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FLEXIBLE BAYESIAN PROPORTIONAL ODD SURVIVAL MODEL WITH SPATIAL RANDOM EFFECT AND COVARIATES INTERACTION STRUCTURES: AN APPLICATION TO UNDER-FIVE MORTALITY RATE

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ABSTRACT
Despite concerted global efforts to reduce child mortality, 5.3 million under-five deaths were reported in 2018, with Nigeria struggling to meet the Sustainable Development Goal (SDG) 3 target of 25 deaths per 1,000 live births by 2030. This study introduced a flexible proportional odds (PO) model, incorporating spatial random effects and covariate interaction Structures, to elucidate the complex relationships between factors influencing Under-Five Mortality (U5M) in Nigeria. Leveraging 2018 Demographic and Health Survey (NDHS) Data, the study evaluated the extended model against existing models, including those without covariate interaction, those without spatial random effects, those with Independent and Identically Distributed (IID) priors, and those with Intrinsic Conditional Autoregressive (ICAR) spatial priors. The results showed that the extended model: PO survival model with ICAR spatial prior and logistic transformed covariates interaction structure (LTCIS) outperformed existing models based on Log Pseudo Marginal Likelihood (LPML), Deviance Information Criterion (DIC), and Watanabe Akaike Information Criterion (WAIC). Key predictors of U5M include maternal age, breastfeeding duration, preceding birth interval, maternal education, wealth index, region, antenatal visits, pregnancy duration, child's sex, twin birth, mosquito net use, and contraceptive use. Notably, interaction effects revealed that higher maternal education levels amplify the benefits of breastfeeding on U5M. This study demonstrated the effectiveness of the proposed PO model in capturing the intricate dynamics of U5M, highlighting geographic and social clustering patterns that can inform targeted interventions to reduce under-five mortality in Nigeria

Keywords: Covariates Interactions Structures, Proportional Odds Model, Spatial Random Effect, Under-Five Mortality,

INTRODUCTION
The number of children dying before their fifth birthday remains alarmingly high, with 5.3 million deaths reported globally in 2018 alone, despite a 59% decline in child mortality rates since 1990 (UNICEF, 2020). However, the African region continues to bear the heaviest burden, with 76 deaths per 1,000 live births, far exceeding the global average. Nigeria, in particular, faces a dire situation, with an under-five mortality (U5M) rate of 132 deaths per 1,000 live births, translating to approximately 920,000 child deaths annually, about 2,500 each day (National Population Commission & ICF International, 2019). This means 4.6 million Nigerian children under five are at risk of not surviving to their fifth birthday, a devastating reality for a country where children make up 17% of its 205 million population (United Nations Population, 2020). In response, the United Nations' Sustainable Development Goals (SDGs) Target 3 aims to end preventable child deaths by 2030, ensuring universal access to quality, affordable healthcare, and reducing under-five mortality to at least 25 deaths per 1,000 live births (United Nations, 2015). Achieving this goal is critical to giving every child a chance to survive and thrive, but Nigeria's struggle highlights the urgent need for improved healthcare systems and societal changes to address this crisis (Abir et al., 2015; Morakinyo & Fagbamigbe, 2017; Yaya et al., 2018).
Survival analysis is a statistical method used to study time-to-event data, widely applied in fields like medicine and social sciences (Henderson & Oman, 2017). The Bayesian parametric proportional odds (PO) model is a flexible tool for survival analysis, especially when the proportional hazards assumption fails (Bennett, 1983; Pettitt, 1982). Recent advancements focus on incorporating spatial dependencies and interactions between variables to capture complex relationships, such as how socioeconomic factors intersect with healthcare access (Li et al., 2019; Li & Xu, 2017). Techniques like frailty and geospatial models help integrate these complexities, offering deeper insights into high-risk areas, such as regions with elevated under-five mortality rates (Kramer & Hogue, 2018; UNICEF, 2020). Bayesian methods are particularly valuable for their ability to incorporate prior knowledge and handle intricate interaction structures, making them essential for understanding and addressing critical issues like child health and well-being (Muller & Ulm, 2003; Clark et al., 2017).
Koissi et al. (2005) employed a Bayesian approach to explore the impact of unobserved family-level factors on child survival times, utilizing data from the demographic and health survey (DHS, 2001) in the Ivory Coast. By applying a proportional hazards model with a multiplicative random effect, the study identified short birth intervals as a significant risk factor for child mortality. Ghilagaber *et al.* (2013) advanced this line of research by examining U5M in Nigeria using a flexible geo-additive Bayesian survival model, drawing on data from the 2003 NDHS. Their analysis revealed that district-level socioeconomic characteristics significantly influenced U5M rates, with notable spatial clustering of high-risk areas. Similarly, Fatima-Tuz-Zahura et al. (2017) investigated infant mortality in Bangladesh using a Log-logistic parametric proportional odds survival model, identifying maternal education, access to healthcare, and birth-related factors as key determinants of infant survival. These studies collectively demonstrated the value of integrating spatial and socioeconomic dimensions into

survival analysis to better understand and address disparities in child mortality.

Building on these insights, more recent studies have further refined the application of spatial and Bayesian methods in child mortality research. Daniel et al. (2021) investigated spatial variations in U5M in Kenya using a spatial Cox Proportional Hazard model with Bayesian inference, revealing significant demographic and socioeconomic disparities across regions. Fagbamigbe and Nnanatu (2022) extended this approach to Nigeria, identified maternal education, poverty, and birth-related factors as critical predictors of U5M. Musa et al. (2024) employed Bayesian Accelerated Failure Time models with spatial dependency to assess U5M risks in Nigeria, highlighting regional variations and the influence of maternal and healthcare factors. Finally, Fenta et al. (2025) applied a spatiotemporal dynamic model across four Sub-Saharan African countries, demonstrated the significant impact of access to water, sanitation, and healthcare on reducing under-five mortality. Together, these studies underscore the importance of advanced statistical methods and spatial analysis in uncovering the multifaceted determinants of child mortality and informing targeted interventions to improve child survival.

The empirical literature reviewed revealed significant gaps in the application of proportional odds models to examine the predictors of under-five mortality in Nigeria. While studies have increasingly employed advanced spatial and Bayesian methods to analyze child mortality, there remains a notable scarcity of research that utilizes proportional odds models, particularly in the Nigerian context. Furthermore, existing studies often overlook the interaction effects of covariates, where the relationship between a given covariate and the outcome is influenced by another covariate. Incorporating such interactions can enhance the flexibility of survival models, enabling them to capture more complex and nuanced relationships between covariates and survival outcomes. For instance, interactions can reveal synergistic effects, where the combined impact of two or more covariates exceeds the sum of their individual effects, providing deeper insights into the multifaceted determinants of under-five mortality. Addressing these gaps, this study introduced a flexible spatial proportional odds survival model that integrated both standard and logistically transformed covariate interactions, offering a more comprehensive framework for understanding and addressing the drivers of child mortality in Nigeria. Thus, the specific objectives of the study are: To extend the spatial proportional odd model by introducing a covariate interaction structure and compare its performance with existing models (models without covariate interaction structures), and to map the differences in U5M in the 36 States of Nigeria.

MATERIALS AND METHODS

The data for the study were extracted from the 2018 NDH Data. The data were extracted and cleaned using STATA statistical software, and the statistical Analysis was performed using the spBayesurv package in R software. In this study, the outcome variable is the time to death within 5 years of a child's birth. A child was considered to be censored if he or she did not die within this period, and the censoring time for this child is taken as 60 months. The event is U5M, an event is observed if the child dies before his or her fifth birthday.

Variables Classification

The careful selection of variables from the 2018 Nigeria DHS data is crucial for ensuring the study's relevance and validity. The variables chosen are those that are directly related to the research hypothesis and are supported by existing literature on the determinants of U5M. By focusing on a targeted subset of variables, researchers can effectively address the study's objectives and provide meaningful insights into the factors influencing U5M.

For this study, the mother's age birth (MAB) and duration of breastfeeding were kept in their metrical forms while the other variables were categorical; Preceding Birth Intervals (PBI) was coded 2 for More than 33 months, 1 for 24 – 33 months and 0 for less 24 months (reference category), Maternal Highest level of Education (MHLE) was coded 3 for those with "Higher education", 2 for those with "secondary education", 1 for those with "primary education" and 0 for those with "No formal education" (Reference category). Wealth Index (WID) was coded 0, 1, 2, 3, and 4 for "poorest (reference category)", "poorer", "Middle", "rich", and "richest" respectively. The region was coded 0, 1, 2, 3, 4, and 5 for "North Central (reference category)", "North East", "North West", "South East", "South-South", and "South West" respectively. Number of antenatal visits (NAV) was coded 0,1, and 2 for those with "None (reference category)", "1 – 8", and "9 and above" respectively. The duration of pregnancy (DOP) was coded "0" for those with less than 37 weeks of DOP (pre-term) and "1" for those with 37 weeks and above DOP (Post-term). Gender of child was coded "0" for male child (reference category) and "1" for female child. Child twin status (CTS) was coded "0" for "singleton (reference category) and "1" for twin. Type of Place of Residence (TPR) was coded "0" for urban (reference category) and "1" for rural. Sleeping under bed net (SUBN) was coded "0" for those that do not sleep under bed net and "1" for those who sleep under a bed net. Contraceptive Use (CU) was coded "0" for non-use (reference category) and "1" for YES. Source of Drinking Water (SDW) and Toilet facility (TF) were coded "0" for unimproved and "1" for improved.

Proportional Odds Survival Model

The odds function indicates how much more likely it is that a particular event will occur for a given period t. As a result, the odds function is denoted by $R(t; M)$, and its mathematical expression is given by the relationship between the cumulative distribution function and its complementary (survival function):

$$R(t;\varphi) = \frac{F(t;\varphi)}{S(t;\varphi)} = \frac{1-\exp\left(-H(t;\varphi)\right)}{exp(-H(t;\varphi)} = \exp\left(H(t;\varphi) - 1\right) \quad (1)$$

Where $R(t;\varphi), F(t;\varphi)$, $S(t;\varphi)$ and $H(t;\varphi)$ are the odds, cumulative density function (CDF), survival and cumulative hazard functions respectively and φ is the vector of distributional parameters. The associated derivative of the odds function is expressed as follows:

$$r(t;\varphi) = \frac{dR(t;\varphi)}{dt} = \frac{h(t:\varphi)}{S(t;\varphi)} = \frac{f(t:\varphi)}{S(t;\varphi)^2} \quad (2)$$

Where $r(t;\varphi), h(t:\varphi)\ and\ f(t:\varphi)$ are the derivatives of odds, hazard rate function (hrf), and probability density function (PDF), respectively.

According to Bennett (1983), the proportional odd model with covariates vector M is defined as:

$$R(t;\beta,M) = \frac{1-S(t;M)}{S(t;M)} = R_0(t)\exp\left(M^T\beta\right) \quad (3)$$

Where $R_0(t)$ is the baseline odd function. The associated derivative of the odds function of the proportional odd model is computed as follows:

$$r(t;\beta,M) = r_0(t)\exp\left(M^T\beta\right) \quad (4)$$

Where $r_0(t)$ is the baseline derivative odds function. The hazard

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rate function (hrf) of the PO model is computed as follows:

$$h(t;\beta,M)=\frac{r_0(t)\exp(M^T\beta)}{1+R_0(t)\exp(M^T\beta)}$$

In terms of baseline hazard, it can be expressed as follows using equation (2):

$$h(t;\beta,M)=\frac{[\frac{h_0(t)}{S_0(t)}]\exp(M^T\beta)}{1+[\frac{F_0(t)}{S_0(t)}]\exp(M^T\beta)}=\frac{h_0(t)\exp(M^T\beta)}{F_0(t)\exp(M^T\beta)+S_0(t)} \quad (5)$$

The survival function (sf) of the PO model is computed as follows:

$$S_0(t;\beta,M)=\left[\frac{1}{1+R_0(t)\exp(M^T\beta)}\right]=\left[\frac{1}{1+[\frac{F_0(t)}{S_0(t)}]\exp(M^T\beta)}\right]=$$
$$[1+[\frac{F_0(t)}{S_0(t)}]\exp(M^T\beta)]^{-1} \quad (6)$$

Log-Logistic Distribution

Supposing that the shape and scale parameters are $\rho>0$ and $\lambda>0$, respectively ,and if the survival time T follows log-logistic (LL) distribution, then the probability density function (pdf), cumulative density function (cdf), survival function (sf), hazard rate function (hrf), and cumulative hazard rate function are computed respectively as follows:

$$\mathrm{f}(t;\rho,\lambda)=\frac{\rho\lambda(\lambda t)^{\rho-1}}{[1+(\lambda t)^\rho]^2},\quad t>0 \quad (7)$$
$$F(t;\rho,\lambda)=\frac{(\lambda t)^\rho}{1+(\lambda t)^\rho}\quad t>0 \quad (8)$$
$$\mathrm{S}(t;\rho,\lambda)=\frac{1}{1+(\lambda t)^\rho}\quad t>0 \quad (9)$$
$$\mathrm{h}(t;\rho,\lambda)=\frac{\rho\lambda(\lambda t)^{\rho-1}}{1+(\lambda t)^\rho}\quad t>0 \quad (10)$$
$$H(t;\rho,\lambda)=\log(1+(\lambda t)^\rho)\quad t>0 \quad (11)$$

The log-logistic distribution is versatile and widely applicable in analysing survival data, particularly for unimodal datasets. Notably, it is closed under both the proportionality odds and accelerated failure time frameworks (Lawless, 2011). The odd function for LL distribution is expressed as:

$$R_{LL}(t;\lambda,\rho)=\frac{F(t;\rho,\lambda)}{\mathrm{S}(t;\rho,\lambda)}=\frac{\frac{(\lambda t)^\rho}{1+(\lambda t)^\rho}}{\frac{1}{1+(\lambda t)^\rho}}=(\lambda t)^\rho \quad (12)$$

The derivative of the odd function is given as below:

$$r_{LL}(t;\lambda,\rho)=R'_{LL}(t;\lambda,\rho)=\rho\lambda(\lambda t)^{\rho-1} \quad (13)$$

If the LL distribution is assumed for the survival time T_i under the PO framework, in equation (4), it then follows that:

$$T_i\sim log-logistic(\lambda^*=\exp(M^T\beta)\lambda^\rho,\rho) \quad (14)$$

The derivative of the odds function for the LL-PO model is rewritten as follows:

$$r_{LL-PO}(t;\beta,M)=r_0(t)\exp(M^T\beta)=$$
$$\rho\lambda(\lambda t)^{\rho-1}\exp(M^T\beta)=\exp(M^T\beta)\lambda^\rho\rho t^{\rho-1}=\rho\lambda^* t^{\rho-1} \quad (15)$$

Where $\exp(M^T\beta)\lambda^\rho=\lambda^*$

This illustration proved that the log-logistic distribution is the only baseline distribution closed under proportional odd model.

Spatial Frailties in Proportional Odds Models

In this study, we introduce the frailties into the proportional odds survival model by modifying (3) to take the form,

$$R(t;\beta,M)=\frac{1-S(t;M)}{S(t;M)}=R_0(t)\exp(M^T\beta+w_i) \quad (15)$$

Where $w_i=n_i+v_i$.

The non-spatial frailty n_i is a random effect which is defined purely based independent and identically distributed (IID) priors, that is $n_i\sim N(0,\sigma^2)$ to account for heterogeneity within clusters (in this study, state) after adjusting for subject specific covariates while the spatial frailty v_i which is defined purely based on Intrinsic Conditional Autoregressive (ICAR) to account for the spatial dependency between clusters (states).

In this study, we further introduced the interactions of covariates by modifying equation (15) to take the form:

$$R(t;\beta,M)=\frac{1-S(t;M)}{S(t;M)}=R_0(t)\exp(M^T\beta+X^T\gamma+w_i) \quad (16)$$

Where the covariates X are subset of M which represent the interactions of covariates and γ is a subset of β which represent the regression coefficient of the interactions of covariates of interest. Interestingly, the extended model can be implemented using spBayesSurv package in R programming software.

For the purpose of this paper, we considered the PO models with Transformed Bernstein Polynomial (TBP) Prior on the baseline survival function $S_0(.)$. Let $\Gamma(c,d)$ denotes a gamma distribution with mean $\frac{c}{d}$ and $N_p(\mu,\Sigma)$ a p-variate normal distribution with mean μ and covariance Σ. The following prior distributions were used:

$$\beta\sim N_p(\beta_0,S_0) \quad (17)$$
$$S_0(.)|\vartheta,\gamma\sim TBP_Y\left(\vartheta,S_\gamma(.)\right),\ \vartheta\sim\Gamma(c_0,d_0),\gamma\sim N_2(\vartheta_0,V_0) \quad (18)$$
$$(w_1,w_2,\dots,w_m)^T|\tau\sim ICAR(\tau^2),\tau^{-2}\sim\Gamma(c_\tau,d_\tau) \quad (19)$$
$$(w_1,w_2,\dots,w_m)^T|\tau\sim IID(\tau^2),\tau^{-2}\sim\Gamma(c_\tau,d_\tau) \quad (20)$$

Where TBP_Y, ICAR and IID refer to the Transformed Bernstein polynomial (TBP) (Chen et al., 2014; Zhou & Hanson 2018) prior, intrinsic conditionally autoregressive (ICAR) (Besag, 1974) prior and independent and identically distributed (IID) Gaussian prior distributions, respectively.

Transformed Bernstein Polynomial Prior

In survival analysis, a wide variety of Bayesian can be used to model the baseline distribution $S_0(.)$ (Muller *et al*, 2015; Zhou & Hanson, 2015). The TBP prior is attractive in that it is centered at a given parametric distribution family and it selects only smooth densities. For a fixed positive integer Y, the prior $TBP_Y(\vartheta,S_\gamma(.))$ is define as:

$$S_0(t)=\sum_{j=1}^{Y}w_yI(S_\gamma(t)|j,Y-j+1,$$
$$W_Y\sim Dirichlet(\vartheta,\dots,\vartheta) \quad (21)$$

Where $W_Y=\left(w_1,w_2,\dots,w_J\right)^T$ is a vector of positive weights, that follows a Dirichlet distribution where $\vartheta>0$ acts like the precision in a Dirichlet process (Ferguson, 1973), controlling how stochastically $S_0(.)$ is relative to $S_\gamma(.)$. $I(.|c,d)$ denotes beta cdf with parameters (c, d), and $S_\gamma(.)$ is a parametric family of baseline distributions. Thus, the log-logistic parametric family implemented in survregbayes function of spBayesSurv package in R were employed in this study as centering distribution, where $\gamma=\rho,\lambda$ are the distributional parameters. Thus, the parametric model provided good starting values for the TBP prior for the semi-parametric PO model. Note that for the semi-parametric model, the $S_\gamma(t)$ always lied in the interval (0,1) for $0<t<\infty$, so the natural prior on $S_0(.)$, termed the TBT prior, is simply:

$S_0(t)=D(S_\gamma(t)|Y,w_Y)$ with density $f_0(t)=$

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$d(S_\gamma(t)|Y, w_Y)f_\gamma(t)$ (22)

Where $f_\gamma(t)$ is the density associated with $S_\gamma(.)$. Clearly, the random distribution $S_0(.)$ is centered at $S_\gamma(.)$, that is, $E[S_0(t)] = S_\gamma(t)$ and $E[f_0(t)] = f_\gamma(t)$.

The weight parameters W_Y adjust the shape of the baseline survival $S_0(.)$ relative to the centering distribution $S_\gamma(.)$. This adaptability makes the TBP prior attractive in its flexibility, but also anchors the random $S_0(.)$ firmly about $S_\gamma(.)$: $w_y = {}^1/_Y$ for $y = 1, \dots, Y$ implies $S_0(t) = S_\gamma(t)$ for $t \geq 0$ (Zhou & Hanson, 2018).

The independently and identically distributed Assumption

An independent and identically distributed (IID) assumption states that the random effects associated with different clusters are independently drawn from the same distribution and are identically distributed when random effects are added to account for clustered or grouped data in statistical models. For example, an IID assumption for the random effect associated with regions (states) implies that the effects of region (states) on U5M are independently drawn from the same distribution and have the same variability. This is relevant in a study examining the determinants of U5M from different regions (States), where each region (states) represents a cluster.

Mathematically, if $n_1, n_2, \dots, n_g$ represent the random effects associated with clusters (for example, region, states) 1, 2, 3, ..., g, an IID prior for these random effects could be represented as: $n_i \sim N(0, \sigma^2)$. Here each n_i follows a normal distribution with mean "0" and variance "σ^2" and these random effects are assumed to be independent of each other. This assumption simplifies the modelling process, particularly in Bayesian hierarchical models, where it allows us to specify a single prior distribution for the random effects across all clusters (Gelman, 2007).

Besag intrinsic conditional Autoregressive Prior

Besag *et al.* (1991) proposed the intrinsic autoregressive model (also known as the conditional autoregressive (CAR) model) to explain the spatial dependencies. The spatial frailty term in (23) is denoted by v_i. Let $e_{ij} = 1$ if area e_i and e_j share a nontrivial border and $e_{ij} = 0$ otherwise. Set $e_{ij} = 0$, then the $G \times G$ matrix $E = [e_{ij}]$ is called the adjacency matrix for the region D. The ICAR prior is defined through the set of all conditional distributions as given below:

$$\pi(ICAR\_Prior) = v_j|\{v_i: i \neq j\} \sim N\left(\bar{v}_j, {}^{\vartheta^2}/_{e_{j+}}\right), j = 1, \dots, G \quad (23)$$

where v_j is the frailty term for j^{th} areal unit, v_i is the set of all spatial random effect except for area j, ϑ^2 is the variance parameter and e_j is the number of neighbors for area j.

In this study, the spatial parameter v_i is a 37×1 vector of spatial effects to account for heterogeneity between states in Nigeria that is $v = (v_1, \dots, v_{37})$, that is, the 37 states in Nigeria including the Federal Capital Territory.

Covariate Interaction Structures

The study adopted two covariates interaction structures as used by Muller and Ulm (2003). That is the standard covariate interaction and the logistically transformed covariates structures.

Standard Covariate Interaction Structure

According to Muller and Ulm (2003), given two covariates M_1 and M_2, the standard covariates interaction structure is given by:

$$f(M_1, M_2) = M_1 * M_2 \quad (24)$$

Logistic Transformed Covariates Interaction Structure

Here, the covariates are transformed logistically before they are multiplied (Muller & Ulm, 2003). Given two continuous covariates M_1 and M_2, the logistically transformed covariate structure is expressed as:

$$f_{tm}(M_1, M_2) = f_{logistic}(M_1) * f_{logistic}(M_2) \quad (25)$$

$$f_{logistic}(M) = \frac{1}{1+\exp(-M)} \quad (26)$$

Where tm denotes transformed before multiplied.

Bayesian Inference

The Likelihood Function

Let us assume that subjects are observed at g distinct spatial locations $g_1, \dots, g_k$. Let us also assume that t_{ij} be the (possibly censored) survival time, for subject j at location g_i and M_{ij} be the corresponding p-dimensional vector of covariates, $i = 1, \dots, k, j = 1, \dots, n_i$; let $n = \sum_{i=1}^k n_i$. Furthermore, let assume that the survival t_{ij} lies in the interval $(c_{ij}, d_{ij}), 0 \leq c_{ij} \leq d_{ij} \leq \infty$. Thus, left censored are of the form $(0, d_{ij})$, right censored (c_{ij}, ∞), interval censored (c_{ij}, d_{ij}) and uncensored $c_{ij} = d_{ij}$. Let $D = \{c_{ij}, d_{ij}, M_{ij}, g_i\}$ be observed data. Assumed $t_{ij} \sim S_{M_{ij}}(t)$ following either proportional hazard, Accelerated Failure Time and Proportional odds models with the TBP prior on $S_0(t)$ defined in (22) and the spatial component w following ICAR or IID. the likelihood for

$$L(W_Y, \gamma, \beta, w) = \prod_{i=1}^k \prod_{j=1}^{n_i} \left[S_{M_{ij}}(c_{ij}) - S_{M_{ij}}(d_{ij})\right]^{I\{c_{ij}<d_{ij}\}} f_{M_{ij}}(c_{ij})^{I\{c_{ij}=d_{ij}\}} \quad (27)$$

MCMC is carried out through an empirical Bayes approach coupled with adaptive Metropolis
samplers (Haario et al., 2001). The posterior density is:

$$p(W_Y, \gamma, \beta, w, \vartheta, \tau^2, \emptyset|D) \propto L(W_Y, \gamma, \beta, w)p(W_Y|\vartheta)p(\vartheta)p(\beta)p(w|\tau^2)p(\tau^2)p(\emptyset) \quad (28)$$

Where each $p(.)$ represents a prior density, and $p(\emptyset)$ is only included for geo-referenced data. Assumed $\gamma \sim N_2(\gamma_0, V_0), \beta \sim N_p(\beta_0, W_0), \vartheta \sim \Gamma(c_\vartheta, d_\vartheta), \tau^{-2} \sim \Gamma(c_\tau, d_\tau)$ and $\phi \sim \Gamma(c_\phi, d_\phi)$.

To determine the running length of an MCMC run, one may first run a short chain without thinning, then use R packages such as coda (Plummer *et al.*, 2006; Flegal *et al.*, 2016) for convergence diagnosis and effective sample size calculations. Regarding the default choice for the hyperparameters, when variable selection is not implemented, we set $\beta_0 = 0, W_0 = 10^{10} I_p, \gamma_0 = \hat{\gamma}, w_0 = 10\hat{w}$, $c_\vartheta = d_\vartheta = 1$, and $c_\tau = d_\tau = 0.001$. Note here we assume a somewhat informative prior on γ to obviate cofounding between W_Y.

Model Comparison

To evaluate and compare the performance of different models, we utilize three established model selection criteria: the Deviance Information Criterion (DIC; Spiegelhalter et al., 2002), Log Pseudo Marginal Likelihood (LPML; Geisser & Eddy, 1979), and Watanabe-Akaike Information Criterion (WAIC; Watanabe, 2010). Each criterion offers a distinct perspective on model evaluation:

DIC assesses model fit (with lower values indicating better fit), whereas LPML and WAIC focus on predictive performance (with higher LPML values and lower WAIC values signifying better predictive accuracy). These criteria can be readily calculated from the MCMC output, facilitating model comparison and selection.

RESULTS

Table 1 showed that the semi-parametric proportional odds survival model with ICAR spatial prior and logistic transformed covariate interactions (LTCIS) emerged as the superior choice for analysing U5M data considered in this study. This model outperforms all other options, including those with no random effects, those with only IID random effects, and those with ICAR random effects only (without the covariates interactions). Its dominance demonstrated by the lowest DIC, WAIC and LPML values, indicating better predictive power. It was also observed that models with ICAR spatial random effect performed better than those without spatial random effect.

Table 1: Semi-Parametric Proportional Odds with LL-TBP prior Model Comparison

Model	**DIC**	**WAIC**	LPML
Models with No Interaction Structures			
PO	57510.03	57485.29	-28742.60
PO with IID	57388.96	57365.40	-28682.65
PO ICAR*	32611.30	32653.04	-16326.48
Models with Standard Covariate Structure			
PO	64498.13	64855.66	-32427.78
PO with IID	55412.61	55410.16	-27705.07
PO ICAR*	29171.35	29319.94	-14659.93
Models with Logistic Transformed Covariate Interaction			
PO	54920.48	54917.70	-27458.84
PO with IID	54556.17	54542.74	-27271.32
PO ICAR**	28647.10*	28810.69*	-14405.30*

Source: Authors Compilation
Note: **Indicates the best fitted model in the sub-group. **Indicate overall best*

Table 2 presents the posterior mean estimates of proportional odd survival models with LL-TBP prior for the baseline survival of $S_0(t)$, ICAR spatial prior and logistically transformed covariates interaction structure. The posterior mean for the maternal age at birth is -0.0217 and the odd ratio (OR) is 0.9785 representing respectively a decrease in odds of U5M for a unit increase in maternal age at birth and the OR of 0.9785 indicates a 2.2% decrease in the odds of U5M.

Table 2: Posterior Mean Estimates of Overall Best Semi-Parametric PO Model

Covariates	Posterior Mean	Odd Ratio	St. Dev	95% CI
Maternal Age at Birth (MAB)	-0.0217	0.9785	0.0047	-0.0316, -0.0131
Duration of Breast Feeding (DBF)	-0.1872	0.1872	0.0049	-0.1961, -0.1782
Preceding Birth Intervals				
< 24	***Ref***			
24 – 33	-0.2300	0.7945	0.0211	-0.2701, -0.1782
> 33	-0.9413	0.3901	0.0406	-1.0128, -0.8535
Maternal Educ. Qualification (MEQ)				
No Formal Education	***Ref***			
Primary	-0.2925	0.7464	0.0993	-0.3496, -0.2507
Secondary	-0.6418	0.5263	0.0829	-0.6943, -0.6112
Higher Education	-0.6729	0.5102	0.1237	-0.7504, -0.4499
Wealth Index				
Poorest	***Ref***			
Poorer	-0.0102	0.9899	0.0281	-0.0617, 0.0462
Middle	-0.1030	0.9021	0.0178	-0.1369, -0.0675
Richer	-0.2508	0.7782	0.0278	-0.3013, -0.1940
Richest	-0.2781	0.7572	0.0201	-0.3181, -0.2397
Region				
North Central	***Ref***			
North East	0.0189	1.0191	0.0936	-0.1376, 0.2063
North West	0.2077	1.2308	0.0451	0.1170, 0.2998
South East	0.1468	1.1581	0.0967	0.0224, 0.3884
South-South	0.1686	1.1836	0.0669	0.0508, 0.3253
South West	0.4337	1.5430	0.1043	0.2549, 0.6522
Number of Antenatal Visits				
None	***Ref***			
1 – 8	-2.0562	0.1279	0.0550	-2.1543, -1.9436
≥9	0.1552	1.1679	0.0933	-0.0527, 0.3162
Duration of Pregnancy				
Pre-Term	***Ref***			
Post-Term	-0.4881	0.6138	0.0827	-0.6431, -0.3202
Gender of Child				
Male	***Ref***			

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Female	-0.1172	0.8894	0.0217	-0.1591, -0.0726
Child Twin Status				
Singleton	***Ref***			
Twin	0.5778	1.7821	0.0496	0.4701, 0.8754
Type of Place of Residence				
Urban	***Ref***			
Rural	-0.0107	0.9894	0.0240	-0.0593, 0.0334
Sleeping Under bed net				
No	***Ref***			
Yes	-0.0806	0.9226	0.0267	-0.1460, -0.0341
Conceptive Use				
No	***Ref***			
Yes	-0.1819	0.8337	0.0376	-0.2606, -0.1075
Source of Drinking Water				
Unimproved	***Ref***			
Improved	0.0148	1.0149	0.0296	-0.0431, 0.0718
Toilet Facility				
Unimproved	***Ref***			
Improved	0.0224	1.0227	0.0209	-0.0198, 0.0662
DBF*MAB	-0.0552	0.9463	0.0036	-0.0622, -0.0481
DBF*MEQ				
DBF*No Formal Education	***Ref***			
DBF*Primary	-0.6075	0.5447	0.0658	-0.8040, -0.5406
DBF*Secondary	-0.6700	0.5117	0.0843	-0.7016, -0.5056,
DBF* Higher Education	-0.6765	0.5084	0.1244	-0.9302, -0.4472
CAR Frailty Variance (τ^2)	0.7198	2.0540	0.1964	0.4224, 1.1680

Source: Author's Compilation

The duration of breast feeding had a negative effect on odds of experiencing U5M, delaying the odds of U5M by 81%. Children whose mothers had preceding birth interval between 24 to 33 months and above 33 months had a decrease in the odds of dying before their fifth birth day by 21% and 61% respectively as compared with the reference category (less than 24 months). Also, children whose mothers had achieved "primary" "secondary" and "higher education" had a decreasing odd of U5M by 25.4%, 47.4% and 49.0% respectively as compared to the reference category (no formal education). Similar results were obtained for wealth index. Children whose household wealth index was poorer, middle, richer and richest had a decreasing odd of dying before their fifth birth day by 1.0%, 9.8%, 22.2% and 24.3% respectively as compared to the reference category (poorest).

Children whose mothers were from the "North-West", "South-East", "South-South" and "South-West" regions have an increasing odd of experiencing U5M as compared to those from "North-Central". This factor increases the odd of U5M by 23.1%, 15.8% 18.4% and 54.3% respectively. Furthermore, children whose mothers had 1 – 8 numbers of antenatal visits had a decreasing odd of U5M by 87.2% as compared to the reference category (no visit). The post-term babies had a decreasing odd of dying before their fifth birth day as compared to the pre-term babies. Also, the "female sex", "living in rural area", "sleeping under bed net" and "contraceptive use" have negative effect on the odds of U5M, decreasing the odds of U5M by 11.1%, 1.1%, 7.7% and 16.6% respectively as compared to their "male sex", "living in urban areas", "not sleeping under bed net" and "not using contraceptive" respectively. The "twin children" have increased odds of dying before fifth birth day by 78.2% as compared to "singletons".

The results of covariate interactions revealed that maternal age at birth and educational level significantly modify the impact of breastfeeding duration on Under-Five Mortality (U5M). The interaction between breastfeeding duration and maternal age at birth showed a negative effect on U5M odds, delaying the odds by 5%, indicating that the older the mother at birth, the less significant the impact of breastfeeding duration on U5M. Additionally, interactions between breastfeeding duration and different educational levels demonstrated a negative effect on U5M odds, with decreased range from 45.5% for primary education to 49.2% for higher education, suggesting that the benefits of breastfeeding on U5M are more pronounced for mothers with higher educational levels.

The mean of the spatial term v which was represented as $\tau^2 = 0.7198$ is significant with a 95% CI (0.4224, 1.1680). This suggested that the inclusion of the random effect was relevant, implying that the risk of U5M was not homogeneous across the thirty-seven states in Nigeria.

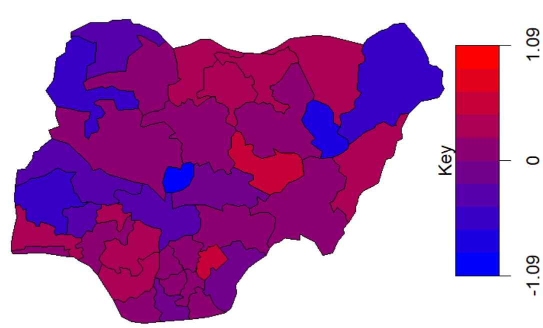

Figure. 1: Posterior mean (Average) Spatial Effect of State Based Survival Risk of U5M

Figure 1 displayed the spatial distribution of state-based U5M in Nigeria based on the overall semi-parametric proportional odds

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model. The regions/states coloured blue indicate regions/states with decreased odds of U5M while those regions coloured red indicates region with increased odds of U5M.

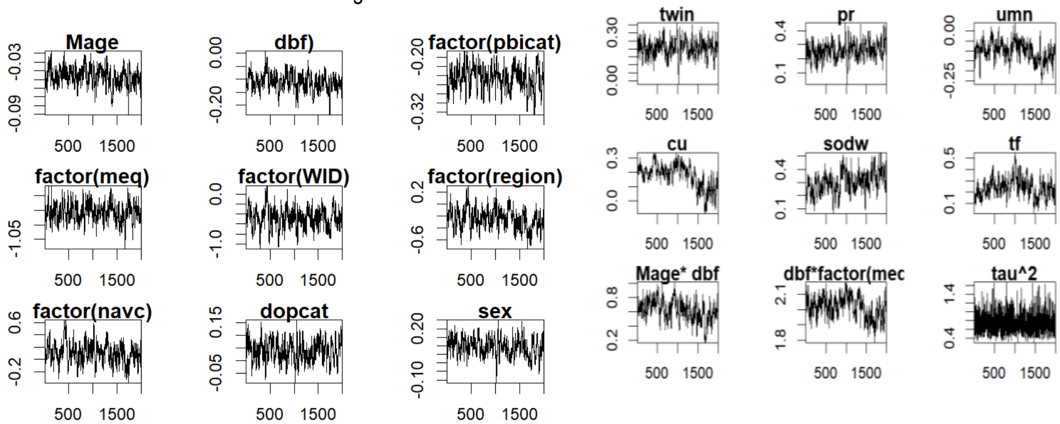

Figure 2: Trace Plot of the PO with ICAR spatial prior and LTCIS

Figure 2 presents the trace plot of the regression coefficient of the PO survival model with ICAR spatial prior and logistically transformed covariates interaction structures for parameters $\beta_1, \ldots, \beta_{17}$ and for the mixing of τ (the variance of the spatial frailty term v) indicating convergence of the chains.

DISCUSSION

This paper investigated factors influencing U5M in Nigeria, using secondary data from the 2018 Nigerian Demographic and Health Survey Data and setting up a survival analysis layout, defining an event as “1” for children that died before their fifth birthday and “0” otherwise. The study used the proportional odds survival model for Bayesian estimation technique.

More interestingly, the study proposed a more flexible PO survival model- a model that incorporate spatial random effect and covariates interaction structures for U5M in Nigeria. Two covariates interaction structures were utilized in this study- Standard Covariates Interaction Structures (SCIS) and Logistically Transformed Covariate Interaction Structures (LTCIS). Three existing models were compared with the proposed model- a PO survival model without spatial random effect, a PO model with IID priors only and a PO model with ICAR spatial priors only. The results revealed that the models with spatial random effects were better than the model that ignores the spatial random effect which was consistent with opinion of Ghilagaber et al. (2013), Daniel et al (2021) Musa et la (2024) and Fenta et al. (2025). However, the proposed PO model with spatial random effect and covariates interaction structures outperformed the existing models with the lowest values of DIC, WAIC and LPML. This suggested that the combination of spatial random effects and covariates interaction structures capture the complexity of the factors influencing U5M in Nigeria.

The incorporation of spatial random effect and covariates interaction structures may affect the posterior mean estimates of fixed effects factors compared to previous studies. The maternal age at birth and duration of breastfeeding significantly decreased the odds of U5M which was consistent with findings of Musa *et al* (2024) and Fatima-Tuz-Zahura et al. (2017). Children whose mothers have more than 24 months of proceeding birth intervals have less odds of U5M as compared to those with less than 24 months. This finding was in line with the submission by Koissi et al. (2005).

The findings of the study revealed further that children whose mothers have achieved at least primary level of education have decreased odds of U5M as compared to those that have no formal education. These findings were in line with findings of Fatima-Tuz-Zahura et al. (2017) and Daniel et al (2021). Similar results were obtained for household wealth index where it was found that children whose household had at least middle wealth quantile have decrease odds of U5M as compared with the reference category (poorer). This finding was in line with the submission of Musa *et al* (2024). Children whose mothers had at least “1 – 8” numbers of antenatal visits had decreases odds of U5M as compared to the reference category “no antenatal visits”. This finding was in line with the submission of Musa *et al* (2024). The “post-term babies”, “female sex”, “living in rural area”, “sleeping under bed net” and “contraceptive use” have negative effect on the odds of U5M. These findings were in agreement with findings from previous study by Musa *et al* (2024) and Fatima-Tuz-Zahura et al. (2017); Daniel *et al* (2021). The findings of the study also revealed that the “twin children” have increase in odds of experiencing U5M as compared to singleton children.

The analysis revealed that a mother's age at childbirth and educational level significantly influence the impact of breastfeeding duration on U5M. Specifically, the interaction between breastfeeding duration and maternal age at birth showed a negative effect on U5M odds, delaying the odds by 5%, indicating that the older the mother at birth, the less significant the impact of breastfeeding duration on U5M. Additionally, interactions between breastfeeding duration and different educational levels demonstrated a negative effect on U5M odds, with decreased range from 45.5% for primary education to 49.2% for higher education, suggesting that the benefits of breastfeeding on U5M are more pronounced for mothers with higher educational levels. These findings indicated that the effect of breastfeeding duration on U5M varies greatly depending on the mother's educational level.

CONCLUSION

This study identified several factors that influence under-five mortality in Nigeria, including maternal age at birth, duration of breastfeeding, preceding birth interval, maternal education, wealth index, region, antenatal visits, duration of pregnancy, child's sex, twin birth, mosquito net use, and contraceptive use. The interaction effects suggested that the benefits of breastfeeding on U5M are more pronounced for mothers with higher education levels. These findings have implications for targeted interventions to reduce under-five mortality in Nigeria.

The study's findings highlighted the importance of integrated interventions that address multiple factors influencing under-five mortality. For instance, programmes that promote breastfeeding, provide maternal education, and improve access to antenatal care and mosquito nets may be more effective in reducing under-five mortality than single-focus interventions. Policymakers and healthcare providers should consider designing and implementing comprehensive programmes that tackle the complex interplay of factors affecting child survival.

The interaction effects between maternal education and duration of breastfeeding, as well as maternal age at birth and duration of breastfeeding, have significant implications for policymakers and healthcare providers. Specifically, breastfeeding interventions should be tailored to mothers' education levels, with more intensive support provided to mothers with lower education levels. This could include additional counseling, peer support groups, and community-based initiatives. Also, healthcare providers should offer age-specific guidance on breastfeeding to mothers, taking into account the mother's age and its interaction with breastfeeding duration. For example, older mothers may require more nuanced guidance on breastfeeding to optimize its benefits for child survival.

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