

FISHER INFORMATION AND QUANTUM LOCALIZATION IN A CALOGERO-LIKE POTENTIAL UNDER VARYING MAGNETIC FIELDS

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ABSTRACT

This research examines the influence of a particle in a quantum-mechanical system constrained in a time-dependent magnetic field and the strengths of a Calogero-like potential. Analytical expressions for the expectation values of the relevant observables were first derived from the exact energy spectrum using the total confinement frequency, which incorporates both the intrinsic confinement and cyclotron frequencies. These expectation values were subsequently employed to evaluate the Fisher information (FI) in both position space (PS) and momentum space (MS), and to assess the effects of the external magnetic fields and the strength of the Calogero-like potential on quantum localization and uncertainty. The expectation values increase with increasing magnetic field strength, resulting in spatial compression and enhanced localization, while stronger Calogero-like potential strengths mimic centrifugal confinement. The examination of FI for both ground and first-excited states across magnetic quantum numbers highlights that localization sharpens with increasing field strength and potential, with observable divergences between quantum states and magnetic quantum numbers. However, the product of the FI was evaluated as the Cramér-Rao bound, showcasing the interplay between quantum uncertainty and information content. A comparison of the ground and first excited states for two magnetic quantum numbers reveals whether the FI product obeys or violates the Cramér-Rao bound, offering insight into quantum stability and control. These results reinforce the controllability of quantum state localization via external field modulation, with implications for quantum control and confinement in potential applications. To the best of our understanding, this is the first time that the analysis of FI for a quantum system governed by a Calogero-like potential in the presence of a varying magnetic field, specifically using a total confinement frequency, is reported. The results provide new insights into quantum localization, information-theoretic uncertainty, and the controlled manipulation of confined quantum states, with potential applications in quantum control, quantum information processing, and nanoscale confinement technologies.

Keywords: Information theory; Fisher information; Expectation values; Energy levels

INTRODUCTION

The heart of every human existence is constantly confronted with uncertainty. The uncertainty, which could be the choice we make or generally every interaction we have, is embedded in a flow of information. The information available in a given system can be

analyzed using information-theoretic measures, with FI and Shannon entropy being prominent. The origin of FI can be traced back as early as the 20th century, when Ronald A. Fisher introduced the concept in statistics. Recently, its applications can be found in different areas such as computer science, physics and information theory. This concept quantifies the content of information in a given system via the probability density. In analyzing the FI for a system, a lower bound is provided via the Cramer-Rao inequality (Fisher, 1925). Previous reports have shown that the lower bound depends on the dimension over which the FI is considered. This concept has a great progressive influence in physics as a result of its linkage to some physical principles, like the uncertainty principle. The study by Frieden and Soffer (1995) shows how physical laws, such as those of quantum and classical mechanics, provide evidence of information-theoretic optimization. FI is seen to control how precisely parameters can be inferred from quantum measurements (Paris 2009).

The quantum counterpart of FI, otherwise called Quantum FI (QFI), offers a bound on parameter estimation through the quantum Cramér-Rao bound. This is of particular importance in quantum metrology, where the goal is to harness quantum phenomena to achieve unprecedented measurement precision (Giovannetti et al., 2011). In statistical mechanics, FI has been connected to entropy production and thermodynamic fluctuations. This can be seen in non-equilibrium thermodynamics, where it has been linked to the Onsager coefficients and interpreted as a measure of structural complexity (Prokopenko et al., 2011). This concept, FI, is strongly linked to the uncertainty principle in quantum mechanics. M. Reginatto (1998) showed that incorporating FI into the action principle leads to the Schrödinger equation, resulting in an interpretation of the information-theoretic measure of quantum dynamics. The versatility and foundational significance of FI bridge statistical inference and fundamental physical laws, opening avenues for novel theoretical developments in both classical and quantum regimes. FI in PS and MS, respectively, are given as (Romera et al. (2005); Romera and Dehesa (2004); Romera et al. (2006)).

$$I_{\rho} = \int \frac{1}{\rho(r)} [\nabla \rho(r)]^2 dr, \quad (1)$$

$$I_{\gamma} = \int \frac{1}{\gamma(p)} [\nabla \gamma(p)]^2 dp. \quad (2)$$

Different authors have reported this concept under different

potential systems (Omugbe et al. (2023); Omugbe et al. (2021); Onate et al. (2022); Onate and Idiodi (2016); Yahya et al. (2015); Falaye et al. (2014); Falaye et al. (2016); Onate and Onyeaju (2021); Idiodi and Onate (2016)). The different studies examined the influence of the potential parameters on the theoretical measures. They validated their results using the Cramer-Rao inequality for the Fisher product, which bounds the product of PS and MS FI from below. In 3D, the Cramer-Rao for the Fisher product, also known as the FI-based uncertainty relation, is given as (Onate et al., 2019)

$$I(\rho)I(\gamma) \geq 36 \quad (3)$$

This equation means that the minimum bound for the product of PS FI and MS FI cannot be lower than 36. However, the results of some studies do not satisfy the conditions, probably due to analytical error or the nature of the potential under consideration. Motivated by the interest in information theory, this study wants to examine the behaviour of FI under a Calogero-like potential. The Calogero-like potential is given as

$$V(r) = \frac{m\omega^2 r^2}{2} + \frac{g}{r^2}, \quad (4)$$

where is the particle's mass, g is the strength of the Calogero-like potential, and ω is the total confinement frequency that comprises the confinement frequency itself and the cyclotron frequency. These frequencies account for the effects of mass, the charge, the particle's frequency, and the magnetic field. **B.** Confinement frequency is a natural frequency at which systems oscillate within a confining potential, such as in a harmonic trap or quantum well. This concept is very important in quantum mechanics and other areas where external potentials spatially confine particles. The total confinement frequency is given by (Onate et al., 2025)

$$\omega = \sqrt{\omega_0^2 + \frac{\omega_c^2}{4}}, \quad (5)$$

where $\omega_0 = 2\pi f_0$, $\omega_c = \frac{qB}{m}$, f_0 is a natural frequency,

q is the particle charge, m is the mass of the particle, and B is the magnetic field. In this potential model, the effect of the magnetic field on a quantum system will be examined without rigorous calculation, compared to when the magnetic field is applied directly to the Schrödinger equation. It allows the study of the 3D Fisher information via expectation values. The study of the FI under the Calogero-like potential has not been reported to the best of our understanding.

Energy Levels

The Schrödinger equation for a system with a non-relativistic energy E_n , reduced Planck's constant $\hbar$, in the presence of a radial potential $V(r)$ given in equation (3) with a wave function

$R_n(r)$ given by

$$-\frac{\hbar^2}{2m} \frac{d^2 R_{n,\ell}(r)}{dr^2} + \frac{m\omega^2 r^4 + 2g}{2r^2} R_{n,\ell}(r) = E_{n,\ell} R_{n,\ell}(r). \quad (6)$$

Equation (6) represents the radial time-independent Schrödinger equation for a particle moving under the influence of the Calogero-like potential. The energy levels of the potential (4) in equation (6) are given by

$$E_{n,\ell} = \sqrt{\omega_0^2 + \frac{\omega_c^2}{4}} \left[2n+1 + \frac{1}{2} \sqrt{(1+2\ell)^2 + \frac{8mg}{\hbar^2}} \right]. \quad (7)$$

Equation (7) gives the quantized energy spectrum of the system. The total confinement frequency combines the intrinsic confinement frequency with the cyclotron frequency generated by the external magnetic field.

Fisher information and a Calogero-like potential.

In terms of the expectation values, (Dehesa et al. (2007); Dehesa et al. (2006); Onate et al. (2022)) define FI for the position space and the momentum space, respectively, as

$$I_\rho = 4 \langle p^2 \rangle - 2(2L+1) |m| \langle r^{-2} \rangle, \quad (8)$$

$$I_\gamma = 4 \langle r^2 \rangle - 2(2L+1) |m| \langle p^{-2} \rangle, \quad (9)$$

Equations (8) and (9) define the Fisher information in position space and momentum space using expectation values. Fisher information quantifies the degree of localization of the quantum probability density. A larger value of position-space Fisher information indicates stronger spatial localization of the particle, whereas momentum-space Fisher information measures localization in momentum space. Where m is the magnetic quantum number, L is the Total angular number, $\langle p^2 \rangle$, $\langle r^2 \rangle$, $\langle r^{-2} \rangle$ and are expectation values. To obtain the expectation values in equations (7) and (8), we define the following identity.

$$2 \langle T \rangle = \left\langle r \frac{dV}{dr} \right\rangle, \frac{dV}{dr} = -\frac{2g}{r^3} + m\omega^2 r. \quad (10)$$

Therefore,

$$r \frac{dV}{dr} = -\frac{2g}{r^2} + m\omega^2 r^2. \quad (11)$$

Hence,

$$2\langle T \rangle = m\omega^2 \langle r^2 \rangle - 2g \langle r^{-2} \rangle. \quad (12)$$

The total energy becomes

$$E = \langle T \rangle + \langle V \rangle = \langle T \rangle + g \langle r^{-2} \rangle + \frac{m\omega^2 \langle r^2 \rangle}{2}. \quad (13)$$

Equations (10)–(13) employ the quantum virial theorem to relate the system's kinetic and potential energies. These relations eliminate the need for direct evaluation of complicated expectation-value integrals by expressing them in terms of the system energy and potential parameters. This analytical approach considerably simplifies the derivation of the expectation values needed to compute Fisher information while preserving the physical dependence on the magnetic field and the Calogero-like potential strength.

Let

$$N = 2n + 1 + \lambda, E = N\hbar\sqrt{\omega_0^2 + \frac{\omega_c^2}{4}}, \text{ and} \quad (14)$$

$$\lambda = \frac{1}{2}\sqrt{(1 + 2\ell)^2 + \frac{8mg}{\hbar^2}},$$

Then, after some simplifications, the expectation values can be written as

$$\langle r^2 \rangle = \frac{\hbar N}{m\sqrt{\omega_0^2 + \frac{\omega_c^2}{4}}}, \quad (15)$$

$$\langle p^2 \rangle = m\hbar N\sqrt{\omega_0^2 + \frac{\omega_c^2}{4}}, \quad (16)$$

$$\langle r^{-2} \rangle = \frac{2m\lambda\sqrt{\omega_0^2 + \frac{\omega_c^2}{4}}}{\hbar N}, \quad (17)$$

$$\langle p^{-2} \rangle = \frac{2\hbar\lambda}{mN\sqrt{\omega_0^2 + \frac{\omega_c^2}{4}}}. \quad (18)$$

These quantities characterize the average spatial extension, kinetic energy, inverse-square confinement, and momentum localization of the particle. Their dependence on the total confinement frequency demonstrates explicitly how increasing the magnetic field or the Calogero potential alters the localization properties of the quantum state. These expectation values serve as the fundamental building blocks for calculating Fisher information. In this study, the magnetic number $|m|$ is taken as either zero ($m = 0$) or one ($m = 1$). Using the expectation values, equations (8) and (9) respectively turn to

$$I_\rho = 16m\hbar(2n + 1 + \lambda)\sqrt{\omega_0^2 + \frac{\omega_c^2}{4}} - 2(2L + 1)|m|\frac{2m\lambda\sqrt{\omega_0^2 + \frac{\omega_c^2}{4}}}{\hbar(2n + 1 + \lambda)}, \quad (19)$$

$$I_\gamma = \frac{16\hbar(2n + 1 + \lambda)}{m\sqrt{\omega_0^2 + \frac{\omega_c^2}{4}}} - 2(2L + 1)|m|\frac{2\hbar\lambda}{m(2n + 1 + \lambda)\sqrt{\omega_0^2 + \frac{\omega_c^2}{4}}}. \quad (20)$$

Equations (19) and (20) constitute the principal analytical models employed in this study. They provide explicit expressions for Fisher information in terms of the magnetic field, confinement frequency, Calogero-like potential strength, and quantum numbers.

RESULTS

Table 1: Expectation values for different magnetic fields and different strengths of inverse-squared potential at the ground state

B	$\langle r^2 \rangle$	$\langle p^2 \rangle$	$\langle r^{-2} \rangle$	$\langle p^{-2} \rangle$	g	$\langle r^2 \rangle$	$\langle p^2 \rangle$	$\langle r^{-2} \rangle$	$\langle p^{-2} \rangle$
0	0.487261	19.236304	8.461796	0.214340	0	0.396633	15.757621	7.563658	0.190384
1	0.485726	19.297115	8.488547	0.213664	1	0.485726	19.297115	8.488547	0.213664
2	0.481205	19.478411	8.568296	0.211676	2	0.555287	22.060669	9.004355	0.226648
3	0.473943	19.776877	8.699588	0.208481	3	0.614351	24.407176	9.350626	0.235364
4	0.464307	20.187318	8.880135	0.204242	4	0.666592	26.482649	9.605761	0.241786
5	0.452740	20.703074	9.107010	0.199154	5	0.713940	28.363717	9.804742	0.246794

Table 2: Expectation values for different magnetic fields at the first excited state

B	$\langle r^2 \rangle$	$\langle p^2 \rangle$	$\langle r^{-2} \rangle$	$\langle p^{-2} \rangle$
0	0.805571	31.802674	5.118239	0.129647
1	0.803033	31.903212	5.134419	0.129238
2	0.795558	32.202941	5.182657	0.128035
3	0.783552	32.696384	5.262070	0.126103

4	0.767621	33.374951	5.371277	0.123539
5	0.748498	34.227632	5.508505	0.120461

Table 3: Fisher information for different magnetic fields at the ground state for two values of the magnetic number

B	$m = 0$			$m = 1$		
	I_γ	I_ρ	$I_\rho I_\gamma$	I_γ	I_ρ	$I_\rho I_\gamma$
	1.949	76.94	149.9	0.663	26.174	17.353

0	044	520	696	006	436	814
	1.942	77.18	149.9	0.660	26.257	17.353
1	904	846	698	917	181	814
	1.924	77.91	149.9	0.654	26.503	17.353
2	820	364	697	765	866	814
	1.895	79.10	149.9	0.644	26.909	17.353
3	772	748	697	884	983	814
	1.857	80.74	149.9	0.631	27.468	17.353
4	228	927	698	772	461	814
	1.810	82.81	149.9	0.616	28.170	17.353
5	960	230	698	034	240	814

Table 4: Fisher information for different values of the strength of the inverse square potential at the ground state for two values of the magnetic number

g	$m=0$			$m=1$		
	I_γ	I_ρ	$I_\rho I_\gamma$	I_γ	I_ρ	$I_\rho I_\gamma$
0	63.03	1.586	100.0	17.64	0.444	7.8400
1	048	534	000	854	230	00
2	77.18	1.942	149.9	26.25	0.660	17.353
3	846	903	697	718	917	814
4	88.24	2.221	196.0	34.21	0.861	29.469
5	268	148	000	655	261	388
	97.62	2.457	239.9	41.52	1.045	43.402
	871	402	130	495	220	715
	105.9	2.666	282.4	48.29	1.215	58.711
	306	368	500	603	655	287
	113.4	2.855	324.0	54.62	1.374	75.111
	549	761	000	642	996	111

Table 5: Fisher information for different magnetic fields at the first excited state for two values of the magnetic number with

B	$m=0$			$m=1$		
	I_γ	I_ρ	$I_\rho I_\gamma$	I_γ	I_ρ	$I_\rho I_\gamma$
0	3.222	127.2	409.9	2.444	96.50	235.8
	284	106	086	406	126	882
	3.212	127.6	409.9	2.436	96.80	235.8
1	132	128	092	702	633	882
2	3.182	128.8	409.9	2.414	97.71	235.8
3	232	118	090	023	582	882
4	3.134	130.7	409.9	2.377	99.21	235.8
5	208	855	090	591	312	882
	3.070	133.4	409.9	2.329	101.2	235.8
	484	998	090	251	721	882
	2.993	136.9	409.9	2.271	103.8	235.8
	992	105	089	224	595	882

DISCUSSION

Table 1 shows the effect of both the magnetic field and the strength of the inverse-squared potential on the various expectation values.

As the magnetic field increases, the expectation values $\langle r^2 \rangle$ and respectively decrease in magnitude, while the expectation value increases in magnitude. This shows that an increase in the magnetic field strength compresses the particle's wave function spatially, decreases the particle's momentum uncertainty, and enhances localization, which, in the long run, increases the kinetic energy linked to. The implication is that higher spatial confinement

leads to higher momentum uncertainty. However, an increase in the strength of the inverse-squared potential increases the magnitude of the expectation values. This means that a change in the inverse-squared potential contributes to confinement behaviour similar to a centrifugal term. This shows that, under higher potential strength and magnetic fields, the system is pushed towards more quantized and energetically spread states.

Table 2 shows the variation of the expectation values at the first excited state against the magnetic field from the cyclotron frequency. The features observed in Table are also observed. However, the numeric values obtained in Table 1 at the ground state are lower than the numeric values obtained at the first excited state in Table 2. The features in this table can be attributed to high-energy states that are more delocalized, and also to the experience of the excited states in a broader spatial distribution, leading to an increase in the magnetic field. The implication is that the first excited states are more sensitive to changes in the field.

Table 3 presents FI in both position space and momentum space and their product at the ground state with two values of the magnetic number ($m=0$ and $m=1$) as a function of the magnetic fields. The Fisher information in position space increases with magnetic field strength for both the absence and the presence of the magnetic number. In contrast, the MS FI for the absence and presence of the magnetic number decreases as the magnetic field increases. However, the FI in the absence of the magnetic number is higher than its counterpart in the presence of a magnetic number. In both cases, the product of FI increases with an increase in the magnetic field. The Fisher product as a function of the magnetic field in the absence of the magnetic number obeys the Heisenberg principle. It satisfies the Cramer-Rao inequality for the Fisher product in 3D by recording a minimum bound of 149.9696, which is far above 36. In contrast, the Fisher product in the presence of a magnetic field obeys the Heisenberg principle. However, it does not satisfy the Cramer-Rao inequality for the Fisher product in 3D, as the highest Fisher product is 17.353814, which is far less than the minimum bound of 36. The FI measures the sharpness or localization of a distribution. An increase in the magnetic field confines the particle more tightly, resulting in sharper distributions and higher FI. The implication is that a system under stronger magnetic fields or with higher quantum numbers has higher information content, suggesting better localization and potentially more stable quantum states.

Table 4 shows FI for PS and MS along with their product as a function of the strength of the Calogero potential (g). For two values of the magnetic quantum number. An increase in g leads to increases in both the FI in PS and MS and their product. An increase in FI as g increases is more pronounced for $m=0$ than

$m=1$. As seen from the Table, the FI for the MS at a lower value of g is higher than its product for $m=1$. These variations are due to a stronger inverse-squared potential, increasing confinement, similar to a centrifugal wall. The implication is that an increase in potential strength refines control over quantum state localization. Table 5 presents the FI as a function of changes in magnetic field in the absence and presence of the magnetic quantum number in the first excited state. In both cases, the PS FI increases, while the FI in MS and the product of FI increase as the magnetic field, via the cyclotron frequency, increases. However, the FI obtained with is higher than that obtained with. This means that the excited states provide tunable localization under external-field control.

CONCLUSION

This study has presented an analytical investigation of the information-theoretic properties of a quantum particle confined by a Calogero-like potential in the presence of a varying magnetic field. By deriving expectation values from the exact energy spectrum and employing them to compute the Fisher information in both position and momentum spaces, the study has provided a quantitative assessment of quantum localization, uncertainty, and confinement in the ground and first excited states. The major findings are summarized as follows:

- 1) The magnetic field reduces the spatial extent of the wave function while increasing the kinetic energy of the particle, demonstrating stronger confinement and improved localization.
- 2) An increase in the inverse-square potential strength acts as an effective centrifugal barrier, leading to stronger confinement and greater localization of the quantum particle.
- 3) Position-space Fisher information increases with increasing magnetic field and potential strength, indicating sharper localization of the particle.
- 4) The first excited state exhibits greater variations in Fisher information with changing magnetic field, indicating enhanced tunability of localization under external field modulation.
- 5) The analytical formulation provides an efficient framework for studying magnetically confined quantum systems.

Overall, this work demonstrates that both the external magnetic field and the Calogero-like potential strength are effective control parameters for manipulating quantum localization and information content. The analytical results presented herein contribute to the understanding of information-theoretic measures in confined quantum systems and provide a solid foundation for future studies.

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