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OPTIMIZING HIRING STRATEGIES: A DYNAMIC PROGRAMMING APPROACH TO CUTTING RECRUITMENT AND OVERSTAFFING COSTS

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ABSTRACT
The aim of this article is to develop a manpower planning model which incorporates fixed recruitment and overstaffing costs at each period of recruitment in order to evaluate total minimum manpower cost at the last period of a planning horizon. The proposed model in this research uses the backward approach which makes it to have lower suboptimal costs in between the stages compared to the existing models which use forward recursive method of dynamic programming. Although the minimum total of manpower cost is the same for both proposed and the existing model algorithms, the suboptimal costs in the proposed model are lower than the corresponding suboptimal manpower costs in the existing model. This is one of the advantages the proposed model has compared to other models in literature, making it possible for policy makers to detect periods where manpower in terms of numbers of staff and the skills needed early enough. Another advantage of the model algorithm is batch recruitment at certain periods of computation. This assists policy makers to carry out recruitment in batches instead of period by period. This will reduce recruitment cost and maximize organization's efficiency and profit.

Keywords: manpower, company, recruitment, overstaffing

INTRODUCTION

Manpower has been defined in Urhoma (2009) and Seckiner et al. (2007) as people involved in the process of manufacturing goods or rendering services in organizations. According to Bontis et al. (1999) and Armstrong (2004), manpower also known as workforce are staffs in a business enterprise which comprises of skills, intelligence, and expertise which determine the market worth of an organization. Assessment of present and future manpower demand in terms of competence, skills and number of staffs to meet organizations' goals over the years has been a challenge to human resource managers (Yu et al. 2004). Manpower planning process involves evaluating both present and future manpower demand in an industry in terms of number of staff, the level of their skills, and competence in order to satisfy the manpower requirements, Bulla and Scot (1994) and Stolletz (2010). Recruitment, promotion and wastage are reported in Ogumeyo and Idisi (2024) as the three factors responsible for staff migration from one grade to another.

Dynamic programming is a multistage decision process in which the decision variables vary from one period to another; hence, these situations are to be "dynamic in nature" Gupta and Hira, 2005; Ogumeyo, 2014). Problems that require the use of a dynamic programming approach usually have a series of interrelated decisions. A mathematical model that studies the cost of strategic manpower planning is reported in Iyer and Felix (2019) and Bakir and Atalik (2021). Their models include training cost and a framework for staff recruitment designed to satisfy the organization's goals. Willcox et al. (2019) developed a financial cost model for training staff through workshops. Meanwhile, a manpower planning model, which considers capacity, lot size, and workforce to determine an ideal ratio for a multi-product and multi-production system, was developed in Sivasurundari et al. (2019). Ezegwu and Ologun (2017) developed a Markov chain model that predicts annual academic staff requirements in a higher institution. Their model aims to evaluate the number of academic staff to be recruited, promoted, and retired annually in a higher institution. Nirmala and Sridevi (2017) developed a stochastic manpower planning model to determine the mean and variance of manpower requirement in an organization.

Manpower planning models to determine the optimum workforce in the aviation industry are studied in Akyurt et al. (2021). The model involves the use of integer programming to determine the number of pilots to be recruited, the type of skills they are required to possess, including their promotion and withdrawal from the industry. Aircraft maintenance personnel model for capacity building is developed in Dijkstra et al. (1991). Petrovc and Kankaras (2020) developed a model to evaluate air traffic control radar personnel in the aviation industry.

The importance of having the right number of personnel in terms of skills and competence in order to achieve the organization's goal cannot be ignored (Ogumeyo, 2014; and Lolli et al., 2019). Yet researchers have not done enough in considering fixed recruitment and over-staffing costs, which are crucial factors in manpower planning. Mathematical models that incorporate costs in their formulation are very crucial in manpower planning. Mathematical models to determine optimal recruitment policies and total minimum cost in manpower planning are reported in Rao (1990), Nirmala and Jeeva (2010), Iyer and Felix (2019), and Willcox et al. (2019). These models assumed that the number of employees required in the present and future periods can be estimated, including the unit overstaffing costs. (a) Rao (1990) uses a forward recursive technique of dynamic programming to determine total minimum cost of the manpower planning problem, which excludes the computation of some sub-decision costs that could be the suboptimal cost of the objective function. (b) The dynamic programming models presented in Rao (1990) and Nirmala and Jeeva (2010), which are in linear programming forms, cannot be solved by manual computation nor a computer program. This is because some of the variables in the objective function do not exist in their linear constraints. In this research work, we transform the models in Rao (1990) and Nirmala and Jeeva (2010) into new

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problems in order to get their dual optimal solution through a backward recursive technique of dynamic programming.

MATERIALS AND METHODS

Assumptions of the Model

Assumptions considered while formulating the proposed model are:

(a) There is a fixed number of staff to be recruited in each period.
(b) Staff in a particular cadre are considered for the recruitment.
(c) The recruitment and overstaffing costs are fixed.
(d) Staff shortage is disallowed

Mathematical Notations

U_j = manpower requirement in period j
k_j = recruitment fixed cost in period j
V_j = overstaffing cost per staff in period j
q_j = number of staff recruited in period j.
x_j = number of staff recruited in an earlier period for the requirement of period j
y_j = Cost of recruitment per recruited employee in period j.
n = Number of periods
r_j = number of staff promoted in period j.
v_j = cost of promotion per staff in period j.
x_{jk} = number of staff employed in an earlier period j for grade k
U_j = number of staff required in period j.
W_j = number of staff qualified for promotion in period j

Model Description

Let n be the number of stages or periods in which recruitment is planned for in a company. Let the recruitment cost and overstaffing costs vary from period to period. The proposed model aims to minimize the total manpower cost in the organization in n-periods when $U_j(t+\delta)$ number of staff is recruited at time $(t+\delta)$ in j period with an overstaffing cost $V_j(t+\delta)$ per staff in time $(t+\delta)$ in j period. The delta δ depicts a small time interval between when a staff member was recruited and the actual time he resumes his official assignment. This period is usually referred to as probation or an induction period. Let $k(t+\delta)$ be the recruitment fixed cost per staff at time $(t+\delta)$ of period j. As delta tends to zero ($\delta \to 0$) $k(t+\delta)$ tends to $K(t)$ $U_j(t+\delta)$ tends to U_j and $V_j(t+\delta)$ tends to $V_j(t)$. Since the model is dynamic in nature, the time duration being considered is partitioned into time intervals which are to be short so that $U_j(t)$, $V_j(t)$ and $k_j(t)$ are assumed to be constants during the time intervals being considered but discontinuous in between the intervals.

Methodology and Material

The method adopted in this research is a dynamic programming technique that uses a backward recursive approach to determine the optimal recruitment cost of a manpower planning problem. The backward recursive method requires solving a given problem by starting from the last period to the first period. The proposed model is an extension of the manpower planning model in Rao (1990) and Ogumeyo (2014), in which the function *F(t)* depicts the minimum cost program of a manpower planning model with a t-period planning horizon. At each period *t*, the sub-cost of the first sub-decision is written as $k_t + F(t-1)$, where $F(t-1)$ denotes the preceding manpower suboptimal cost. The other sub-costs at period *t* are obtained by using equation (1) as follows:

$$c_t = k_t + \sum_{q=j}^{t-1} \sum_{p=q+1}^{t} V_q U_p + F(t-1), t = 1,2,3,\ldots,z-1$$

(1)

Hence, the suboptimal manpower cost at the last stage or period *t* is stated as

$$F(t) = \min_{1\le z\le n}\left[\min_{1\le t<z}\left[k_t + \sum_{q=j}^{t-1}\sum_{p=q+1}^{t} V_q U_p + F(t-1)\right] + k_t + F(t-1)\right]$$

(2)

Table 1 applies to the proposed model described in this research.

Table 1: Fixed recruitment and overstaffing costs

Periods	No. of staff (U_j)	Fixed shipment cost k_j (₦)	Overstaffing cost V_j (₦)
1 2 3 ⋮ n	U_1 U_2 U_3 ⋮ U_n	k_1 k_2 k_3 ⋮ k_n	V_1 V_2 V_3 ⋮ Vn

Given the values of $U_j(t)$, $V_j(t)$ and $k_j(t)$ of a manpower planning problem, the objective of this research is to formulate a manpower planning model which determines the optimal quantities U_j, V_j and k_j that minimize the total manpower cost over a given period of time. That is

$$Minimize \;\; H = \sum_{j=1}^{n}\left[k_j \delta q_j + y_j q_j + v_j x_j\right] \quad \ldots\ldots(1)$$

s.t.

$$\sum_{j=1}^{i} x_j = \sum_{j=1}^{i} U_j, \quad i = 1, 2, \cdots, n. \quad \ldots\ldots(2)$$

$$x_i \ge 0, \;\; j = 1(1)n \quad \ldots\ldots(3)$$

The objective function in Equation (1) is the total recruitment cost. Equation (2) is the linear constraints, and Equation (3) is the non-negativity constraints.

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It can be observed from Equation (1) that

$$\delta q_j = \begin{cases} 1 & \text{if } q_j > 0 \\ 0 & \text{if } q_j = 0 \end{cases} \quad \text{......(4)}$$

$and\ v_j x_j$ is the overstaffing cost if we take $x_0 = x_n$.

This is similar to Nirmala and Jeeva's (2010) dynamic programming model in linear programming form, stated in system (5) as follows:

$$\left.\begin{aligned} &Minimize\ H = \sum_{j=1}^{n}\left[k_j \delta q_j + y_j q_j + l_j r_j + v_j x_j\right], \\ &subject\ to\ \sum_{j-1}^{i} x_{jk} = \sum_{j-1}^{i} U_{jk},\ i = 1, 2, \cdots, n;\ k = 1, 2 \\ &\sum_{j=1}^{i} r_j = \sum_{j=1}^{i} W_j,\ i = 1, 2, \cdots, n \end{aligned}\right\} \quad (5)$$

The linear constraints in equations (1) to (3) and system (5) possess the attribute of a dynamic programming system, hence they fall into the category of dynamic programming models. Although the variable $(q_j, j = 1,2,3....n)$ in the objective functions in systems (3) and (5) do not exist in their constraints hence the simplex method cannot be used to solve them. If we are considering employment in a particular cadre, the formulation of the proposed model starts from equation (1). Hence, the objective function can be stated as

$$Minimize\ \ H = \sum_{j=1}^{n}\left[k\delta q_j + y_j q_j + v_j x_j\right] \quad (6)$$

We have that $\sum_{j=1}^{n} y_j q_j$ is a constant in equation (6), and the variable cost of recruitment is constant according to the model assumptions. But $\sum_{j=1}^{n} y_j q_j$ is a constant in the sense that the point at which it is applied depends on the earlier period at which employment took place, and not necessarily for all *j*. Moreover, $q_j x_j$ = 0 for all *j*. That is the reason why the objective function in equation (6) becomes:

$$Minimize\ \ H = \sum_{j=1}^{n}\left[k\delta q_j + v_j x_j\right]$$

i.e. $$Minimize\ H = K + \sum_{j=1}^{n} v_j x_j \quad (7)$$

where $$K = \sum_{j=1}^{n} k\delta q_j \quad (8)$$

K is a known fixed cost for all periods in equation (7).
Hence, equation (7) can be expressed as:

$$Minimize\ \ H = \sum_{j=1}^{n} v_j x_j \quad (9)$$

Subject to the constraints:

$$\sum_{j=1}^{i} x_j \geq \sum_{j=1}^{i} U_j,\quad i = 1, 2, \cdots, n. \quad (10)$$

From Equation (10), the use of the inequality '≥' depends on whether overstaffing is allowed. Hence, the dynamic programming model in linear programming to determine the periodic recruitments (x_j) when the number of staff to be recruited $U_j\ (j = 1,2.....n)$ is known can be expressed as follows:

$$\text{Minimize}\ \ H = \sum_{j=1}^{n} U_j x_j \quad (11)$$

Subject to

$$\sum_{j=1}^{i} x_j \geq \sum_{j=1}^{i} U_j\ \ ,\ i = 1, 2, \cdots, n \quad (12)$$

$$x_j \geq 0\ \ ,\ \ j = 1, 2, \cdots, n \quad (13)$$

The total recruitment cost is the objective function stated in Equation (11). Equation (12) is the set of linear constraints, while equation (13) is the set of non-negativity constraints. The proposed dynamic programming model stated in equations (11)-(13) is now solvable since the variables in the constraints and objective function are the same. The model denoted in equations (11)–(13) applies to Table 1 and can further be expressed to give system (14), which is the primal dynamic model:

The Primal Dynamic Model

$$\left.\begin{array}{ll} Min\ H = v_1x_1 + v_2x_2 + v_3x_3 + \cdots + v_nx_n & \\ s.t. & \\ x_1 & \geq U_1 \\ x_1 + x_2 & \geq U_1 + U_2 \\ x_1 + x_2 + x_3 & \geq U_1 + U_2 + U_3 \\ x_1 + x_2 + x_3 + x_4 & \geq U_1 + U_2 + U_3 + U_4 \\ \cdots\cdots\cdots & \\ x_1 + x_2 + x_3 + \cdots + x_n & \geq U_1 + U_2 + U_3 + \cdots + U_n \\ x_1, x_2, \cdots, x_n \geq 0 & \end{array}\right\} \quad (14)$$

In order to get a dynamic programming model solution, we form the dual of the dynamic programming model in system (14). System (15) is the dual corresponding to the DP model stated in (14).

Dual Dynamic Programming Model

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$$\left.\begin{array}{l} Max\ \ G = U_1\sum_{i=1}^{n} e_i + U_2\sum_{i=2}^{n} e_i + U_3\sum_{i=3}^{n} e_i + \cdots + U_n\sum_{i=n}^{n} e_i \\ s.t. \\ \quad e_1 + e_2 + e_3 + \cdots + e_n \le v_1 \\ \qquad e_2 + e_3 + \cdots + e_n \le v_2 \\ \qquad\quad e_3 + \cdots + e_n \le v_3 \\ \qquad\qquad \cdots\cdots\cdots\cdots \\ \qquad\qquad\qquad e_n \le v_n \\ \quad e_i \ge 0, \quad i = 1(1)n \end{array}\right\} \tag{15}$$

The dual variables are the e_i's.

The system (15) can be rewritten as stated in system (16). That is:

$$\left.\begin{array}{l} Max\ \ G = U_1\sum_{i=1}^{n} e_i + U_2\sum_{i=2}^{n} e_i + U_3\sum_{i=3}^{n} e_i + \cdots + U_n\sum_{i=n}^{n} e_i \\ s.t. \\ \quad \sum_{i=k}^{n} e_i \le v_k\,, \quad k = 1,2\ldots\ldots n \\ \quad e_i \ge 0\,, \quad i = 1,2\ldots\ldots\ldots n \end{array}\right\} \tag{16}$$

The matrix linear constraint coefficients for the primal and dual dynamic programming models in (14) and (15) are denoted in Fig. 1 and Fig. 2, respectively.

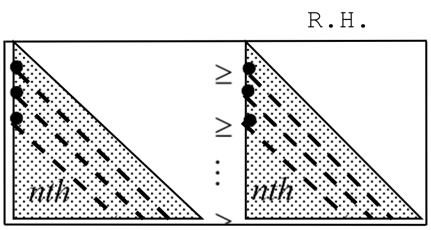

Figure 1: Primal matrix.

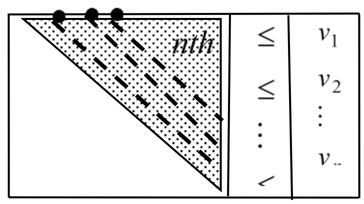

Figure 2: Dual matrix.

A primal sub-problem of (14) can be obtained by deleting the first constraint, while the sub-problem corresponding dual can be determined if we delete the first column in system (15). By following this procedure, we obtain n sub-problems for a given number of periods in the manpower planning problem as depicted by the partitions in Figs 1 and 2. We first determine the dual suboptimal solution of the last nth period and continue till we get to the first suboptimal solution, which is the dual DP problem of the original primal DP problem.

To solve any of the dual sub-problems stated in (15), we need to start from the nth sub-problem and redefine the variables of the dual as follows:

Let $E_k = \sum_{i=k}^{n} e_i\,,\ k = 1,2\ldots.n.$ (17)

To ensure that E_k are non-negative, we impose additional constraints in equation (18), since non-negativity of E_k does not imply that $e_i \ge 0,\ \forall\ i \quad i = 1\ldots..n$. That is

$$E_k \ge E_{k+1}\,,\ k = 1,2\ldots\ldots\ldots.n-1 \tag{18}$$

Note $E_n \ge E_{n+1}$ is the same as $E_n \ge 0$ because $E_{n+1} = 0$ as period $(n+1)$ does not exist.

The dual dynamic programming problem in (16) now becomes:

$$\left.\begin{array}{l} Max\ \ G = \sum_{k=1}^{n} U_k E_k \\ s.t. \\ \quad E_k \le v_k\,, \quad k = 1\ldots\ldots n \\ \quad E_k \ge E_{k+1}\,,\ \ k = 1,2\ldots.n-1 \\ \quad E_k \ge 0, \quad k = 1,2\ldots..n \end{array}\right\} \tag{19}$$

The backward recursive approach can be used to solve the dual DP problem starting from the nth period. That is

$$\begin{array}{l} Max\ \ G = U_n E_n \\ s.t. \\ \left.\begin{array}{l} E_n \le v_n \\ E_n \ge 0 \end{array}\right\} \end{array}$$

The interval $0 \le E_n \le v_n$ is the interval solution.

Since $w,\ and\ R_k$ are known in Table 1, and the dual variables are to be maximized, $E_n = v_n$ or $E_n = \max(v_n\ ,\ 0) = v_n$.

The $(n-1)^{th}$ dual sub-problem is:

$$\left.\begin{array}{l} Max\ \ G = U_{n-1}E_{n-1} \\ s.t. \\ \quad E_{n-1} \le v_{n-1} \\ \quad E_{n-1} \ge E_n = v_n \\ \quad E_{n-1} \ge 0 \end{array}\right\} \tag{20}$$

We can obtain an interval solution from the constraints in system

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(20) if $E_n = v_n \le v_{n-1}$ and in the interval $v_n \le E_{n-1} \le v_{n-1}$. That is $E_{n-1} = v_{n-1}$. If

$$v_k \le v_{k-1} \ , \ (k = 2........n) \tag{21}$$

$$\text{then } E_k = v_k \ (k = 1,2..........n) \tag{22}$$

If we substitute for v_k in the dual objective function, we have:

$$G = v_1u_1 + v_2u_2 + v_3U_3 + \cdots + v_nU_n$$
$$= v_1x_1 + v_2x_2 + v_3x_3 + \cdots + v_nx_n \tag{23}$$

The dual objective function is equal to the primal objective function by the Duality Theorem. If the condition stated in (21) is satisfied, then the solution is $x_j = U_j$ and U_j are given in Table 1. If the condition in equation (21) is not satisfied, a computer program may be required to obtain an optimal solution, especially for large-sized problems.

Numerical Example

The data in Table 2 depicts the number of staff, the fixed recruitment, and the overstaffing costs of an organization for a ten-year planning horizon. Determine the periodic recruitment schedule throughout the period that will give the minimum total manpower cost using the model developed in Section 3.

Table 2: Fixed recruitment and overstaffing costs

Year N	No. of Staff required R	Fixed Recruitment Cost k , (₦)	Overstaffing cost v (₦)
1	74.0	71800	130
2	35.0	70700	110
3	47.0	68800	140
4	62.0	71600	150
5	20.0	69800	140
6	90.0	74100	160
7	51.0	68500	130
8	30.0	70600	100
9	43.0	67900	110
10	35.0	71400	150

The DP model is as earlier stated in Equations (11) to (13). That is

$$\text{Minimize } H = \sum_{j=1}^{n} U_j x_j \tag{11}$$

Subject to

$$\sum_{j=1}^{i} x_j \ge \sum_{j=1}^{i} U_j \ , \ i = 1, 2, \cdots, n \tag{12}$$

$$x_j \ge 0 \ , \ j = 1, 2, \cdots, n \tag{13}$$

The given manpower problem is formulated as a dynamic programming problem, thus:

$$\textit{Minimize } z = 130x_1 + 110x_2 + 140x_3 + 150x_4 + 140x_5 + 160x_6 + 130x_7 + 100x_8 + 110x_9 + 150x_{10}$$

s.t.

$$x_1 \ge 740$$
$$x_1 + x_2 \ge 1090$$
$$x_1 + x_2 + x_3 \ge 1560$$
$$x_1 + x_2 + x_3 + x_4 \ge 2180$$
$$x_1 + x_2 + x_3 + x_4 + x_5 \ge 2380$$
$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 \ge 3280$$
$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 \ge 3790$$
$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 \ge 4090$$
$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 \ge 4520$$
$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} \ge 4870$$
$$x_j \ge 0, \quad j = 1,2........10$$

Since the unit overstaffing costs v_k do not satisfy the condition earlier stated in Equation (21). That is

$$v_k \le v_{k-1}, \quad k = 2.,3,...........n$$

(21) The given problem cannot be solved by the backward recursive approach of the DP technique. Consequently, we use the Program Full Simplex in Ogumeyo (2014).

RESULTS AND DISCUSSION

When Full Simplex is applied to the numerical example in the last section, we obtained the optimal solution after seventeen iterations. The optimal solution is found in Tableau 17, shown in Appendix A of this article. The optimal solution is N53970 in terms of the original variables. Three decision variables that give the optimal recruitment policies are $x_1 = 74.0$, $x_2 = 305.0$, and $x_8 = 108.0$, and they form the objective function value. The total number of staff to be recruited in periods 1, 2, and 8 is $x_1 + x_2 + x_8 =$ 740 + 1080 + 3050. This implies that recruitment should be carried out in periods 1, 2, and 8. In period 1, 740 staff should be recruited, while in periods 2 and 3, we recruit 1080 and 3050 staff, respectively. This will give the total minimum manpower cost of ₦53,970 million for the ten-year plan. If the optimal values x_1, x_2 *and* x_8 are substituted into the objective function, we obtain $v_1x_1 + v_2x_2 + v_8x_8$ = ₦53,970 million, which is equivalent to the value of the objective function in the optimal tableau.

Conclusion

A manpower planning model, which incorporates fixed recruitment and overstaffing costs at each period of recruitment in order to evaluate the total minimum manpower cost at the last period of a planning horizon, has been developed. The proposed model uses the backward recursive approach of dynamic programming to obtain suboptimal costs that are lower between the stages compared to the existing models, which use the forward recursive method. Although the minimum total of manpower cost is the same for both the proposed and the existing model algorithms, the suboptimal costs in the proposed model are lower than the

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corresponding suboptimal manpower costs in the existing model. This is one of the advantages the proposed model has compared to other models in the literature, making it possible for policy makers to detect periods where manpower in terms of numbers of staff and the skills are needed early enough. Another advantage of the model algorithm is batch recruitment at certain periods of computation. For example, the given ten-year planning horizon problem requires recruitment to be carried out in only three periods. That is in periods 1, 2, and 8. This makes it possible for policy makers to carry out recruitment in batches instead of period by period. This will reduce recruitment costs and maximize the organization's efficiency and profit.

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APPENDIX A

ITERATION 17

BASE VAR.	VALUE	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	X11	X12	X13	X14	X15	X16	X17	X18	X19	X20
X1	740.0	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
X2	3050.0	0.00	1.00	1.00	1.00	1.00	1.00	1.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	-1.00	0.00	0.00	0.00
X12	2700.0	0.00	0.00	1.00	1.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	-1.00	0.00	0.00	0.00
X13	2230.0	0.00	0.00	0.00	1.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	-1.00	0.00	0.00	0.00
X14	1610.0	0.00	0.00	0.00	0.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	-1.00	0.00	0.00	0.00
X15	1410.0	0.00	0.00	0.00	0.00	0.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	-1.00	0.00	0.00	0.00
X16	510.0	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	-1.00	0.00	0.00	0.00
X8	1080.0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	-1.00
X18	780.0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	-1.00
X19	350.0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	-1.00
z	-53970.0	0.00	0.00	3.00	4.00	3.00	5.00	2.00	0.00	1.00	5.00	2.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	10.00