

DYNAMIC ANALYSIS OF FLUID-CONVEYING CARBON NANOPIPES USING A NOVEL SPECTRAL ELEMENT FORMULATION

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ABSTRACT

The dynamic behaviour of fluid-conveying double-walled carbon nanotubes (DWCNTs) is important in nanoscale fluid-transport systems because fluid–structure interaction, interwall forces, size-dependent effects and environmental loading can significantly influence their vibration characteristics. Therefore, this paper aims to control the dynamic vibration of flowing fluid through DWCNT systems to enhance their integrity and reliability. However, this study develops a spectral-element formulation for the dynamic analysis of a water-conveying double-walled nano-pipe subjected to an elastic foundation. The formulation incorporates Eringen's nonlocal elasticity theory, linear van der Waals interaction between the inner and outer walls, Knudsen-number-dependent slip flow, Pasternak elastic foundation, and linear and nonlinear Winkler foundation effects. The governing equation is derived using Hamilton's principle and transformed into a frequency-domain spectral-element formulation from which the dynamic response and natural frequencies are obtained. Numerical simulations examine how the major governing parameters influence the nano-pipe's vibration characteristics. The results show that increasing the Pasternak and Winkler foundation stiffnesses increases the natural frequency while reducing the displacement amplitude and suppressing sustained oscillations. Similarly, increasing the van der Waals interaction strength and geometric length also reduces the vibration amplitudes and enhances the dynamic stability of the double-walled system. The combined variation of van der Waals interaction and Knudsen number produces a gradual increase in the first natural frequency from approximately 32.45 GHz to 32.95 GHz within the investigated parameter range. In contrast, increasing the Knudsen number also increases the displacement amplitude, although the oscillations progressively decay with time. These results demonstrate that interwall interaction, nanoscale fluid-flow effects, geometric length and foundation stiffness are important in predicting and controlling the dynamic behaviour of fluid-conveying DWCNTs. The proposed spectral-element framework provides an efficient computational approach for dynamically assessing and designing reliable nanoscale water-transport systems.

Keywords: Nanopipe, Pasternak Foundation, Knudsen Number, Winkler Foundation, Spectral Element Method

INTRODUCTION

Carbon nanotubes (CNTs) are significant nanoscale structural components due to their superior mechanical rigidity, strength-to-weight ratio, thermal properties and function as molecular-level channels. These properties have led to their investigation and use for nanoelectromechanical systems (NEMS), nano-resonators,

sensing devices, energy systems and nanofluidic applications. The hollow cylindrical configuration of CNTs is also appealing for nanoscale fluid transport, where the nanotube's mechanical response is intrinsically linked to the motion of the enclosed fluid. Numerous experimental and theoretical studies have shown that the mechanical and vibrational properties of CNTs are strongly influenced by their surroundings and internal interactions (Cui *et al.*, 2023; Gandoura *et al.*, 2023; Vassilev & Valchev, 2024).

In addition to fluid inertia, fluid flow-induced forces are significant. They can greatly affect the natural frequencies and stability characteristics of fluid-conveying CNTs, making dynamic analysis of this system especially important. The dynamic response in fluid-conveying nanotubes is coupled with fluid motion and varies with flow velocity, fluid parameters and structural stiffness. Recently, Vassilev and Valchev (2024) studied the dynamics and stability of cantilevered DWCNTs carrying fluid in an elastic medium and explicitly accounted for the effects of the internal fluid, the surrounding medium and the van der Waals force between the nanotube walls.

However, at the nanoscale, classical local elasticity may be insufficient, as mechanical response may depend on internal length scales and long-range interactions. Therefore, nonlocal elasticity theory has emerged as an important model for CNTs and other nanostructures. Storz *et al.* (2024) developed a nonlocal strain-gradient anisotropic shell model for SWCNT vibration and showed that the nonlocal parameter and material parameters affect the natural frequencies. Similarly, Ramazani *et al.* (2024) showed that nonlocal elasticity significantly affects the stability of fluid-conveying curved CNTs and that thermal load and fluid velocity can alter the stability region, both of which can interact with the small-scale parameter. Recently, Tepe (2025) reported that the natural frequencies of SWCNTs with an elastic foundation decrease with increasing nonlocal parameter and structural rigidity, and that the natural frequencies increase with increasing foundation stiffness. Chen *et al.* (2023) explored the transverse vibration of DWCNTs resting in an elastic medium, accounting for inner- and outer-tube van der Waals interactions and thermal and electrical effects. Gandoura *et al.* (2023) also investigated DWCNT vibrations based on Timoshenko beam theory and found that the natural frequencies of DWCNTs depend strongly on geometrical and modal parameters.

The van der Waals (vdW) interaction between adjacent nanotube walls is a coupling mechanism that distinguishes DWCNTs from SWCNTs. The vdW force provides an interwall restoring mechanism and can modify the effective stiffness and modal characteristics of the coupled system. Tepe (2026) presented a semi-analytical model of DWCNTs with inter-tube vdW interaction and Winkler foundation effects, also in a nonlocal elasticity

framework. More recently, experimental studies on DWCNTs have further confirmed that inter-shell interactions play a significant role in the dynamic mechanical response of these structures, highlighting the importance of interwall coupling in nanoscale structural models, as Yue *et al.* (2025) reported.

The Knudsen number (Kn) is another parameter associated with fluid-conveying nanotubes that measures the relative importance of the molecular mean free path to the characteristic dimension of the channel through which the fluid flows. At the nanoscale, continuum no-slip boundary conditions may not be valid, and fluid-wall interactions can alter flow properties. This is important for CNTs, as their extremely small internal diameter may lead to flow regimes where molecular-scale effects play a role in effective fluid-structure interaction. CNT models also account for nanoscale flow and environmental coupling, a topic studied recently. Selvamani *et al.* (2026) studied stochastic nonlocal vibration of thermopiezoelectric fluid-conveying CNTs, explicitly considering the Knudsen number, surface characteristics and small-scale effects. Another important contribution to the dynamic response is from the surrounding elastic medium. In applications, CNTs can be placed in a polymeric matrix, deposited on an elastic substrate, or otherwise bound by other materials. The Winkler foundation is the usual representation of elastic support in the transverse direction. At the same time, the Pasternak model extends it to include shear-layer interaction and provides a more general, two-parameter description of the surrounding medium. Tepe (2025) showed that SWCNT natural frequencies increase with elastic-foundation stiffness and decrease with the nonlocal parameter. Recent research on reinforced structural systems using CNTs has confirmed the role of Winkler/Pasternak support in determining nanoscale vibration characteristics (Li *et al.*, 2024). Ebrahimi (2024) studied fluid-carrying CNTs on nonlinear elastic foundations using nonlocal Euler–Bernoulli theory and showed that fluid viscosity, foundation characteristics, the nonlocal parameter and excitation characteristics can lead to strongly nonlinear dynamic behaviour.

Recently, Chen *et al.* (2023) introduced the effect of temperature gradients into the nonlocal vibration analysis of a DWCNT embedded in an elastic medium and found that the thermal environment can affect DWCNT vibration characteristics. Similarly, the temperature-dependent vibrational behaviour of isolated single- and double-walled CNTs has also been confirmed experimentally, further highlighting the importance of thermal conditions in nanoscale dynamic analysis (Feng *et al.*, 2025).

The same applies to the modelling framework used to analyse these coupled effects. For large-parameter studies, direct experimental characterization and atomistic simulations remain time-consuming, so numerical and semi-analytical approaches that cover the continuum of possible states remain appealing. Tepe (2024) showed that the initial-value method and the approximate transfer-matrix approach can efficiently analyse nonlocal SWCNTs, offering a computational alternative for nanoscale structures. Tepe (2025) then used a similar transfer-matrix approach to evaluate the free vibration of SWCNTs supported by elastic foundations, showing that the technique can be a useful tool for finding natural frequencies under various boundary conditions.

Higher-order and generalized continuum theories are therefore gaining importance in modern nanoscale mechanics. In their recent review, Barretta *et al.* (2023) summarized recent advances in nonlocal elasticity for nanostructures. They noted the need to develop a suitable size-dependent elasticity formulation to predict

nanoscale mechanical behaviour. Arad and Aydogdu (2023) also showed that nonlocal stress-gradient elasticity can be used to study the torsional dynamics of axially graded viscoelastic CNTs, highlighting the importance of nanoscale constitutive parameters in nanotube dynamics.

While these advances are important, a key research gap remains. Recent years have investigated the individual effects of combinations of fluid flow, nonlocality, elastic foundations, thermal loading, interwall interaction and nonlinear effects with success. Vassilev and Valchev (2024) focused on the dynamics and stability of fluid-conveying DWCNTs on a nonlinear elastic foundation with vdW interaction. However, systematic dynamic studies that represent linear vdW interaction, along with the Knudsen number, nonlocal parameter, Pasternak elastic support, linear Winkler stiffness, nonlinear Winkler stiffness, thermal-magnetic loading, and distributed harmonic force in a unified matrix-based formulation remain comparatively limited.

These gaps matter because these parameters affect different aspects of the dynamic system and can have opposing or reinforcing effects. The vdW interaction modifies interwall coupling, the Knudsen number modifies nanoscale fluid-wall interaction, the nonlocal parameter introduces size-dependent constitutive behaviour, the Pasternak parameter modifies foundation shear interaction, the linear and nonlinear Winkler parameters induce elastic restoring forces, and temperature variation introduces thermally induced mechanical effects. This treatment of individual mechanisms can therefore mask each mechanism's relative contribution to the overall dynamic response. Advances in DWCNT modelling over the last few years, especially when accounting for nonlocality and elastic foundations, have enabled a systematic investigation of these effects (Chen *et al.*, 2023; Vassilev & Valchev, 2024; Tepe, 2026).

Accordingly, this study proposes a dynamic model of a fluid-conveying double-walled carbon nanotube with linear van der Waals interaction, Knudsen-number effects, nonlocal elasticity, Pasternak elastic support, linear Winkler foundation stiffness, nonlinear Winkler foundation stiffness, thermal-magnetic loading, and distributed harmonic force. The spectral element method is used to solve the dynamic system and obtain the natural frequencies and dynamic properties. This paper emphasizes systematic parametric investigations of the individual physical mechanisms.

MATHEMATICAL FORMULATION

Mathematical Model

This paper investigates a system of double-walled nanopipes for water supply resting on a Pasternak elastic foundation under the influence of a longitudinal magnetic field and thermal load, as shown in Figure 1. Therefore, the following assumptions are taken into consideration:

- i. The double-walled nanotube is considered as a homogeneous and isotropic elastic material
- ii. Assuming small transverse deflections in the displacement field $u(x, t)$ in order to apply Euler–Bernoulli beam theory
- iii. Size-dependent effects are accounted for, so that Eringen's nonlocal elasticity theory is applicable
- iv. Slip boundary flow condition is applied since the radius of the pipe is a characteristic length scale, then the definition of Knudsen number is used.

- v. Linear van der Waals force is considered to take proper care of vibrations between the pipes
- vi. Pasternak foundation is considered to incorporate shear interaction effects, thereby providing a more accurate representation of the surrounding medium.

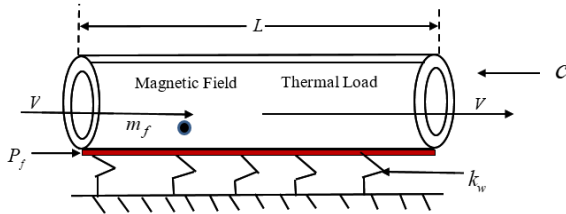


Figure 1: A Schematic Representation of Double-Walled Nano-pipe for Water Supply System on Pasternak-Elastic Medium

In Figure 1, the parameter P_f is the coefficient of the Pasternak-elastic medium acting on the system, V is the velocity correction factor, m_f is the fluid mass, L is the nanotube length, C is the linear van der Waals force, and k_w denotes the linear and nonlinear Winkler foundation.

However, based on the assumptions above and adopting the extended form of Hamilton's principle, the governing partial differential equation of motion of a double-walled nano-pipe for a water supply system is obtained as:

$$\left. \begin{aligned} (EI+h)\frac{\partial^4 u}{\partial x^4} + (V^2 m_f - \Pi_0 - EA\alpha_x \Delta T - \eta AH_x^2 - P_f)\frac{\partial^2 u}{\partial x^2} + 2Vm_f \frac{\partial^2 u}{\partial x \partial t} + \\ (m_p + m_f)\frac{\partial^2 u}{\partial t^2} + k_w u + cu - (e_0 a)^2 \frac{\partial^2}{\partial x^2} \left((V^2 m_f - \Pi_0 - EA\alpha_x \Delta T - \eta AH_x^2 - P_f)\frac{\partial^2 u}{\partial x^2} + 2Vm_f \frac{\partial^2 u}{\partial x \partial t} + (m_p + m_f)\frac{\partial^2 u}{\partial t^2} + k_w u + cu \right) = f(x,t) - \\ (e_0 a)^2 \frac{\partial^2 f(x,t)}{\partial x^2} \end{aligned} \right\} \quad (1)$$

But the Winkler foundation ($k_w w$) is defined as:

$$k_w w = k_1 u + k_3 u^3 \quad (2)$$

where k_1 and k_3 are the coefficients of the linear and nonlinear Winkler foundation, respectively. Thus, applying equation (2) to (1) yields:

$$\left. \begin{aligned} (EI+h)\frac{\partial^4 u}{\partial x^4} + (V^2 m_f - \Pi_0 - EA\alpha_x \Delta T - \eta AH_x^2 - P_f)\frac{\partial^2 u}{\partial x^2} + 2Vm_f \frac{\partial^2 u}{\partial x \partial t} + \\ (m_p + m_f)\frac{\partial^2 u}{\partial t^2} + (k_1 + c)u + k_3 u^3 - (e_0 a)^2 \frac{\partial^2}{\partial x^2} \left((V^2 m_f - \Pi_0 - EA\alpha_x \Delta T - \eta AH_x^2 - P_f)\frac{\partial^2 u}{\partial x^2} + 2Vm_f \frac{\partial^2 u}{\partial x \partial t} + (m_p + m_f)\frac{\partial^2 u}{\partial t^2} + (k_1 + c)u + k_3 u^3 \right) = \\ f(x,t) - (e_0 a)^2 \frac{\partial^2 f(x,t)}{\partial x^2} \end{aligned} \right\} \quad (3)$$

where

$$\left. \begin{aligned} h = \frac{1}{8} E \pi t_0 (d_{in}^3 + d_{out}^3) \\ \Pi_0 = 2\tau_0 (d_{in} + d_{out}) \end{aligned} \right\} \quad (4)$$

$$f(x,t) = \frac{F_0}{L} \sin(\omega t + \phi) \quad (5)$$

$$V = VCF = \frac{V_{arg-slip}}{V_{arg-(non-slip)}} = (1 + a_k k_n) \left(\frac{-\sigma_v}{\sigma_v} \right) \left(\frac{k_n}{1 + k_n} \right) + 1 \quad (6)$$

$$a_k = \frac{2a_0}{k_n} \tan^{-1}(a_1 k_n^B) \quad (7)$$

$$a_0 = \frac{64}{3\pi - \frac{12\pi}{b}} \quad (8)$$

with the Clamped-Free (C-F) supported boundary conditions given as:

$$\left. \begin{aligned} u(x,t)|_0^L = \frac{\partial u}{\partial x} \Big|_0^L = 0 \\ \frac{\partial^2 u}{\partial x^2} \Big|_0^L = \frac{\partial^3 u}{\partial x^3} \Big|_0^L = 0 \end{aligned} \right\} \quad (9)$$

where $u(x,t)$ is the transverse displacement, η is the

magnetic field permeability, H_x is the magnetic field strength,

A is the cross-sectional area of the tube, E is the Young's modulus of the elasticity, I is the second moment of area of the tube, h is the additional flexural rigidity of the tube, Π_0 is the distributed load relation, α_x is the coefficient of thermal expansion, ΔT is the temperature change, m_p is the pipe mass, $f(x,t)$ is the distributed Harmonic force per unit length,

F_0 is the constant amplitude (magnitude) of the distributed harmonic force, ω is the angular frequency of the oscillation field per unit length, ϕ is the phase angle (the initial phase at $t = 0$), t_0 is the thickness of the surface layer, τ_0 is the residual surface stress, $(e_0 a)$ is the nonlocal material constant of the Eringen's theory, k_n is the Knudsen number, b is the general slip coefficient, σ_v is the tangential momentum accommodation coefficient and d_{in} , d_{out} are the inner and outer diameter of the CNT respectively.

Spectral Element Discretization
 The entire carbon nanotube system is discretized into beam elements. Each element is a two-node clamped-free (C-F) supported beam element with three (3) degrees of freedom: two

are longitudinal (axial) displacement, two are transverse (lateral) displacement, and the remaining two are angular displacement (rotational angle). The truss element has a total of six (6) degrees of freedom per element, as shown in Figure 2. The shape function uses a cubic interpolation function called the Hermite polynomial.

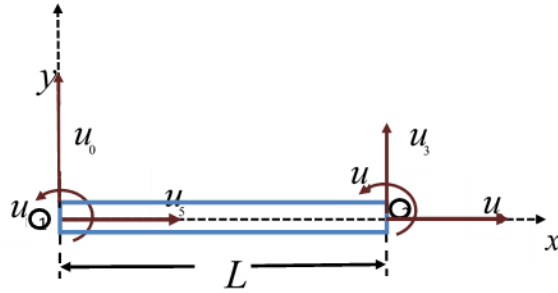


Figure 2: A Nodal Displacement of Two-Node Clamped-Free (C-F) Truss Element

Spectral Element Solution of the Model Equation

First, the flowchart that describes the general procedure of the spectral element method for the model solution is shown in Figure 3 for better understanding of the solution below:

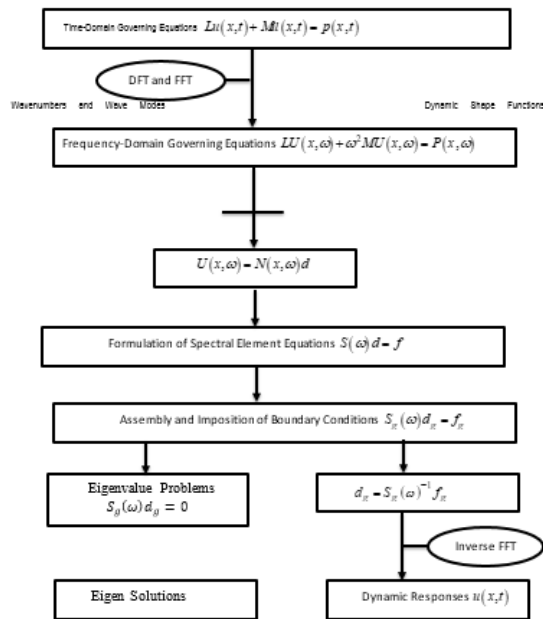


Figure 3: The Flowchart of the Spectral Element Method

Now, let the dynamic response of the governing equation (4) be written in spectral form as:

$$u(x, t) = \frac{1}{N} \sum_{j=0}^{N-1} U_j(x, \omega_j) e^{i\omega_j t} \quad (10)$$

where $U_j(x, \omega_j)$ is the spectral component of the

displacement field $u(x, t)$

Then, differentiating equation (9) twice and substituting into (8), ignoring the subscripts j for brevity, gives the frequency-domain equation of the system as:

$$\left. \begin{aligned} & (EI + h) \frac{d^4 U}{dx^4} + (V^2 m_f - \Pi_0 - EA\alpha_x \Delta T - \eta AH_x^2 - P_f) \frac{d^2 U}{dx^2} + 2i\omega V m_f \frac{dU}{dx} - \\ & \omega^2 (m_p + m_f) U + (k_1 + c) U + k_3 U^3 - (e_0 a)^2 \left((V^2 m_f - \Pi_0 - EA\alpha_x \Delta T - \eta AH_x^2 - P_f) \frac{d^4 U}{dx^4} + \right. \\ & \left. 2i\omega V m_f \frac{d^3 U}{dx^3} - \omega^2 (m_p + m_f) \frac{d^2 U}{dx^2} + (k_1 + c) \frac{d^2 U}{dx^2} + \right. \\ & \left. k_3 \frac{d^2 U^3}{dx^2} \right) - F(x, \omega) + (e_0 a)^2 \frac{d^2 F(x, \omega)}{dx^2} = 0 \end{aligned} \right\} \quad (11)$$

Moreover, the Clamped-Free (C-F) supported boundary conditions become:

$$\left. \begin{aligned} U(x, \omega) \Big|_0^L &= \frac{\partial U(x, \omega)}{\partial x} \Big|_0^L = 0 \\ \frac{\partial^2 U(x, \omega)}{\partial x^2} \Big|_0^L &= \frac{\partial^3 U(x, \omega)}{\partial x^3} \Big|_0^L = 0 \end{aligned} \right\} \quad (12)$$

Now, taking the weak form of equation (11) by adopting the weighted-integral statement as:

$$\left. \begin{aligned} & (EI + h) \int_0^L \frac{d^4 U}{dx^4} \delta U dx + (V^2 m_f - \Pi_0 - EA\alpha_x \Delta T - \eta AH_x^2 - P_f) \int_0^L \frac{d^2 U}{dx^2} \delta U dx + \\ & 2i\omega V m_f \int_0^L \frac{dU}{dx} \delta U dx - \omega^2 (m_p + m_f) \int_0^L U \delta U dx + (k_1 + c) \int_0^L U \delta U dx + \\ & k_3 \int_0^L U^3 \delta U dx - (e_0 a)^2 \left((V^2 m_f - \Pi_0 - EA\alpha_x \Delta T - \eta AH_x^2 - P_f) \int_0^L \frac{d^4 U}{dx^4} \delta U dx - \right. \\ & \left. 2i\omega V m_f (e_0 a)^2 \int_0^L \frac{d^3 U}{dx^3} \delta U dx + \omega^2 (e_0 a)^2 (m_p + m_f) \int_0^L \frac{d^2 U}{dx^2} \delta U dx - \right. \\ & \left. (e_0 a)^2 (k_1 + c) \int_0^L \frac{d^2 U}{dx^2} \delta U dx - (e_0 a)^2 k_3 \int_0^L \frac{d^2 U^3}{dx^2} \delta U dx - \int_0^L F(x, \omega) \delta U dx + \right. \\ & \left. (e_0 a)^2 \int_0^L \frac{d^2 F(x, \omega)}{dx^2} \delta U dx = 0 \right\} \quad (13) \end{aligned}$$

where δU is an arbitrary variation in $U(x, \omega)$. Then, applying integration by parts and the clamped-free supported boundary conditions in equation (12) to (13) gives:

$$\left. \begin{aligned} & (EI + h) \int_0^L \frac{d^2 U}{dx^2} \delta U'' dx - (V^2 m_f - \Pi_0 - EA\alpha_x \Delta T - \eta AH_x^2 - P_f) \int_0^L \frac{dU}{dx} \delta U' dx + \\ & i\omega V m_f \int_0^L \left(\frac{dU}{dx} \delta U - U \delta U' \right) dx - \omega^2 (m_p + m_f) \int_0^L U \delta U dx + (k_1 + c) \int_0^L U \delta U dx + \\ & k_3 \int_0^L U^3 \delta U dx - (e_0 a)^2 \left((V^2 m_f - \Pi_0 - EA\alpha_x \Delta T - \eta AH_x^2 - P_f) \int_0^L \frac{d^2 U}{dx^2} \delta U'' dx + \right. \\ & 2i\omega V m_f (e_0 a)^2 \int_0^L \frac{d^2 U}{dx^2} \delta U' dx - \omega^2 (e_0 a)^2 (m_p + m_f) \int_0^L \frac{dU}{dx} \delta U' dx + \\ & (e_0 a)^2 (k_1 + c) \int_0^L \frac{dU}{dx} \delta U' dx + (e_0 a)^2 k_3 \int_0^L \frac{dU^3}{dx} \delta U' dx - \int_0^L F(x, \omega) \delta U dx - \\ & \left. (e_0 a)^2 \int_0^L \frac{dF(x, \omega)}{dx} \delta U' dx = 0 \right\} \quad (14) \end{aligned}$$

But the displacement field of the spectral element component $U_j(x, \omega_j)$ is given as:

$$U_j(x, \omega_j) = N_j(x, \omega_j) d_j \quad \text{for } (j = 0, 1, 2, \dots, N-1) \quad (15)$$

where $N_j(x, \omega_j)$ is the dynamic shape function and d_j is the nodal DOFs vector; then applying equation (15) to (14) gives:

$$\left\{ \begin{aligned} & \left((EI + h) \int_0^L N''^T N'' dx - (V^2 m_f - \Pi_0 - EA\alpha_x \Delta T - \eta AH_x^2 - P_f) \int_0^L N'^T N' dx + \right. \\ & i\omega V m_f \int_0^L (N^T N' - N'^T N) dx - \omega^2 (m_p + m_f) \int_0^L N^T N dx + (k_1 + c) \int_0^L N^T N dx + \\ & k_3 \int_0^L N^T N^3 dx - (e_0 a)^2 (V^2 m_f - \Pi_0 - EA\alpha_x \Delta T - \eta AH_x^2 - P_f) \int_0^L N''^T N'' dx + \\ & 2i\omega V m_f (e_0 a)^2 \int_0^L N'^T N'' dx - \omega^2 (e_0 a)^2 (m_p + m_f) \int_0^L N'^T N' dx + \\ & (e_0 a)^2 (k_1 + c) \int_0^L N'^T N' dx + (e_0 a)^2 k_3 \int_0^L N'^T N'^3 dx \Big) d = \int_0^L N^T F dx + \\ & (e_0 a)^2 \int_0^L N'^T F' dx \end{aligned} \right\} \quad (16)$$

Therefore, the spectral element equation for the system of double-walled nano-pipe for a water supply system is derived from equation (16) as:

$$S(\omega) d = F(\omega) \quad (17)$$

where

$$\left\{ \begin{aligned} S(\omega) = & \left((EI + h) \int_0^L N''^T N'' dx - \left(V^2 m_f - \Pi_0 - EA\alpha_x \Delta T - \eta AH_x^2 - P_f \right) \int_0^L N'^T N' dx + \right. \\ & \omega V m_f \int_0^L (N^T N' - N'^T N) dx - \omega^2 (m_p + m_f) \int_0^L N^T N dx + \\ & (k_1 + c) \int_0^L N^T N dx + k_3 \int_0^L N^T N^3 dx - (e_0 a)^2 (V^2 m_f - \Pi_0 - \\ & EA\alpha_x \Delta T - \eta AH_x^2 - P_f) \int_0^L N''^T N'' dx + 2i\omega V m_f (e_0 a)^2 \int_0^L N'^T N'' dx - \\ & \omega^2 (e_0 a)^2 (m_p + m_f) \int_0^L N'^T N' dx + (e_0 a)^2 (k_1 + c) \int_0^L N'^T N' dx + \\ & (e_0 a)^2 k_3 \int_0^L N'^T N'^3 dx \end{aligned} \right\} \quad (18)$$

$$F(\omega) = \int_0^L N^T F dx + (e_0 a)^2 \int_0^L N'^T F' dx \quad (19)$$

Hence, the matrix $S(\omega)$ is called the system's spectral element matrix.

Thus, equation (17) becomes:

$$d = S(\omega)^{-1} F(\omega) \quad (20)$$

Then, substituting equation (19) into (14) gives the dynamic response of the spectral component as:

$$U_j(x, \omega_j) = N_j(x, \omega_j) S(\omega_j)^{-1} F(\omega_j) \quad (21)$$

Hence, substituting equation (21) into (10) gives the complete dynamic response of the system as:

$$u(x, t) = \frac{1}{N} \sum_{j=0}^{N-1} N_j(x, \omega_j) S(\omega_j)^{-1} F(\omega_r) e^{i\omega_j t} \quad (22)$$

However, the spectral element matrix in equation (18) can further

be simplified into a finite element matrix as:

$$[M] \ddot{Y}(t) + [C] \dot{Y}(t) + [K] Y(t) = 0 \quad (23)$$

where

$$[M] = -\omega^2 (m_p + m_f) \int_0^L N^T N dx - \omega^2 (m_p + m_f) (e_0 a)^2 \int_0^L N'^T N' dx \quad (24)$$

$$[C] = (i\omega V m_f) \int_0^L (N^T N' - N'^T N) dx + 2i\omega V m_f (e_0 a)^2 \int_0^L N'^T N'' dx \quad (26)$$

$$\left\{ \begin{aligned} [K] = & (EI + h) \int_0^L N''^T N'' dx - \left(V^2 m_f - \Pi_0 - EA\alpha_x \Delta T - \eta AH_x^2 - P_f \right) \int_0^L N'^T N' dx + \\ & (k_1 + c) \int_0^L N^T N dx + k_3 \int_0^L N^T N^3 dx - (e_0 a)^2 (V^2 m_f - \Pi_0 - EA\alpha_x \Delta T - \\ & \eta AH_x^2 - P_f) \int_0^L N''^T N'' dx + (e_0 a)^2 (k_1 + c) \int_0^L N'^T N' dx + (e_0 a)^2 k_3 \int_0^L N'^T N'^3 dx \end{aligned} \right\} \quad (27)$$

Therefore, after performing the differentiation and integration in equations (24) – (27) and evaluating, it gives:

$$\text{Mass Matrix } [M] = [M_1] + [M_2] \quad (28)$$

where

$$[M_1] = \frac{-\omega^2 (m_p + m_f) L}{420} \begin{bmatrix} 140 & 0 & 0 & 70 & 0 & 0 \\ 0 & 156 & 22L & 0 & 54 & -13L \\ 0 & 22L & 4L^2 & 0 & 13L & -3L^2 \\ 70 & 0 & 0 & 140 & 0 & 0 \\ 0 & 54 & 13L & 0 & 156 & -22L \\ 0 & -13L & -3L^2 & 0 & -22L & 4L^2 \end{bmatrix} \quad (29)$$

$$[M_2] = \frac{-\omega^2 (e_0 a)^2 (m_p + m_f)}{30L} \begin{bmatrix} 30 & 0 & 0 & -30 & 0 & 0 \\ 0 & 36 & 3L & 0 & -36 & 3L \\ 0 & 3L & 4L^2 & 0 & -3L & -L^2 \\ -30 & 0 & 0 & 30 & 0 & 0 \\ 0 & -36 & -3L & 0 & 36 & -3L \\ 0 & 3L & -L^2 & 0 & -3L & 4L^2 \end{bmatrix} \quad (30)$$

$$\text{Damping Matrix } [C] = [C_1] + [C_2]$$

(31)

where

$$[C_1] = \frac{(i\omega V m_f)}{30} \begin{bmatrix} 0 & 0 & 0 & 30 & 0 & 0 \\ 0 & 0 & 6L & 0 & 30 & -6L \\ 0 & -6L & 0 & 0 & 6L & -L^2 \\ -30 & 0 & 0 & 0 & 0 & 0 \\ 0 & -30 & -6L & 0 & 0 & 6L \\ 0 & 6L & L^2 & 0 & -6L & 0 \end{bmatrix}$$

$$[C_2] = \frac{2i\omega V m_f (e_0 a)^2}{2L} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & -2 \\ 0 & -2 & -L & 0 & 2 & -L \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -2 & 0 & 0 & 2 \\ 0 & 2 & L & 0 & -2 & L \end{bmatrix} \quad (32)$$

Stiffness Matrix

$$[K] = [K_1] + [K_2] + [K_3] + [K_4] + [K_5] \quad (34)$$

where

$$[K_1] = \frac{\left((EI + h) - (e_0 a)^2 (V^2 m_f - \Pi_0 - EA\alpha_x \Delta T - \eta A H_x^2 - P_f) \right)}{L^3} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 12 & 6L & 0 & -12 & 6L \\ 0 & 6L & 4L^2 & 0 & -6L & 2L^2 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -12 & -6L & 0 & 12 & -6L \\ 0 & 6L & 2L^2 & 0 & -6L & 4L^2 \end{bmatrix} \quad (35)$$

$$[K_2] = \frac{\left((e_0 a)^2 (k_1 + c) - (V^2 m_f - \Pi_0 - EA\alpha_x \Delta T - \eta A H_x^2 - P_f) \right)}{30L} \begin{bmatrix} 30 & 0 & 0 & -30 & 0 & 0 \\ 0 & 36 & 3L & 0 & -36 & 3L \\ 0 & 3L & 4L^2 & 0 & -3L & -L^2 \\ -30 & 0 & 0 & 30 & 0 & 0 \\ 0 & -36 & -3L & 0 & 36 & -3L \\ 0 & 3L & -L^2 & 0 & -3L & 4L^2 \end{bmatrix} \quad (36)$$

$$[K_3] = \frac{(k_1 + c)L}{420} \begin{bmatrix} 140 & 0 & 0 & 70 & 0 & 0 \\ 0 & 156 & 22L & 0 & 54 & -13L \\ 0 & 22L & 4L^2 & 0 & 13L & -3L^2 \\ 70 & 0 & 0 & 140 & 0 & 0 \\ 0 & 54 & 13L & 0 & 156 & -22L \\ 0 & -13L & -3L^2 & 0 & -22L & 4L^2 \end{bmatrix} \quad (37)$$

$$[K_4] = k_3 L \begin{bmatrix} \frac{1}{5} & 0 & 0 & \frac{1}{20} & 0 & 0 \\ 0 & \frac{191}{715} & \frac{7L^3}{8580} & 0 & \frac{801}{20020} & \frac{-3L^3}{8008} \\ 0 & \frac{131L}{4290} & \frac{L^4}{6435} & 0 & \frac{191L}{17160} & \frac{-L^4}{10296} \\ \frac{1}{20} & 0 & 0 & \frac{1}{5} & 0 & 0 \\ 0 & \frac{801}{20020} & \frac{3L^3}{8008} & 0 & \frac{191}{715} & \frac{-7L^3}{8580} \\ 0 & \frac{-191L}{17160} & \frac{-L^4}{10296} & 0 & \frac{-131L}{4290} & \frac{L^4}{6435} \end{bmatrix} \quad (38)$$

$$[K_5] = \frac{(e_0 a)^2 k_3}{L} \begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & \frac{87}{77} & \frac{3L^3}{1540} & 0 & \frac{-87}{77} & \frac{3L^3}{1540} \\ 0 & \frac{-197}{1540} & \frac{L^4}{462} & 0 & \frac{-397}{1540} & \frac{-L^4}{4620} \\ -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & \frac{-87}{77} & \frac{-3L^3}{1540} & 0 & \frac{87}{77} & \frac{-3L^3}{1540} \\ 0 & \frac{397}{1540} & \frac{-L^4}{4620} & 0 & \frac{197}{1540} & \frac{L^4}{462} \end{bmatrix} \quad (39)$$

Natural Frequency of the System

Analysing the natural frequency of the system, the external excitation force is assumed to be zero, that is ($F = 0$) which is called free vibration. This shows the system's intrinsic dynamics in terms of natural frequencies, damping and undamped factors and mode shapes.

Thus, considering the global equation of motion of the system as in equation (23), given as:

$$[M]\ddot{Y}(t) + [C]\dot{Y}(t) + [K]Y(t) = 0 \quad (40)$$

$$\Rightarrow \ddot{Y}(t) = -[M]^{-1}[C]\dot{Y} - [M]^{-1}[K]Y \quad (41)$$

However, to obtain the natural frequency of equation (41) in the presence of a damping term, it must be transformed to first-order state-space equations using the assumption given as:

$$x_1 = Y \text{ and } x_2 = \dot{Y} \quad (42)$$

Then, applying equation (42) to (41) gives:

$$\left. \begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -[M]^{-1}[C]x_2 - [M]^{-1}[K]x_1 \end{aligned} \right\} \quad (43)$$

Now, equation (43) is expressed in matrix form as:

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -[M]^{-1}[K] & -[M]^{-1}[C] \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \quad (44)$$

$$\Rightarrow \dot{X} = \begin{bmatrix} 0 & 1 \\ -[M]^{-1}[K] & -[M]^{-1}[C] \end{bmatrix} X \quad (45)$$

where

$$\dot{X} = \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} \text{ and } X = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \quad (46)$$

Also, equation (45) can be expressed in a more compact form as:

$$\dot{X} = AX \quad (47)$$

where

$$A = \begin{bmatrix} 0 & 1 \\ -[M]^{-1}[K] & -[M]^{-1}[C] \end{bmatrix} \quad (48)$$

Now, the standard State-space eigenvalue problem of the system in the presence of a damping factor is given as:

$$|A - \omega I| = 0 \quad (49)$$

Hence, equation (49) is the characteristic equation of the eigenvalue problem of the damped system. The square roots of the eigenvalues are the natural frequencies, and the eigenvector represents the mode shape of the system. The eigenvalues are computed using MATLAB.

RESULTS AND DISCUSSION

Dynamic Analysis of the System

The dynamic effect of linear van der Waals force, linear and nonlinear Winkler Spring, Pasternak foundation and Knudsen number are presented in Figures 4 to Figure 11

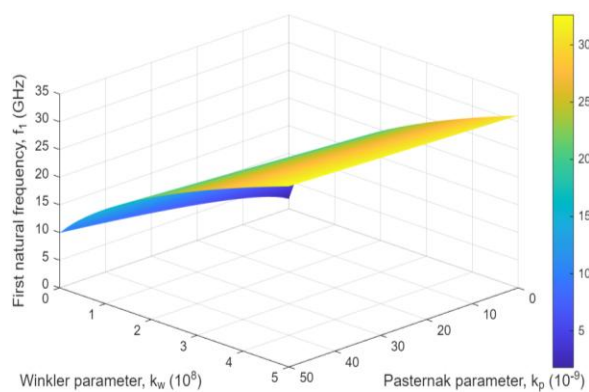


Figure 4: Effect of Pasternak and Winkler Foundation on natural frequency

Figure 4 presents the combined effect of the Pasternak foundation parameter, k_p and the Winkler foundation parameter k_w on the first natural frequency of the water-conveying double-walled nano-pipe. The results show that the natural frequency increases as foundation stiffness increases, with the highest frequencies occurring when both k_p and k_w are large. This is because stronger elastic support increases the nano-pipe's effective stiffness and, consequently, its resistance to vibration. Practically, this result matters for designing nanoscale water-transport systems where the nano-pipe may interact with a surrounding supporting material. Increasing foundation stiffness can shift the operating frequency away from potentially damaging low-frequency excitations, reducing the likelihood of excessive vibration and improving the dynamic reliability of the nano-pipe during water transport.

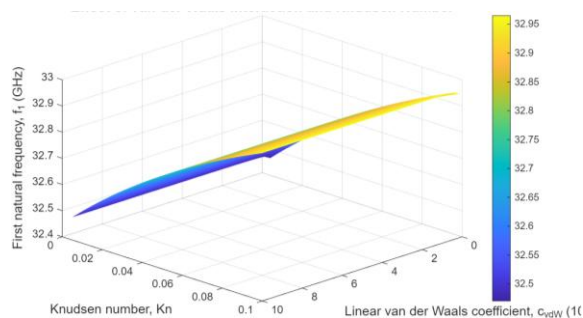


Figure 5: Effect of Van der Waals Interaction and Knudsen Number on natural frequency

Figure 5 illustrates the combined effect of the linear van der Waals coefficient C_{vdw} and Knudsen number k_n on the first natural frequency of the water-conveying double-walled nano-pipe. The surface shows a gradual increase in natural frequency, from approximately 32.45 GHz to 32.95 GHz, as both parameters increase.

The result indicates that the nano-pipe becomes slightly more resistant to vibration as the van der Waals interaction and Knudsen number increase. In practical nanoscale water-transport applications, this behaviour matters because interactions between the two nanotube walls and nanoscale fluid-flow characteristics can influence the pipe's vibration response. Proper consideration of these effects can therefore help in selecting operating conditions that minimize undesirable vibrations and enhance the dynamic reliability of nano-pipes used for water conveyance.

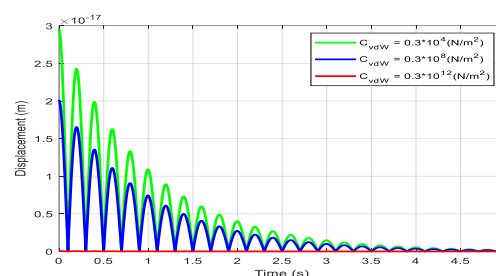


Figure 6: Effect of liner van der Waals Interaction on displacement amplitude

Figure 6 shows the effect of van der Waals interaction force on the displacement behaviour of a double-walled nano-pipe for a water supply system. The observation reveals that the displacement amplitude rapidly decreases as the van der Waals parameter increases. The oscillatory motion gradually decays with time for all values of the van der Waals parameter, indicating the presence of a damping mechanism such as energy dissipation within the system due to the combined effects of elastic foundation support, magnetic field interaction, and fluid-structure coupling. The rapid attenuation of displacement at higher van der Waals coefficients suggests that strong intertube interactions improve the nanotube's structural integrity and reduce the possibility of resonance-induced instability.

Therefore, as the stiffness increases, the nan-pipe becomes more resistant to transverse deformation and vibration, resulting in smaller displacement amplitudes and enhanced dynamic stability. This behavior is important in nano-electromechanical systems (NEMS), where enhanced stiffness and vibration suppression improve precision, durability, and operational reliability under thermal and electromagnetic conditions.

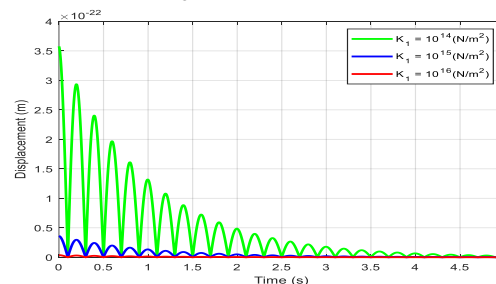


Figure 7: Effect of linear Winkler Spring on displacement amplitude

Figure 7 presents the impact of the linear Winkler spring on the vibrational amplitude of the displacement response of the water supply nano-pipe system. As the linear Winkler stiffness increases, the displacement amplitude decreases, and the nano-pipe's oscillatory motion is significantly suppressed.

Furthermore, stronger linear Winkler support enhances energy absorption and suppresses sustained vibration. With fluid transport, thermal loading, and magnetic field interaction, a larger linear foundation coefficient stabilizes the double-walled nano-pipe by reducing sensitivity to flow-induced excitation and external disturbances. Practically, this finding demonstrates that selecting appropriate Winkler foundation stiffness can improve the mechanical durability and vibration resistance of nanotube-based systems used in nano-actuators, resonators, and advanced structural applications where vibration control is essential.

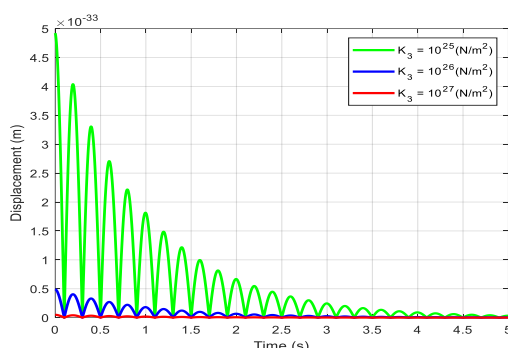


Figure 8: Effect of Nonlinear Winkler Spring on displacement amplitude

Figure 8 depicts the effects of the nonlinear Winkler spring on the displacement profile of a double-walled nano-pipe conveying water supply system. The results show that displacement amplitude decreases significantly as the nonlinear Winkler stiffness parameter increases. The pronounced decay of oscillation over time indicates that nonlinear Winkler effects provide enhanced stabilization under combined thermal, magnetic, and fluid-flow loading conditions.

Practically, in nano-engineering applications, increasing nonlinear foundation stiffness can effectively suppress vibrations, protect structures, and improve dynamic performance in double-walled nano-pipe-based nano-fluidic channels and nano-electromechanical devices.

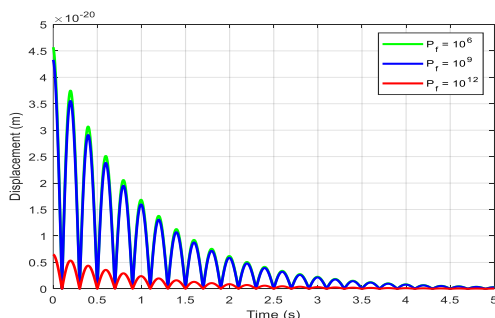


Figure 9: Effect of Pasternak foundation on displacement amplitude

Figure 9 shows the dynamic effect of the Pasternak elastic foundation on the system's displacement profile for different values. It clearly shows that a decrease in the vibrational amplitude of the displacement profile and a gradual decay of oscillations lead to an increase in the Pasternak parameter. The gradual attenuation of oscillations over time further shows that the Pasternak foundation acts as an efficient vibrational-control mechanism capable of dissipating energy transferred from internal fluid motion and environmental load. This confirms that the fluid-conveying double-walled nano-pipe remains dynamically stable for all Pasternak parameter values considered.

Physically, the Pasternak foundation introduces an additional shear interaction between the nanotube and the surrounding elastic medium, unlike conventional foundation models that account only for vertical resistance. Therefore, a nano-fluidic transport system based on a double-walled nano-pipe embedded in a stiffer Pasternak-type elastic medium is highly effective in reducing vibration, preventing flow-induced instability, and improving structural reliability and performance.

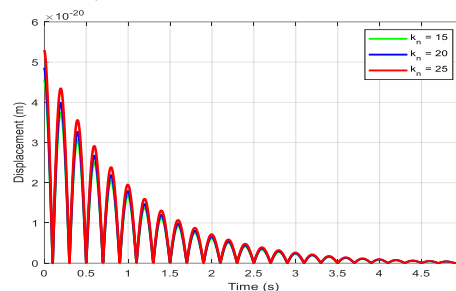


Figure 10: Effect of Knudsen number on displacement amplitude

Figure 10 depicts the time-history response of the displacement profile of a double-walled nano-pipe for different Knudsen numbers. This result indicates that increasing the Knudsen number leads to a gradual rise in the system's displacement amplitude. Despite the increase in displacement amplitude with Knudsen number, the oscillations gradually decay with time for all Knudsen numbers, indicating stable dynamic behaviour within the considered range.

These findings show that higher Knudsen numbers intensify the transient vibration response. This underscores the importance of accounting for Knudsen number effects when predicting dynamic responses, as higher Knudsen numbers can lead to larger displacements that may affect serviceability in fluid-conveying and nanoscale structural systems.

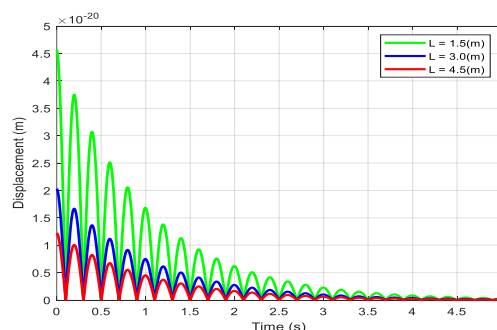


Figure 11: Effect of the Geometric Length on Displacement Profile

Figure 11 illustrates the time-history displacement response for different values of geometric length of the double-walled nano-pipe. The results show that as the geometric length of the nano-pipe increases, the vibrational amplitude of the displacement profile decreases. The oscillatory decay indicates that, despite the destabilizing influence of the flowing fluid, the system remains dynamically stable for all three geometric lengths considered. This means that increasing the nano-pipe geometric length in the presence of a Pasternak foundation improves vibration suppression and flow-induced stability. Therefore, this behaviour is desirable for nano-fluidic devices, nano-sensors and DWCNT-based nano-pipes, because reduced vibration lowers the risk of flow-induced instability, buckling or resonance, thereby enhancing the system's operational reliability and lifespan.

Conclusion

This study developed a dynamic analysis of a double-walled carbon nanotube for a water supply system under coupled fluid-flow and elastic-foundation effects. The formulation incorporates Eringen's nonlocal elasticity, linear van der Waals interaction, Knudsen-number-dependent slip effects, Pasternak elastic support, and linear and nonlinear Winkler foundation effects. The governing dynamic equation was established using Hamilton's principle and transformed into a frequency-domain equation by applying Discrete Fourier Transform (DFT) theory. Thereafter, the weighted-integral statement was applied and evaluated. Then, the Fast Fourier Transform (FFT) algorithm was used to compute the system dynamics and determine the natural frequencies and time-dependent displacement response.

The numerical results show that the stiffness of the surrounding elastic medium significantly influences the dynamic behaviour of the nano-pipe. Increasing the Pasternak and Winkler foundation parameters increases the first natural frequency and reduces the displacement amplitude, indicating improved resistance to transverse vibration. Linear and nonlinear Winkler foundations also suppress vibrations effectively, while stronger Pasternak interaction enhances nano-pipe stability by adding shear resistance from the surrounding medium.

The van der Waals interaction between the inner and outer walls also plays an important role in the dynamic response. Increasing the van der Waals interaction coefficient increases the effective interwall stiffness and reduces the displacement amplitude, thereby improving the dynamic stability of the double-walled system. Furthermore, the combined effect of the van der Waals coefficient and Knudsen number produces a gradual increase in the first natural frequency from approximately 32.45 GHz to 32.95 GHz over the investigated parameter range.

The results further show that increasing the Knudsen number also increases the displacement amplitude of the nano-pipe, reflecting the importance of nanoscale fluid-wall interaction, slip-flow and geometric length effects. Nevertheless, the oscillations progressively decay with time for all investigated Knudsen-number values, indicating stable transient behaviour within the considered parameter range.

Also, increasing the geometric length of the double-walled nano-pipe suppresses the vibrational amplitude of the system's displacement response, and the oscillatory decay implies that, despite the destabilizing influence of the flowing fluid, the system remains dynamically stable for all three geometric lengths considered. Thus, this behaviour is desirable for nano-fluidic

devices, nano-sensors, and nano-pipes based on DWCNT, because reduced vibration lowers the risk of flow-induced instability and buckling, thereby enhancing the system's operational reliability and lifespan.

Overall, the findings confirm that interwall van der Waals interaction, nanoscale fluid-flow effects and foundation stiffness must be considered in the dynamic modelling of fluid-conveying DWCNTs. Appropriate selection of foundation properties and operating conditions can control vibration, reduce the risk of flow-induced instability, and improve the reliability of nanoscale fluid-transport systems. The developed spectral-element framework therefore provides a useful computational tool for investigating coupled dynamic effects in nano-pipe systems and provides a basis for future studies involving nonlinear vibration, thermal stability, fluid-flow-induced instability and more advanced nanoscale fluid models.

Recommendations

Based on the findings of this study, the following recommendations are proposed:

1. Optimization of foundation parameters: The Pasternak and Winkler foundation parameters should be carefully optimized for specific nano-pipe applications. Excessively high foundation stiffness should not automatically be assumed optimal; instead, select an appropriate stiffness range to achieve effective vibration suppression while avoiding undesirable resonance within the operating-frequency range.
2. Improved fluid-flow modelling: Future research should incorporate temperature-dependent fluid properties, viscosity variations and more comprehensive nanoscale slip-flow models to provide a more realistic representation of water transport through nano-pipes. Molecular or atomistic approaches may also be considered where continuum assumptions become inadequate.
3. Experimental validation: Experimental investigations should be undertaken to validate the predicted natural frequencies and displacement responses. Such validation would strengthen the applicability of the proposed spectral-element formulation to practical nanoscale fluid-transport systems.

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