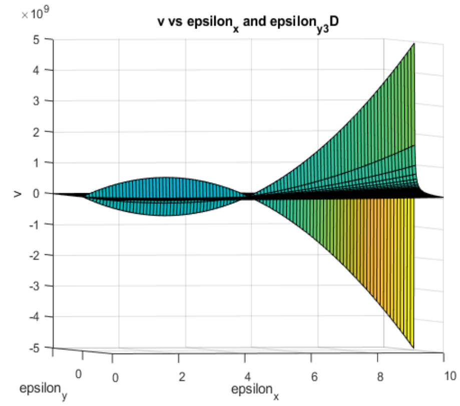

Figure 17 Graph of v/ε_x & ε_y $in\ 3D$

For steady flow $\frac{\partial u}{\partial t}=0$

$$\therefore\ v=\left\{\frac{1}{\varepsilon_y\,\rho F_0}\left[\frac{n_B-C_B}{n_\tau-C_\tau}\right]-u\frac{\varepsilon_x}{\varepsilon_y}\right\}=\left\{\frac{1}{\varepsilon_y\,\rho F_0}\left[\frac{n_B-C_B}{n_\tau-C_\tau}\right]-y\varepsilon_x[n_\tau-C_\tau]\right\}$$

(68)

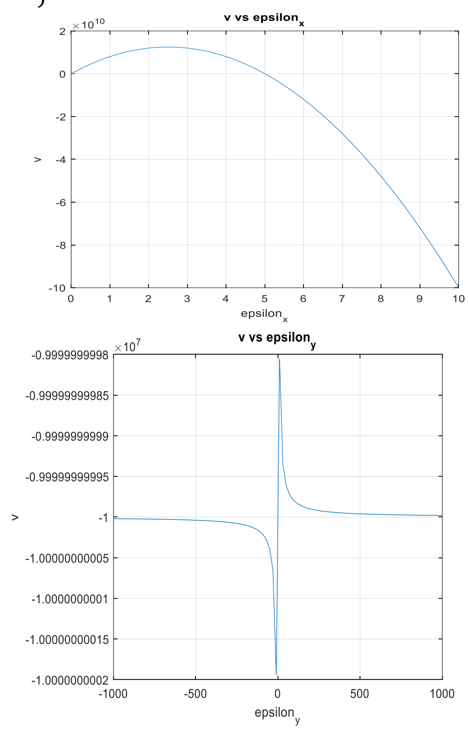

Figure 18 Graph of v/ε_x & v/ε_y

From equations 56 and 54

$$-\nabla P=-\frac{1}{F_0}[n_P+C_P]=\frac{1}{F_0}[n_B-C_B]-\frac{1}{F_0}[n_\tau-C_\tau]$$

$$\Rightarrow -\frac{\partial p}{\partial x}=\rho\left[\frac{\partial u}{\partial t}+[u\varepsilon_x+v\varepsilon_y][n_\tau-C_\tau]\right]-\frac{1}{F_0}[n_\tau-C_\tau],$$

$\mathbb{R}^d: x[0,\infty], t>0$

$$\Rightarrow -\frac{\partial p}{\partial x}=\rho\frac{\partial u}{\partial t}+[n_\tau-C_\tau]\left[\rho[u\varepsilon_x+v\varepsilon_y]-\frac{1}{F_0}\right]$$

(69)

For a steady flow, $\frac{\partial u}{\partial t}=0$

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$$\Rightarrow -\frac{\partial p}{\partial x} = [n_\tau - C_\tau]\left[\rho[u\varepsilon_x + v\varepsilon_y] - \frac{1}{F_0}\right]$$
(70)
For a steady, laminar and fully developed flow
$$[u\varepsilon_x + v\varepsilon_y] = 0$$
$$\Rightarrow -\frac{\partial p}{\partial x} = [n_\tau - C_\tau]\left[-\frac{1}{F_0}\right]$$
$$\Rightarrow -\frac{\partial p}{\partial x} = -\frac{1}{F_0}[n_\tau - C_\tau]$$
(71)
For a Cottle flow,
$$0 = -\frac{\partial p}{\partial x} + \mu\frac{\partial^2 u}{\partial y^2}$$
From (55)
$$\mu\frac{\partial^2 u}{\partial y^2} = \frac{1}{\mu F_0}[n_\tau - C_\tau]$$
Integrating
$$\iint \mu\frac{\partial^2 u}{\partial y^2}\partial y\partial y = \iint \frac{1}{\mu F_0}[n_\tau - C_\tau]\partial y\partial y$$
$$u = \frac{y^2}{\mu F_0}[n_\tau - c_\tau] + c_1 y + c_2 \qquad \left.\begin{array}{c} at\ u = 0, y = 0 \\ c_2 = 0 \\ at\ u = u_\propto, y = h \\ c_1 = \frac{u_\propto}{h} + \frac{h}{\mu F_0}[n_\tau - c_\tau] \end{array}\right\}$$
As $\partial y \to 0$
Substituting;
$$\frac{u_\propto}{u} = \frac{y}{h} + \frac{h^2}{u_\propto \mu F_0}[-(n_\tau - c_\tau)]\frac{y}{h}\left(1 - \frac{y}{h}\right)$$
(72)

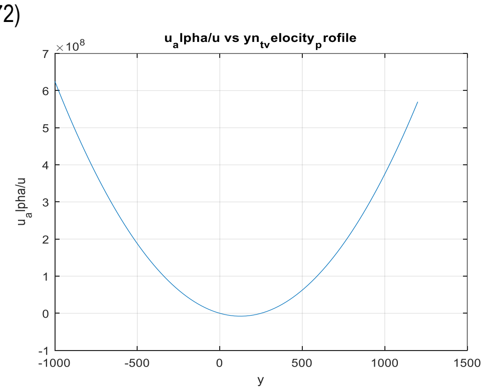


Figure 19 Graph of $\frac{u_\propto}{u}/y$

Recall, (71)
$$-\frac{\partial p}{\partial x} = -\frac{1}{F_0}[n_\tau - C_\tau]$$
$$\Rightarrow \frac{u_\propto}{u} = \frac{y}{h} + \frac{h^2}{u_\propto \mu}\left[-\frac{\partial p}{\partial x}\right]\frac{y}{h}\left(1 - \frac{y}{h}\right)$$
(73)

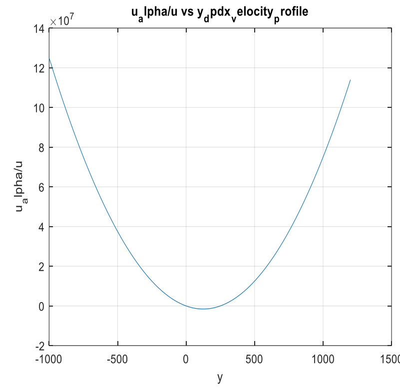
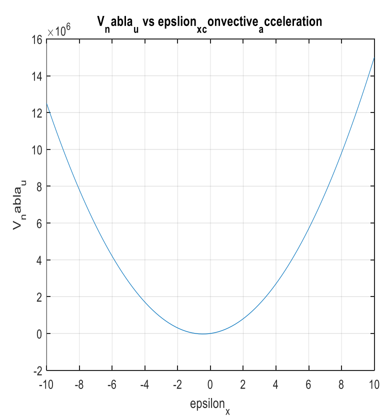


Figure 20 Graph of $\frac{u_\propto}{u}/y$

$$\Rightarrow \frac{u_\propto}{u} = \frac{y}{h} + \frac{h^2}{u_\propto \mu F_0}[-(n_P + c_P)]\frac{y}{h}\left(1 - \frac{y}{h}\right)$$
(74)

Solutions for Local and Convective Terms of Acceleration in the NSE.

From (7)
$$\rho\frac{\mathcal{D}u}{\mathcal{D}t} = \rho\left[\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right]$$
$$\Rightarrow \rho\frac{\mathcal{D}u}{\mathcal{D}t} = \rho\left[\frac{\partial u}{\partial t} + \vec{V}\nabla u\right]$$
Substituting values of the convective terms;
$$\rho\frac{\mathcal{D}u}{\mathcal{D}t} = \rho\left[\frac{\partial u}{\partial t} + y\varepsilon_x[n_\tau - C_\tau]y\varepsilon_x[n_\tau - C_\tau] + \frac{1}{2}\left\{\frac{1}{\varepsilon_y\,\rho F_0}\left[\frac{n_B - C_B}{n_\tau - C_\tau}\right] - y\varepsilon_x[n_\tau - C_\tau]\right\}y\varepsilon_x[n_\tau - C_\tau]\right]$$
$$\Rightarrow \rho\frac{\mathcal{D}u}{\mathcal{D}t} = \rho\left[\frac{\partial u}{\partial t} + \frac{[n_B - C_B]}{2\rho F_0} + \frac{y\varepsilon_x\varepsilon_y[n_\tau - C_\tau]^2}{2}\right]$$
(75)
$$\Rightarrow \vec{V}\nabla u = \frac{[n_B - C_B]}{2\rho F_0} + \frac{y\varepsilon_x\varepsilon_y[n_\tau - C_\tau]^2}{2},$$ is the convective acceleration. (76)



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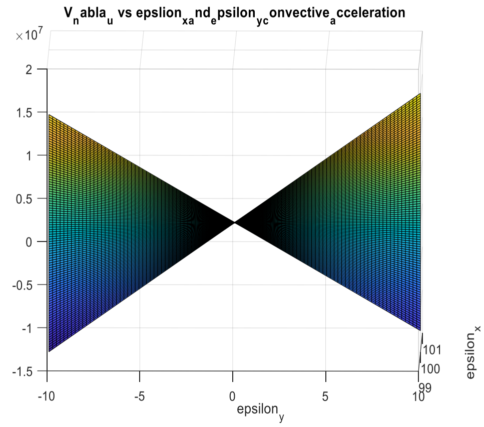


Figure 21 Graph of $\vec{V}\nabla u$ / ε_x & $\vec{V}\nabla u/\varepsilon_x, \varepsilon_y$

To find the local acceleration;
From equation (69),

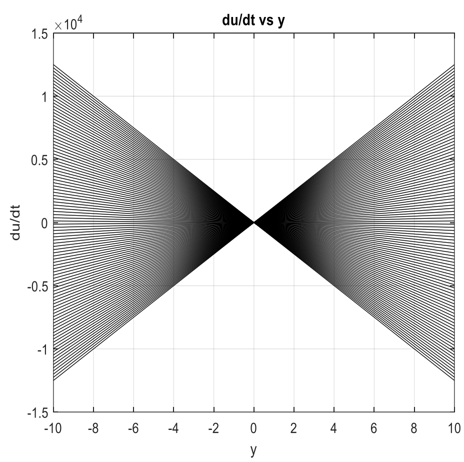
$$\frac{\partial u}{\partial t} = -\frac{\partial p}{\rho \partial x} - [n_\tau - C_\tau]\left[[u\varepsilon_x + v\varepsilon_y] - \frac{1}{\rho F_0}\right], \quad \text{for } \mathbb{R}^d: x[0,\infty], t > 0$$

$$\Rightarrow \frac{\partial u}{\partial t} = -\frac{\partial p}{\rho \partial x} - [n_\tau - C_\tau]\left[\frac{[n_B - C_B]}{2\rho F_0[n_\tau - C_\tau]} + \frac{y\varepsilon_x\varepsilon_y[n_\tau - C_\tau]}{2} - \frac{1}{\rho F_0}\right]$$

Recall. $-\frac{\partial p}{\rho \partial x} = -\frac{1}{F_0}[n_P + C_P]$

$$\Rightarrow \frac{\partial u}{\partial t} = -\frac{1}{\rho F_0}[n_P + C_P] - \frac{[n_B - C_B]}{2\rho F_0} - \frac{y\varepsilon_x\varepsilon_y[n_\tau - C_\tau]^2}{2} + \frac{[n_\tau - C_\tau]}{\rho F_0}$$

$$\Rightarrow \frac{\partial u}{\partial t} = -\frac{1}{\rho F_0}\left[\frac{[n_B - C_B]}{2} + [n_P + C_P] - [n_\tau - C_\tau]\right] - \frac{y\varepsilon_x\varepsilon_y[n_\tau - C_\tau]^2}{2}, \quad \text{local acceleration} \qquad (77)$$



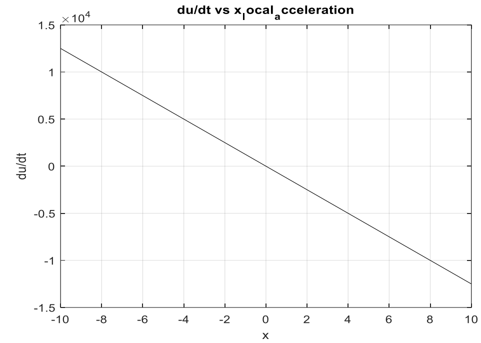


Figure 22 Graph of $\frac{\partial u}{\partial t}/y$ & $\frac{\partial u}{\partial t}/x$

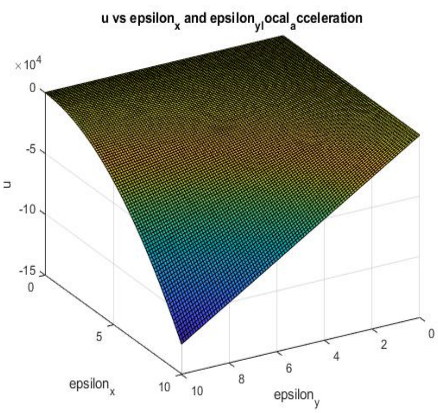


Figure 23: Graph of $\frac{\partial u}{\partial t}/\varepsilon_x, \varepsilon_y$

Substituting (76) into (77)

$$\rho\frac{\mathcal{D}u}{\mathcal{D}t} = \rho\left[-\frac{1}{\rho F_0}\left[\frac{[n_B - C_B]}{2} + [n_P + C_P] - [n_\tau - C_\tau]\right] - \frac{y\varepsilon_x\varepsilon_y[n_\tau - C_\tau]^2}{2} + \frac{[n_B - C_B]}{2\rho F_0} + \frac{y\varepsilon_x\varepsilon_y[n_\tau - C_\tau]^2}{2}\right]$$

$$\Rightarrow \rho\frac{\mathcal{D}u}{\mathcal{D}t} = \rho\left[-\frac{1}{\rho F_0}[[n_P + C_P] - [n_\tau - C_\tau]]\right]$$

$$\Rightarrow \rho\frac{\mathcal{D}u}{\mathcal{D}t} = -\frac{1}{\rho F_0}[[n_P + C_P] - [n_\tau - C_\tau]]$$

$$\Rightarrow \rho\frac{\mathcal{D}u}{\mathcal{D}t} = -\frac{1}{\rho F_0}[n_P + C_P] + [n_\tau - C_\tau]$$

$$\Rightarrow \rho\frac{\mathcal{D}u}{\mathcal{D}t} = \frac{1}{F_0}[n_B - C_B] = -\frac{1}{\rho F_0}[n_P + C_P] + [n_\tau - C_\tau] \quad \Leftrightarrow (56)$$

Vorticity Transport Equation
From the solutions in (65) and (67)

CURL 1:

$$u = \frac{x^2\left[y - \frac{h}{x}\right]}{\mu F_0\left[y - \frac{h}{l}\right]}[n_\tau - C_\tau],$$

$$v = \frac{1}{2}\left\{\frac{\mu F_0\left[x - \frac{l}{h}\right]}{y\rho F_0\left[x - \frac{l}{y}\right]}\cdot\left[\frac{n_B - C_B}{n_\tau - C_\tau}\right] - \frac{yx\left[y - \frac{h}{x}\right]}{\mu F_0\left[y - \frac{h}{l}\right]}\cdot[n_\tau - C_\tau]\right\}$$

$$w = 0$$

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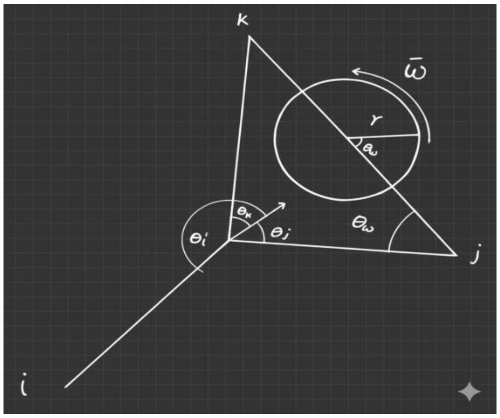


Figure 24

$$\varpi = \overline{\nabla} \times \begin{pmatrix} u \\ v \\ w \end{pmatrix}$$

$$\varpi = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{x^2\left[y-\frac{h}{x}\right]}{\mu F_0\left[y-\frac{h}{l}\right]}[n_\tau - C_\tau] & \frac{1}{2}\left\{\frac{\mu F_0\left[x-\frac{l}{h}\right]}{y\rho F_0\left[x-\frac{l}{y}\right]}\cdot\left[\frac{n_B - C_B}{n_\tau - C_\tau}\right] - \frac{yx\left[y-\frac{h}{x}\right]}{\mu F_0\left[y-\frac{h}{l}\right]}\cdot[n_\tau - C_\tau]\right\} & 0 \end{vmatrix}$$

$$\varpi = k\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

$$\Rightarrow \varpi = k\left\{\frac{1}{\mu F_0\left[x-\frac{l}{y}\right]^2\left[y-\frac{h}{l}\right]^2[n_\tau - C_\tau]}\left(\frac{1}{2}\mu^2 {F_0}^2\rho y[n_B - C_B]\left[\frac{-l(h+y)}{yh}\right]\left[y-\frac{h}{l}\right]^2 - \frac{1}{2}y^2[n_\tau - C_\tau]^2\left[x-\frac{l}{h}\right]^2\left[y-\frac{h}{l}\right] - hx^2[n_\tau - C_\tau]^3\left[\frac{1}{x}-\frac{1}{y}\right]\left[x-\frac{l}{y}\right]^2\right)\right\}$$

∴ The components of rotation are;

$\varpi_x = 0$

$\varpi_y = 0$

$$\varpi_z = \left\{\frac{1}{\mu F_0\left[x-\frac{l}{y}\right]^2\left[y-\frac{h}{l}\right]^2[n_\tau - C_\tau]}\left(\frac{1}{2}\mu^2 {F_0}^2\rho y[n_B - C_B]\left[\frac{-l(h+y)}{yh}\right]\left[y-\frac{h}{l}\right]^2 - \frac{1}{2}y^2[n_\tau - C_\tau]^2\left[x-\frac{l}{h}\right]^2\left[y-\frac{h}{l}\right] - hx^2[n_\tau - C_\tau]^3\left[\frac{1}{x}-\frac{1}{y}\right]\left[x-\frac{l}{y}\right]^2\right)\right\}$$

(78)

Recall, (38)

$$\frac{1}{r}\left\{\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right\} = (\varpi.\overline{\nabla})u + \upsilon\nabla^2\varpi$$

Substituting equations (76)-(77) (solutions of local and convective acceleration) into the vorticity transport equation (38)

$$\frac{1}{r}\left\{-\frac{1}{\rho F_0}\left[\frac{[n_B - C_B]}{2} + [n_P + C_P] - [n_\tau - C_\tau]\right] - \frac{y\varepsilon_x\varepsilon_y[n_\tau - C_\tau]^2}{2} + \frac{[n_B - C_B]}{2\rho F_0} + \frac{y\varepsilon_x\varepsilon_y[n_\tau - C_\tau]^2}{2}\right\} = \frac{[n_\tau - C_\tau]}{\mu F_0}\left\{\varpi_x\left[\frac{x\left[y-\frac{h}{x}\right]}{\left[y-\frac{h}{l}\right]}\right] + \varpi_y\left[\frac{y\left[x-\frac{l}{y}\right]}{\left[x-\frac{l}{h}\right]}\right]\right\} + \upsilon\left\{\frac{\partial^2\left(\frac{\partial u_z}{\partial y} - \frac{\partial u_y}{\partial z}\right)}{\partial^2 x} + \frac{\partial^2\left(\frac{\partial u_z}{\partial x} - \frac{\partial u_x}{\partial z}\right)}{\partial^2 y} + \frac{\partial^2\left(\frac{\partial u_y}{\partial x} - \frac{\partial u_x}{\partial y}\right)}{\partial^2 z}\right\} \quad (79)$$

Also recall, (42)

$$(3-\beta)\ln\gamma = \ln\alpha^3 - \ln\left\{\frac{\left\{\frac{\rho}{\{(\varpi.\overline{\nabla})u + u\nabla^2\varpi\}}\left\{\frac{\partial u}{\partial t} + \bar{u}\nabla u\right\}\right\}^2 L}{d\theta r_v^2}\right\}$$

Substituting (76) and (77) into the Natural logarithmic function of vorticity transport equation gives;

$$\Rightarrow \quad \gamma^{(3-\beta)}L\rho\left\{-\frac{1}{\rho F_0}\left[\frac{[n_B - C_B]}{2} + [n_P + C_P] - [n_\tau - C_\tau]\right] - \frac{y\varepsilon_x\varepsilon_y[n_\tau - C_\tau]^2}{2} + \frac{[n_B - C_B]}{2\rho F_0} + \frac{y\varepsilon_x\varepsilon_y[n_\tau - C_\tau]^2}{2}\right\}^2 = \alpha^3 d\theta r_v^2\left\{\frac{[n_\tau - C_\tau]}{\mu F_0}\left\{\varpi_x\left[\frac{x\left[y-\frac{h}{x}\right]}{\left[y-\frac{h}{l}\right]}\right] + \varpi_y\left[\frac{y\left[x-\frac{l}{y}\right]}{\left[x-\frac{l}{h}\right]}\right]\right\} + \upsilon\left\{\frac{\partial^2\left(\frac{\partial u_z}{\partial y} - \frac{\partial u_y}{\partial z}\right)}{\partial^2 x} + \frac{\partial^2\left(\frac{\partial u_z}{\partial x} - \frac{\partial u_x}{\partial z}\right)}{\partial^2 y} + \frac{\partial^2\left(\frac{\partial u_y}{\partial x} - \frac{\partial u_x}{\partial y}\right)}{\partial^2 z}\right\}\right\}^2 \quad (80)$$

https://dx.doi.org/10.4314/swj.v20i4.46

To find the pressure and momentum of a fluid within a reference frame (x,y)

Let (x,y) be the coordinates of maximum range x = R and maximum height or altitude y = h.

From (54)

$$-\nabla P = -\frac{1}{F_0}[n_P + C_P]$$

$$\Rightarrow -\frac{\partial p}{\partial x} = -\frac{1}{F_0}[n_P + C_P]$$

Integrating

$$-\int \frac{\partial p}{\partial x}.\partial x = -\int_0^R \frac{1}{F_0}[n_P + C_P]\ \partial x$$

As $\partial x \to 0$

$$P = \frac{1}{F_0}[n_P + C_P]R \quad (81)$$

$\Rightarrow n_P = \frac{PF_0}{R}$, at $C_P = 0$

From, (66) , (65) and (51)

$$n_\tau = \frac{u}{y\varepsilon_y}, C_\tau = 0$$

$$n_\tau = \frac{u}{x\varepsilon_x}, C_\tau = 0$$

$$n_B = -n_P + n_\tau$$

$$\Rightarrow n_B = -\frac{PF_0}{R} + \frac{u}{x\varepsilon_x}$$

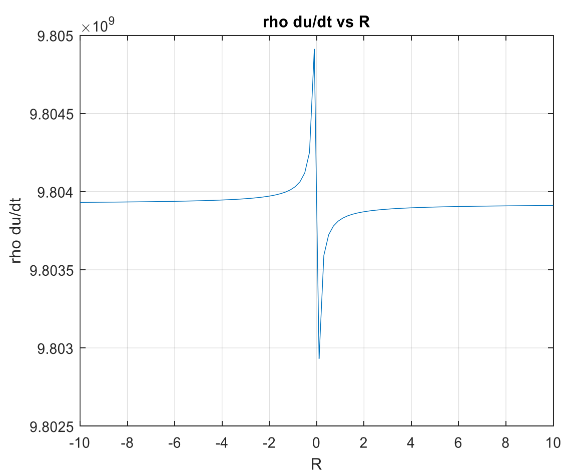
$\Rightarrow n_B = -\frac{PF_0}{R} + \frac{u}{R\varepsilon_x}$, at $x = R$

$$\Rightarrow \quad n_B = \frac{1}{R}\left\{-PF_0 + \frac{u}{\varepsilon_x}\right\} \quad (82)$$

$$\frac{1}{F_0}[n_B] = \rho\frac{\mathcal{D}u}{\mathcal{D}t}, C_B = 0$$

$$\Rightarrow n_B = \rho\frac{\mathcal{D}u}{\mathcal{D}t} = \frac{1}{F_0 R}\left\{-PF_0 + \frac{u}{\varepsilon_x}\right\}$$

$$\therefore \quad \rho\frac{\mathcal{D}u}{\mathcal{D}t} = \frac{1}{F_0 R}\left\{-PF_0 + \frac{u}{\varepsilon_x}\right\} \quad (83)$$



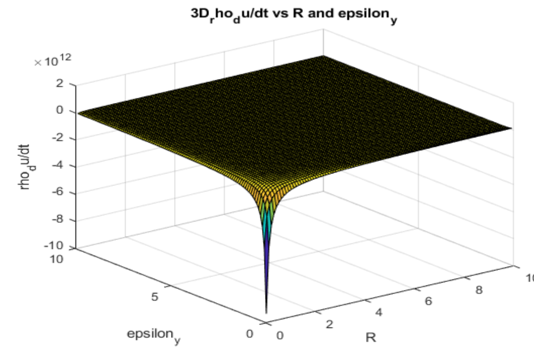


Figure 25 $\rho\frac{\mathcal{D}u}{\mathcal{D}t}/R$, $\rho\frac{\mathcal{D}u}{\mathcal{D}t}/\varepsilon_y$

Similarly,

$$\rho\frac{\mathcal{D}u}{\mathcal{D}t} = \frac{1}{F_0}\left\{-\frac{PF_0}{R} + \frac{u}{y\varepsilon_y}\right\}, \quad \text{in y-direction} \quad (84)$$

Solutions to the parameters in the linear and non-linear terms on Torus $\mathbb{R}^3/\mathbb{Z}^3$.

The shear force divergence as a linear term with its solution given in Lemma 4, due to internal friction (viscosity) in a fluid is integrated on **Torus** $\mathbb{R}^3/\mathbb{Z}^3$, yielding the solution to the divergence of velocity u in 3-dimensions. Substituting the solution of the divergence of u, into the non-linear term yield the solutions of; v *and* w, local acceleration, convective acceleration and pressure gradient. The solutions yield the subsequent graphs and the vorticity transport equation.

Considering the linear term from "Lemma 4":

$$\mu\nabla^2 u = \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) = \frac{1}{F_0}[n_\tau - C_\tau]$$

$$\Rightarrow \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = \frac{1}{\mu F_0}[n_\tau - C_\tau]$$

Integrating,

$$\iiint\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right)\partial x\partial y\partial z = \iiint_{000}^{xyz} \frac{1}{\mu F_0}[n_\tau - C_\tau]\partial x\partial y\partial z$$

As, $\partial x \longrightarrow 0, \partial y \longrightarrow 0$ and $\partial z \longrightarrow 0$

$$u = \frac{x^2y^2z^2[n_\tau - C_\tau]}{\mu F_0[y^2z^2 + x^2z^2 + x^2y^2]} \quad (85)$$

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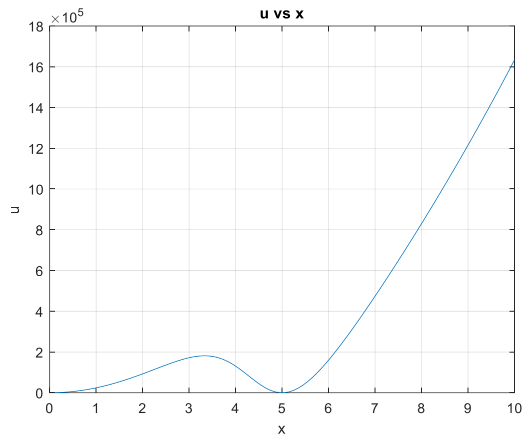


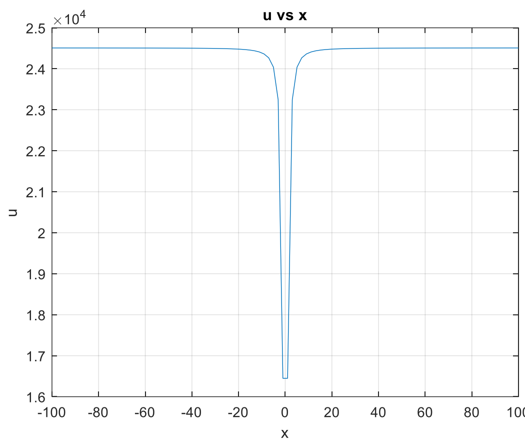
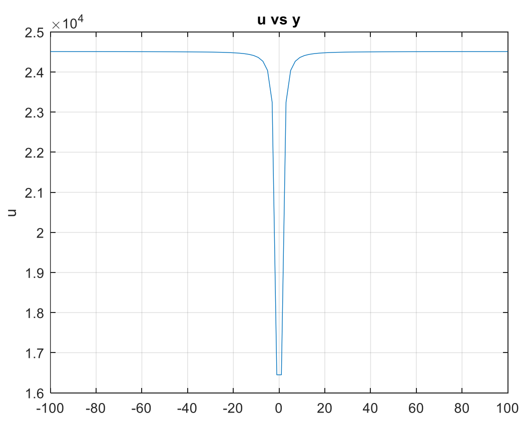


Figure 26: Graph of u/x & u/y



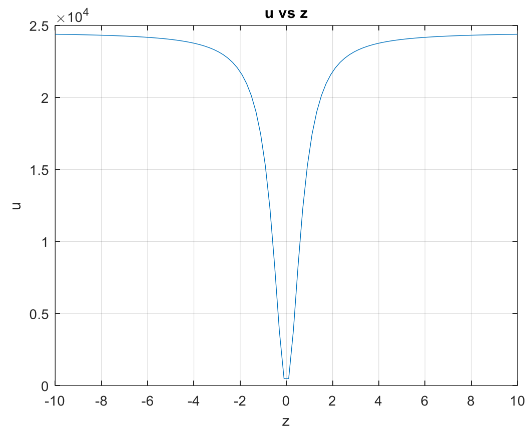


Figure 27 : Graph of u/y & u/z

To find the change in velocity on torus $[x, y, z]^T$.

$$\frac{\partial u}{\partial x} = \frac{2xy^4z^4[n_\tau - C_\tau]}{\mu F_0[y^2z^2 + x^2z^2 + x^2y^2]^2}$$
(86)

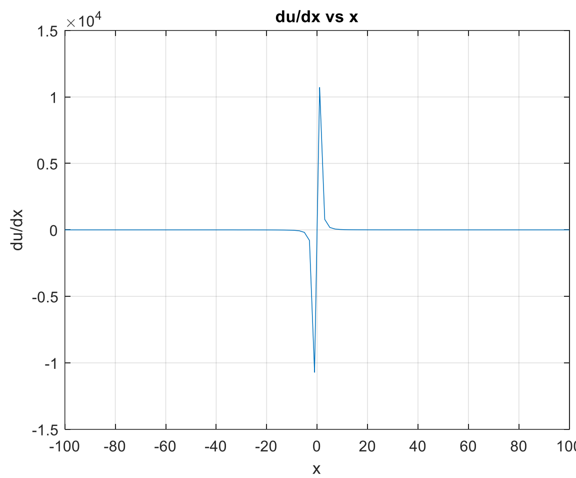


Figure.. 28 Graph of $\frac{\partial u}{\partial x}$ / x

$$\frac{\partial u}{\partial y} = \frac{2yx^4z^4[n_\tau - C_\tau]}{\mu F_0[y^2z^2 + x^2z^2 + x^2y^2]^2} \quad (87)$$

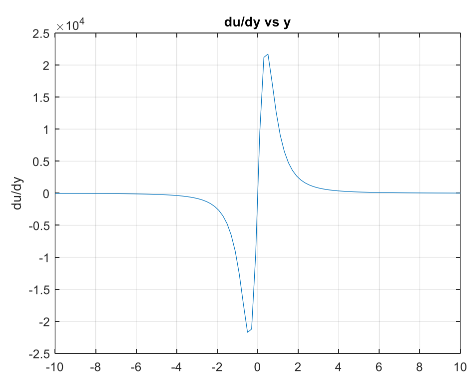


Figure 29 Graph of $\frac{\partial u}{\partial y}$ / y

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$$\frac{\partial u}{\partial z} = \frac{2zx^4y^4[n_\tau - C_\tau]}{\mu F_0[y^2z^2+x^2z^2+x^2y^2]^2} \quad (88)$$

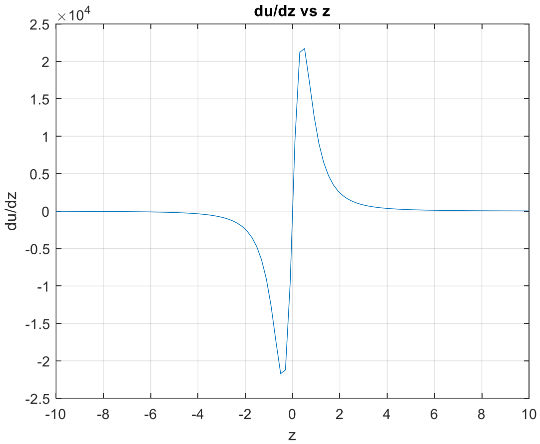

Figure 30 Graph of $\frac{\partial u}{\partial z} / z$

Integrating "u" on torus $[l, h, k]^T$

$$\int \frac{\partial u}{\partial x}\partial x = \int_0^l \frac{2xy^4z^4[n_\tau - C_\tau]}{\mu F_0[y^2z^2+x^2z^2+x^2y^2]^2}\partial x$$

As $\partial x \to 0$

$$\int du = \int \frac{2xy^4z^4[n_\tau - C_\tau]}{\mu F_0[y^2z^2+x^2z^2+x^2y^2]^2}dx$$

$$u = \frac{y^4z^4[n_\tau - C_\tau]}{\mu F_0[y^2+z^2][y^2z^2+x^2z^2+x^2y^2]}\ln[y^2z^2+x^2z^2 + x^2y^2] + c$$

$c = \ln A$

At $x = l, y = h, z = k, u = u_\propto$

$$c = u_\propto - \frac{h^4k^4[n_\tau - C_\tau]}{\mu F_0[h^2+k^2][h^2k^2+l^2k^2+l^2h^2]}\ln[h^2k^2 + l^2k^2 + l^2h^2]$$

$$\ln A = u_\propto - \frac{h^4k^4[n_\tau - C_\tau]}{\mu F_0[h^2+k^2][h^2k^2+l^2k^2+l^2h^2]}\ln[h^2k^2 + l^2k^2 + l^2h^2]$$

$$\Rightarrow A = e^{u_\propto - \frac{h^4k^4[n_\tau - C_\tau]}{\mu F_0[h^2+k^2][h^2k^2+l^2k^2+l^2h^2]}\ln[h^2k^2+l^2k^2+l^2h^2]}$$

(89)

$$u = \frac{y^4z^4[n_\tau - C_\tau]}{\mu F_0[y^2+z^2][y^2z^2+x^2z^2+x^2y^2]}\ln[y^2z^2+x^2z^2+x^2y^2] + u_\propto - \frac{h^4k^4[n_\tau - C_\tau]}{\mu F_0[h^2+k^2][h^2k^2+l^2k^2+l^2h^2]}\ln[h^2k^2+l^2k^2+l^2h^2]$$

(90)

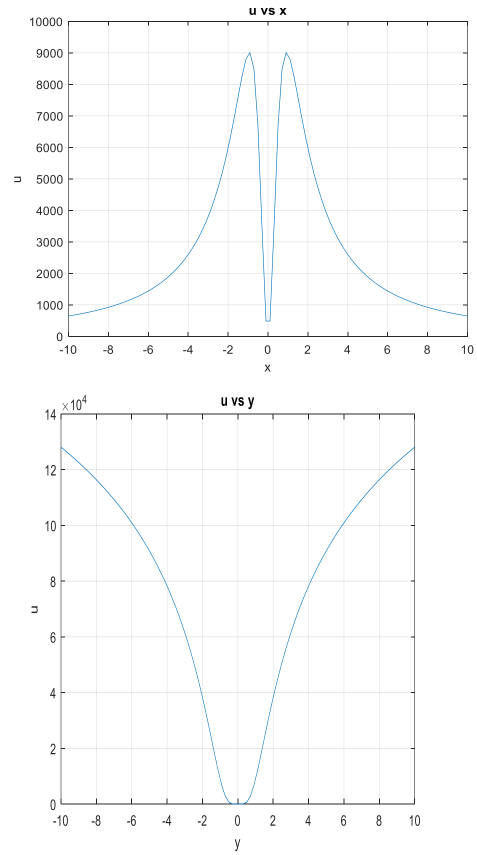

Figure 31 Graph of u / x & u / y

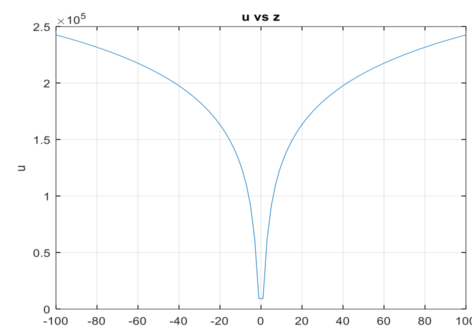

Figure 32 Graph of u / z

$$\frac{u}{u_\propto} = \frac{y^4z^4[n_\tau - C_\tau]}{u_\propto \mu F_0[y^2+z^2][y^2z^2+x^2z^2+x^2y^2]}\ln[y^2z^2+x^2z^2+x^2y^2] - \frac{h^4k^4[n_\tau - C_\tau]}{\mu F_0[h^2+k^2][h^2k^2+l^2k^2+l^2h^2]}\ln[h^2k^2+l^2k^2+l^2h^2] + 1$$

(91)

$$\frac{u}{u_\propto} = \frac{[n_\tau - C_\tau]}{u_\propto \mu F_0}\left[\frac{y^4z^4[n_\tau - C_\tau]}{[y^2+z^2][y^2z^2+x^2z^2+x^2y^2]}\ln[y^2z^2 + x^2z^2 + x^2y^2] - \frac{h^4k^4[n_\tau - C_\tau]}{[h^2+k^2][h^2k^2+l^2k^2+l^2h^2]}\ln[h^2k^2+l^2k^2+l^2h^2]\right]$$

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$$\frac{u}{u_\propto} = \frac{[n_\tau - C_\tau]}{u_\propto \mu F_0}\left[\ln \frac{[y^2z^2+x^2z^2+x^2y^2]^{\frac{y^4z^4[n_\tau - C_\tau]}{[y^2+z^2][y^2z^2+x^2z^2+x^2y^2]}}}{[h^2k^2+l^2k^2+l^2h^2]^{\frac{h^4k^4[n_\tau - C_\tau]}{[h^2+k^2][h^2k^2+l^2k^2+l^2h^2]}}}\right] + 1$$

(92)

To find the velocities in the non-linear term in 3-dimensions.

$\rho\frac{\mathcal{D}u}{\mathcal{D}t} = \frac{1}{F_0}(n_B - C_B)$, from equation (53)

$$\Rightarrow \rho\left[\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right] = \frac{1}{F_0}(n_B - C_B)$$

$$\Rightarrow \left[\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right] = \frac{1}{\rho F_0}(n_B - C_B)$$

Let, $\beta = \frac{[n_\tau - C_\tau]}{\mu F_0[y^2z^2+x^2z^2+x^2y^2]}$, $\varphi = \frac{[n_\tau - C_\tau]}{\mu F_0[y^2z^2+x^2z^2+x^2y^2]^2}$

$$\Rightarrow \frac{\partial u}{\partial x} = \frac{2xy^4z^4[n_\tau - C_\tau]}{\mu F_0[y^2z^2+x^2z^2+x^2y^2]^2} = 2xy^4z^4\varphi$$

$$\Rightarrow \frac{\partial u}{\partial y} = \frac{2yx^4z^4[n_\tau - C_\tau]}{\mu F_0[y^2z^2+x^2z^2+x^2y^2]^2} = 2yx^4z^4\varphi$$

$$\Rightarrow \frac{\partial u}{\partial x} = \frac{2zx^4y^4[n_\tau - C_\tau]}{\mu F_0[y^2z^2+x^2z^2+x^2y^2]^2} = 2zx^4y^4\varphi$$

$$u = \frac{x^2y^2z^2[n_\tau - C_\tau]}{\mu F_0[y^2z^2+x^2z^2+x^2y^2]} = x^2y^2z^2\beta$$

(93)

$$\Rightarrow \frac{\partial u}{\partial t} + u[2xy^4z^4\varphi] + v[2yx^4z^4\varphi] + w[2zx^4y^4\varphi] = \frac{1}{\rho F_0}[n_B - c_B]$$

$$\Rightarrow \frac{\partial u}{\partial t} + 2xyz\varphi[y^3z^3u + x^3z^3v + x^3y^3w] = \frac{1}{\rho F_0}[n_B - c_B]$$

$$\frac{\partial u}{\partial t} = \frac{1}{\rho F_0}[n_B - c_B] - 2xyz\varphi[y^3z^3u + x^3z^3v + x^3y^3w],$$

Local acceleration (94)

$$2xyz\varphi[y^3z^3u + x^3z^3v + x^3y^3w] = \frac{\partial u}{\partial t} - \frac{1}{\rho F_0}[n_B - c_B],$$

Convective acceleration (95)

$$\Rightarrow y^3z^3u + x^3z^3v + x^3y^3w = \frac{1}{2xyz\varphi\rho F_0}[n_B - c_B] - \frac{1}{2xyz\varphi}\frac{\partial u}{\partial t}$$

$$\Rightarrow x^3z^3v + x^3y^3w = \frac{1}{2xyz\varphi\rho F_0}[n_B - c_B] - \frac{1}{2xyz\varphi}\cdot\frac{\partial u}{\partial t} - y^3z^3u$$

$$z^3v + y^3w = \frac{1}{2x^4yz\varphi\rho F_0}[n_B - c_B] - \frac{1}{2x^4yz\varphi}\cdot\frac{\partial u}{\partial t} - \frac{y^3z^3u}{x^3}$$

$$z^3v + y^3w = \frac{1}{2x^4yz\varphi\rho F_0}[n_B - c_B] - \frac{2yx^4z^4}{2x^4yz}\cdot\frac{\partial y}{\partial u}\cdot\frac{\partial u}{\partial t} - \frac{y^3z^3u}{x^3}$$

$$z^3v + y^3w = \frac{1}{2x^4yz\varphi\rho F_0}[n_B - c_B] - z^3v - \frac{y^3z^3u}{x^3}$$

$$2z^3v + y^3w = \frac{1}{2x^4yz\varphi\rho F_0}[n_B - c_B] - \frac{y^3z^3u}{x^3}$$

$$\Rightarrow 2z^3v + y^3w = \frac{1}{2x^4yz\varphi\rho F_0}[n_B - c_B] - \frac{y^5z^5\beta}{x}$$

Solving for v and w simultaneously between any two points; 1 and 2 in a flow field,

Let, $\quad 2{z^3}_1v + {y^3}_1w = \frac{[n_B - c_B]}{2{x^4}_1y_1z_1\varphi\rho F_0} - \frac{{y^5}_1{z^5}_1\beta_1}{x_1}$

(1)

$$2{z^3}_2v + {y^3}_2w = \frac{[n_B - c_B]}{2{x^4}_2y_2z_2\varphi\rho F_0} - \frac{{y^5}_2{z^5}_2\beta_2}{x_2}$$

(2)

$$\Delta k = \begin{vmatrix} 2{z^3}_2 & {y^3}_1 \\ 2{z^3}_2 & {y^3}_2 \end{vmatrix} = 2[{z^3}_1{y^3}_2 - {z^3}_2{y^3}_1]$$

$$\Delta v = \begin{vmatrix} \frac{[n_B - c_B]}{2{x^4}_1y_1z_1\varphi\rho F_0} - \frac{{y^5}_1{z^5}_1\beta_1}{x_1} & {y^3}_1 \\ \frac{[n_B - c_B]}{2{x^4}_2y_2z_2\varphi\rho F_0} - \frac{{y^5}_2{z^5}_2\beta_2}{x_2} & {y^3}_2 \end{vmatrix}$$

$$= {y^3}_2\left[\frac{[n_B - c_B]}{2{x^4}_1y_1z_1\varphi\rho F_0} - \frac{{y^5}_1{z^5}_1\beta_1}{x_1}\right] - {y^3}_1\left[\frac{[n_B - c_B]}{2{x^4}_2y_2z_2\varphi\rho F_0} - \frac{{y^5}_2{z^5}_2\beta_2}{x_2}\right]$$

$$\Delta w = \begin{vmatrix} 2z_1^3 & \frac{[n_B - c_B]}{2{x^4}_1y_1z_1\varphi\rho F_0} - \frac{{y^5}_1{z^5}_1\beta_1}{x_1} \\ 2z_2^3 & \frac{[n_B - c_B]}{2{x^4}_2y_2z_2\varphi\rho F_0} - \frac{{y^5}_2{z^5}_2\beta_2}{x_2} \end{vmatrix}$$

$$= 2{z^3}_1\left[\frac{[n_B - c_B]}{2{x^4}_2y_2z_2\varphi\rho F_0} - \frac{{y^5}_2{z^5}_2\beta_2}{x_2}\right] - 2{z^3}_2\left[\frac{[n_B - c_B]}{2{x^4}_1y_1z_1\varphi\rho F_0} - \frac{{y^5}_1{z^5}_1\beta_1}{x_1}\right] -$$

$$v = \frac{\Delta v}{\Delta k} = \frac{1}{2[{z^3}_1{y^3}_2 - {z^3}_2{y^3}_1]}\left\{ {y^3}_2\left[\frac{[n_B - c_B]}{2{x^4}_1y_1z_1\varphi\rho F_0} - \frac{{y^5}_1{z^5}_1\beta_1}{x_1}\right] - {y^3}_1\left[\frac{[n_B - c_B]}{2{x^4}_2y_2z_2\varphi\rho F_0} - \frac{{y^5}_2{z^5}_2\beta_2}{x_2}\right]\right\} \quad (96)$$

$$w = \frac{\Delta w}{\Delta k} = \frac{1}{{z^3}_1{y^3}_2 - {z^3}_2{y^3}_1}\left\{ {z^3}_1\left[\frac{[n_B - c_B]}{2{x^4}_1y_1z_1\varphi\rho F_0} - \frac{{y^5}_1{z^5}_1\beta_1}{x_1}\right] - {z^3}_2\left[\frac{[n_B - c_B]}{2{x^4}_2y_2z_2\varphi\rho F_0} - \frac{{y^5}_2{z^5}_2\beta_2}{x_2}\right]\right\} \quad (97)$$

Pressure gradient

$$-\nabla p = -\frac{1}{F_0}[n_P + C_P] = \frac{1}{F_0}[n_B + C_B] - \frac{1}{F_0}[n_\tau + C_\tau]$$

$$-\nabla p = \rho\left[\frac{\partial u}{\partial t} + 2xyz\varphi[y^3z^3u + x^3z^3v + x^3y^3w]\right] - \frac{1}{F_0}[n_\tau + C_\tau]$$

$$\Rightarrow -\frac{\partial p}{\partial x} = \rho\left[\frac{\partial u}{\partial t} + 2xyz\varphi[y^3z^3u + x^3z^3v + x^3y^3w]\right] - \frac{1}{F_0}[n_\tau + C_\tau] \quad (98)$$

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Vorticity Transport Equation
From the solutions in (93), (86), (87) and (88)

CURL 2:

$$u = \frac{x^2y^2z^2[n_\tau - C_\tau]}{\mu F_0[y^2z^2 + x^2z^2 + x^2y^2]}$$

$$\frac{\partial u}{\partial x} = \frac{2xy^4z^4[n_\tau - C_\tau]}{\mu F_0[y^2z^2 + x^2z^2 + x^2y^2]^2}$$

$$\frac{\partial u}{\partial y} = \frac{2yx^4z^4[n_\tau - C_\tau]}{\mu F_0[y^2z^2 + x^2z^2 + x^2y^2]^2}$$

$$\frac{\partial u}{\partial z} = \frac{2zx^4y^4[n_\tau - C_\tau]}{\mu F_0[y^2z^2 + x^2z^2 + x^2y^2]^2}$$

$$u_x = \frac{y^4z^4[n_\tau - C_\tau]}{\mu F_0[y^2 + z^2]^2[y^2z^2 + x^2z^2 + x^2y^2]^2}\ln[y^2z^2 + x^2z^2 + x^2y^2] + C_x$$

$$u_y = \frac{x^4z^4[n_\tau - C_\tau]}{\mu F_0[x^2 + z^2]^2[y^2z^2 + x^2z^2 + x^2y^2]^2}\ln[y^2z^2 + x^2z^2 + x^2y^2] + C_y$$

$$u_z = \frac{x^4y^4[n_\tau - C_\tau]}{\mu F_0[x^2 + y^2]^2[y^2z^2 + x^2z^2 + x^2y^2]^2}\ln[y^2z^2 + x^2z^2 + x^2y^2] + C_z$$

$$\varpi = \overline{\nabla} \times \begin{pmatrix} u_x \\ u_y \\ u_z \end{pmatrix}$$

$$\varpi = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u_x & u_y & u_z \end{vmatrix}$$

The components of rotation are:

$$\varpi_x = \left(\frac{\partial u_z}{\partial y} - \frac{\partial u_y}{\partial z}\right) = \frac{x^4[n_\tau - C_\tau]}{\mu F_0[y^2z^2+x^2z^2+x^2y^2]^2}\left\{\frac{y^4[2yx^2+2yz^2]}{[x^2+y^2]} - \frac{z^4[2zx^2+2zy^2]}{[x^2+z^2]} + \frac{y^3\{[y^2z^2+x^2z^2+x^2y^2][4x^2+4y^2-2y^2]-2y^2[x^2+y^2][x^2+z^2]\}}{[x^2+y^2]^2} - \frac{z^3\{[y^2z^2+x^2z^2+x^2y^2][4x^2+4z^2-2z^2]-2z^2[x^2+y^2][x^2+z^2]\}}{[x^2+z^2]^2}\right\}\ln[y^2z^2 + x^2z^2 + x^2y^2] \quad (99)$$

$$\varpi_y = \left(\frac{\partial u_z}{\partial x} - \frac{\partial u_x}{\partial z}\right) = \frac{y^4[n_\tau - C_\tau]}{\mu F_0[y^2z^2+x^2z^2+x^2y^2]^2}\left\{\frac{x^4[2xy^2+2xz^2]}{[x^2+y^2]} - \frac{z^4[2zx^2+2zy^2]}{[y^2+z^2]} + \frac{x^3\{[y^2z^2+x^2z^2+x^2y^2][4x^2+4y^2-2x^2]-2x^2[x^2+y^2][y^2+z^2]\}}{[x^2+y^2]^2} - \frac{z^3\{[y^2z^2+x^2z^2+x^2y^2][4y^2+4z^2-2z^2]-2z^2[x^2+y^2][y^2+z^2]\}}{[y^2+z^2]^2}\right\}\ln[y^2z^2 + x^2z^2 + x^2y^2] \quad (100)$$

$$\varpi_z = \left(\frac{\partial u_y}{\partial x} - \frac{\partial u_x}{\partial y}\right) = \frac{z^4[n_\tau - C_\tau]}{\mu F_0[y^2z^2+x^2z^2+x^2y^2]^2}\left\{\frac{x^4[2xy^2+2xz^2]}{[x^2+z^2]} - \frac{y^4[2yx^2+2yz^2]}{[y^2+z^2]} + \frac{x^3\{[y^2z^2+x^2z^2+x^2y^2][4x^2+4z^2-2x^2]-2x^2[x^2+z^2][y^2+z^2]\}}{[x^2+z^2]^2} - \frac{y^3\{[y^2z^2+x^2z^2+x^2y^2][4y^2+4z^2-2y^2]-2y^2[y^2+z^2][x^2+z^2]\}}{[y^2+z^2]^2}\right\}\ln[y^2z^2 + x^2z^2 + x^2y^2] \quad (101)$$

Recall, (38)

$$\frac{1}{r}\left\{\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right\} = (\varpi.\overline{\nabla})u + v\nabla^2\varpi$$

Substituting equations (99), (100) and (101) (solutions to the partial derivatives) into the vorticity transport equation;

$$\frac{1}{r}\left\{\frac{1}{\rho F_0}[n_B - c_B] - 4xyz\varphi[y^3z^3u + x^3z^3v + x^3y^3w]\right\} = \frac{2xy[n_\tau - C_\tau]}{\mu F_0[y^2z^2+x^2z^2+x^2y^2]^2}\{y^3z^3\varpi_x + x^3z^3\varpi_y + x^3y^3\varpi_z\} + v\left\{\frac{\partial^2\left(\frac{\partial u_z}{\partial y} - \frac{\partial u_y}{\partial z}\right)}{\partial^2 x} + \frac{\partial^2\left(\frac{\partial u_z}{\partial x} - \frac{\partial u_x}{\partial z}\right)}{\partial^2 y} + \frac{\partial^2\left(\frac{\partial u_y}{\partial x} - \frac{\partial u_x}{\partial y}\right)}{\partial^2 z}\right\}$$

(102)

Recall (42)

$$(3-\beta)\ln\gamma = \ln\alpha^3 - \ln\left\{\frac{\left\{\frac{\rho}{\{(\varpi.\overline{\nabla})u + u\nabla^2\varpi\}}\left\{\frac{\partial u}{\partial t} + \bar{u}\nabla u\right\}\right\}^2 L}{d\theta r_v^2}\right\}$$

Substituting (94) into (95) into the Natural logarithmic function of vorticity transport equation gives;

$$\Rightarrow \quad \gamma^{(3-\beta)}L\rho\left\{\frac{1}{\rho F_0}[n_B - c_B] - 4xyz\varphi[y^3z^3u + x^3z^3v + x^3y^3w]\right\}^2 = \alpha^3 d\theta r_v^2\left\{\frac{2xy[n_\tau - C_\tau]}{\mu F_0[y^2z^2+x^2z^2+x^2y^2]^2}\{y^3z^3\varpi_x + x^3z^3\varpi_y + x^3y^3\varpi_z\} + v\left\{\frac{\partial^2\left(\frac{\partial u_z}{\partial y} - \frac{\partial u_y}{\partial z}\right)}{\partial^2 x} + \frac{\partial^2\left(\frac{\partial u_z}{\partial x} - \frac{\partial u_x}{\partial z}\right)}{\partial^2 y} + \frac{\partial^2\left(\frac{\partial u_y}{\partial x} - \frac{\partial u_x}{\partial y}\right)}{\partial^2 z}\right\}\right\}^2$$

(103)

CONCLUDING REMARKS

The derived solutions provide exact expressions for velocity, acceleration, pressure gradients, and vorticity, particularly on a toriodal domain. The logarithmic model of vortex formation highlights the role of energy, geometry, and spin in turbulent flows. By bridging fluid dynamics with quantum mechanics and relativity, this work contributes a novel analytical path to one of the Clay Millennium problems.

The linear term (Ω) of the Navier-Stokes equation expressed as a function of its quantum number, provides a solution for the PDEs. By differentiating the function, $\lim_{\Delta n_\Omega \to 0}\frac{d\Omega}{dn_\Omega} = \frac{f(n_\Omega + dn_\Omega) - f(n_\Omega)}{dn_\Omega} = F_m$, was obtained from Lemma 1. Furthermore, a solution of the form, $\nabla^2 u = \frac{1}{\mu F_0}[n_\tau - C_\tau]$, was obtained for the Shearing force. The momentum force per volume and the pressure gradient in the same manner, this led to a new simplified form of the Navier-Stokes equation. By integration from both sides of the equation $\forall\, n_\tau \in \mathbb{Z}$ and $\mathbb{R}_+^d = \{(x_1, x_2, \ldots, x_d) \in \mathbb{R}^d : d > 0\}$, yields tractable solution for $u \cong \frac{n_\tau F_m}{\mu}u(x_1^{d-1}, x_2^{d-1}, \ldots, x_d^{d-1})$, it rate of change in $[\mathbb{R}^d/\mathbb{Z}^d : d > 0,\ \ t > 0]$ and the pressure gradient, $\frac{\partial p}{\partial x_i}, \mathbb{R}^d : x[0, \infty]$. Substituting the aforementioned solutions into the substantial derivative of the momentum force also provides tractable solutions for the Local and convective accelerations on torus. Plotting $u(x, y, z, t)$, against space and time from the aforementioned equations yields; parabolic curves, asymptotic curves, inverted curves, linear graphs and also the topology of the flows as shown in figures; 17, 23, 24, 25 and 26. The general quantum number (n) of the Navier-Stokes equation determines, the states of equilibrium, smoothness and turbulence (chaos) in a fluid continuum. The existential of smooth solutions is a function of positive values of the general quantum number "n" of the Navier-Stokes equation at any given time. Turbulence is a function of the negative values of "n" at any given time. When $n = 0$ the fluid is said to be at equilibrium (in between Smoothness and Outburst), at $n < 0$ the fluid is in a state of chaos (outburst) whose magnitude increases as n is decreasing (going down the slope of the graph in fig. 6), at this condition the formation of vorticity follows a natural logarithmic function showing mechanical/thermal energy, vibrational energy and vorticity term and its diffusion $((3-\beta)\ln\gamma = \ln\left[\phi^3\sqrt{\frac{mv^2\sigma}{[T-fT^2(w+v)]}}\right] - \ln\left\{\frac{\left\{\frac{\rho}{\{(\varpi.\overline{\nabla})u+u\nabla^2\varpi\}}\left\{\frac{\partial u}{\partial t}+\overline{u}\nabla u\right\}\right\}^2 L}{d\theta r_v^2}\right\})$, and also when $n > 0$ the fluid is in a state of smoothness whose magnitude increases with increase in n (going up the slope of the graph in fig. 5). At $n = 0$ the fluid is said to be in equilibrium.

This approach not only deepens theoretical understanding but also holds potential applications in weather prediction, aviation safety, quantum fluid modeling, and advanced computational fluid dynamics. Future work will focus on computing specific minimum forces for different fluid types and validating these models with empirical data.

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