

A LOWER BOUND FOR MIXED INTEGER LINEAR PROGRAMMING OF SOLID WASTE ASSIGNMENT PROBLEM

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ABSTRACT

The increasing volume of municipal solid waste generated in Kaduna Metropolis, coupled with the availability of only two operational disposal sites, has resulted in inefficient waste transportation, long distances, and high transportation costs. This study aims to develop a lower bound for a Mixed Integer Linear Programming (MILP) model of the Kaduna Municipal Solid Waste Disposal Assignment Problem (KMSWDAP) using the Lagrangian Relaxation (LR) technique to simplify the optimization process and obtain a cost-effective waste transportation plan. The study formulates the practical assignment of waste from collection points to disposal sites through different conveyance types as a multi-conveyance capacitated transportation-flow model. The complicating demand constraints are relaxed using LR to generate lower bounds for the objective function. The model was implemented in AMPL and evaluated using the supplied Lagrangian subgradient procedure. After 28 subgradient iterations, the best lower bound was ₦180,783,542.83, and the best feasible upper bound was ₦225,100,000.00, giving an optimality gap of 19.69%. The resulting transportation flows, Number of trips by conveyance type, lower-bound subgradient iterations, and Convergence were subsequently analyzed. The optimized transportation plan identifies the selected waste-allocation routes from the seven collection points to the two disposal sites using three conveyance types. The computational results demonstrate how Lagrangian relaxation can provide a lower bound for the capacitated transportation-flow formulation and support route and vehicle allocation decisions. Because the run terminated at the maximum number of iterations rather than the target gap, the reported lower bound is not claimed to be the proven optimal objective value of the original MILP.

Keywords: Municipal Solid Waste Management; Kaduna Metropolis; Mixed Integer Linear Programming (MILP); Lagrangian Relaxation (LR); Capacitated Transportation-Flow Model; Waste Disposal Assignment Problem (KMSWDAP)..

INTRODUCTION

Municipal solid waste management constitutes one of the most crucial health and environmental problems facing governments in African cities (Puopiel, F., 2010). This is because even though these cities are using 20-50% of their budget for solid waste management, only 20-80% of waste is collected. The uncollected waste, or illegally dumped waste, constitutes a disaster for human health and the environment. Most cities in Nigeria practice the open dump system of waste disposal, which is in a more or less uncontrolled manner since the system is not highly engineered. It poses numerous challenges to both public health and the environment (Ajibade, 2005).

To allow timely removal and disposal of solid waste, a city is typically divided into many sectors which serve as collection points (temporary sites), and solid waste removal and disposal operations are carried out concurrently at each collection point. The waste in each collection point is transported to a government-approved disposal site, often by trucks. Solid waste ploughed from streets contains biodegradables, non-biodegradables, recyclables and inert materials. In many cities, one of the most common cost-effective methods of solid waste disposal is to dump the waste at the outskirts of the city, where the recyclable materials are separated from it, and then the remainder is burnt.

The municipal waste-loading operation must be completed within a specific time period, based on the total waste accumulation. All the waste from each collection point is transported by trucks to its assigned waste disposal site weekly. About 3 types of trucks are used in Kaduna State to haul waste to disposal sites. The three types of trucks are compactors, tippers (six tires), and tippers (four tires). The Kaduna municipal solid waste disposal assignment problem (KMSWDAP) would create a pattern to assign different types of trucks for the collection and disposal of solid waste in order to minimize the cost of transportation of solid waste, which is an important strategic problem.

Recent optimization literature further demonstrates the relevance of mathematical programming to waste transportation and routing. Komilis (2008) developed mixed-integer linear models for optimizing municipal solid waste haul and transfer costs. Sulemana et al. (2018) reviewed mathematical programming and GIS approaches for optimizing solid waste collection and transportation. Asefi et al. (2019) formulated an integrated mixed-integer framework for cost-effective solid waste management with heterogeneous fleet considerations. More recently, Hess et al. (2024) surveyed waste collection routing problems and identified vehicle type, facility, demand, and sustainability characteristics as important modelling features. These studies support the use of optimization for waste transportation while also showing that the present study's focus on lower-bound generation through Lagrangian relaxation provides a distinct methodological emphasis.

THE MATHEMATICAL MODEL

Kaduna metropolis is divided into seven sectors to make the collection and removal of solid waste easier and more efficient. Each sector is labeled from C1-C7. The city has only two dump sites located in the north and south, represented by D1 and D2, as shown in Tables 1 and 2, and three conveyance types are used in transporting solid waste from collection to disposal sites, i.e., compactors, tipper (six-tire), and tipper (four-tire).

Table 1: Solid waste collection sectors

C1	Central Market Kaduna State Secretariat Magajin gari Lagos street Kano road Leventis roundabout Ahmadu Bello Way
C2	Rigasa Panteka Unguwan Kaji Barakallahu Mando
C3	Sabon Tasha Kakuri (industrial area) Television Garage Sabo Garage Narayi junction Tirkania Command junction Makera
C4	Kawo Badarawa Unguwan Dosa Malali Unguwan Sarki Unguwan Shanu Kabala
C5	Unguwan rimi Gonin Gora Barnawa Extension Millennium City Unguwan yelwa
C6	Tudun wada Nasarawa Unguwan sunusi Kurmin Mashi Rigasa (partially) Tudun nupawa Sabon Gari
C7	Badiko Unguwan Mu'azu Kabala Unguwan Rimi (partially)

Table 2: Solid waste disposal sites

D1- Disposal site 1	Northern (Mai Adua Site)
D2- Disposal site 2	Southern (Narayi)

Mathematical Model proposed for KMSWDAP

In this study, the term "assignment problem" refers to the practical decision of allocating waste flows from collection points to disposal sites through specific conveyance types. It is not intended to imply the classical one-to-one assignment problem, in which binary variables assign each origin to a single destination. In the present formulation, the decision variable represents the quantity of waste transported from a collection point to a disposal site using a

conveyance type, and is continuous/non-negative. Consequently, the mathematical structure of KMSWDAP is more accurately characterized as a multi-conveyance capacitated transportation-flow model, with route costs, waste volumes, disposal demands, route capacities, and distances. The model will be formulated in consideration of the waste flow in Kaduna State. A schematic diagram of the waste flow through the different facilities of the proposed KMSWDAP in Kaduna State is shown in Figure1. The residential, commercial and municipal service sources will dispose of their waste in the collection bins located in front of them. Wheelbarrows are responsible for transferring the waste from the waste bins to the collection points. Assigned trucks will be responsible for waste transfer from the collection points to the disposal sites.



Figure 1: Schematic diagram of solid waste flow

Mathematical Formulation

Notations

Sets

i-set of collection points (C1,C2,C3,C4,C5,C6,C7)

j-set of disposals (Disposal site 1, Disposal site2)

k-set of conveyance types (Compactor, tipper (six-tire), tipper (four-tire))

Parameters

$C_{ik} \geq 0$: Volume of solid waste available at collection point i for conveyance k

$D_{jk} \geq 0$: Volume of solid waste required at disposal j for conveyance k

$T_{ijk} \geq 0$: Transport cost per collection of solid waste from collection point i to disposal j using conveyance k

$A_{ijk} \geq 0$: Maximum volume allowed to be transported over nodes (i,j) by conveyance k

d_{ij} : Distance between collection point i and disposal site j

d_{max} : Maximum allowed transportation distance.

Decision Variables

$E_{ijk} \geq 0$: Volume to be transported from collection point i to disposal site j using conveyance k

$N_{ijk} \geq 0$ Number of trips from collection point i to disposal sites using j conveyance k

Assumptions

1. The model assumes a linear cost structure for transportation.
2. Transported quantities E_{ijk} are measured in tons.
3. The amount of waste to be transported E_{ijk} is non-negative.
4. The collection points and disposal sites are assumed to be fully operational.
5. There is no disruption in transport routes.

The Objective function

Minimize the total transportation cost:

$$(KMSWDP) \quad W = \text{Min} \sum_{i \in \text{COLL}} \sum_{j \in \text{DIS}} \sum_{k \in \text{CON}} T_{ijk} E_{ijk} \quad (1)$$

Subject to:

$$\sum_{j \in DIS} E_{ijk} = C_{ik} \quad \forall i \in COLL, k \in CON \quad (2)$$

$$\sum_{i \in COLL} E_{ijk} = D_{jk} \quad \forall j \in DIS, k \in CON \quad (3)$$

$$\sum_{k \in CON} E_{ijk} \leq A_{ijk} N_{ijk} \quad \forall i \in COLL, j \in DIS, k \in CON \quad (4)$$

$$E_{ijk} = 0, \text{ If } d_{ij} \geq d_{max} \quad (5)$$

$$N_{ijk} = 0, \text{ If } d_{ij} \geq d_{max} \quad (6)$$

$$E_{ijk} \geq 0, N_{ijk} \in \mathbb{Z}_{\geq 0},$$

Lagrangian relaxation of the model

$$W(\lambda) = \text{Min} \sum_{i \in COLL} \sum_{j \in DIS} \sum_{k \in CON} T_{ijk} E_{ijk} + \lambda_{jk} (D_{jk} - E_{ijk}) \quad (7)$$

Subject to:

$$\sum_{j \in DIS} E_{ijk} = C_{ik} \quad \forall i \in COLL, k \in CON \quad (8)$$

$$\sum_{k \in CON} E_{ijk} \leq A_{ijk} N_{ijk} \quad \forall i \in COLL, j \in DIS, k \in CON \quad (9)$$

$$E_{ijk} = 0, \text{ If } d_{ij} \geq d_{max} \quad (10)$$

$$N_{ijk} = 0, \text{ If } d_{ij} \geq d_{max} \quad (11)$$

$$E_{ijk} \geq 0, N_{ijk} \in \mathbb{Z}_{\geq 0},$$

Subgradient Optimization of the Model

The Lagrangian dual function is

$$L(E, \lambda) = \sum_{i \in COLL} \sum_{j \in DIS} \sum_{k \in CON} T_{ijk} E_{ijk} + \lambda_{jk} (D_{jk} - E_{ijk}) \quad (12)$$

Simplify the Lagrangian dual and rearrange terms

$$L(E, \lambda) = \sum_{i \in COLL} \sum_{j \in DIS} \sum_{k \in CON} (T_{ijk} - \lambda_{jk}) E_{ijk} + \lambda_{jk} D_{jk} \quad (13)$$

The subgradient of the dual function is given by

$$g_{ij}(\lambda) = D_{jk} - \sum_{i \in COLL} E_{ijk}^* \quad (14)$$

Where

E_{ijk}^* is the optimal solution of the inner minimization

$g_{ij}(\lambda)$ = Demand – Supplied quantity

This quantity measures the difference between demand and quantity supplied through the following conditions:

1. If $g_{ij}(\lambda) > 0$, demand exceeds supply, so the multiplier should be increased.
2. If $g_{ij}(\lambda) < 0$, supply exceeds demand, so the multiplier should be reduced.
3. If $g_{ij}(\lambda) = 0$, the relaxed constraint is satisfied.

In the AMPL implementation, the initial step-size coefficient is $\Gamma_0 = 0.8$, And a reduction factor of 0.05. The code reduces Γ only when the specified reduction interval is reached.

$$\lambda_{jk}^{(t+1)} = \max\{0, \lambda_{jk}^{(t)} + \alpha_t g_{jk}^{(t)}\} \quad (15)$$

The Lagrangian multipliers are subsequently updated according to: where α_t is the step size at iteration t , Γ_t is the step-size coefficient, Best UB is the best feasible upper bound obtained so far, CurrLB_t is the current Lagrangian objective value (lower-bound candidate), and $\|g^{(t)}\|^2$ is the squared norm of the subgradient. If $\|g^{(t)}\| \leq 10^{-8}$, the step size is set to zero.

$$\alpha_t = \Gamma_t (\text{BestUB} - \text{CurrLB}_t) / \|g^{(t)}\|^2 \quad (16)$$

The computational implementation uses a Polyak-like step-size rule. Provided that $\|g^{(t)}\| > 10^{-8}$, the step size is calculated by:

$$\|g^{(t)}\|^2 = \sum_j \sum_{i \in DIS} \sum_{k \in CON} [g_{ijk}^{(t)}]^2 \quad (17)$$

The squared Euclidean norm of the subgradient is calculated as:

$$g_{jk}^{(t)} = D_{jk} - \sum_i E_{ijk}^{(t)} \quad (18)$$

At iteration t , the subgradient associated with the relaxed demand constraint is defined as:

Step-size Rule for the Subgradient Method

Subgradient iteration formula:

$$\lambda_{jk}^{(t+1)} = \lambda_{jk}^{(t)} + \alpha^{(t)} (D_{jk} - \sum_{i \in COLL} E_{ijk}^{(t)}) \quad (19)$$

Where

$\alpha^{(t)}$: Step size

t : Iteration index

Optimality Gap

For the minimization problem with optimal solution W^* , A value LB is a lower bound if $LB \leq W^*$. The research seeks to minimize total cost:

$$W = \text{Min} \sum_{i \in COLL} \sum_{j \in DIS} \sum_{k \in CON} T_{ijk} E_{ijk} \quad (20)$$

A lower bound is any value guaranteed to be less than or equal to the true minimum cost W^* .

The optimality gap is the difference between the upper bound and the lower bound, given as:

$$\text{Gap} = \frac{UB - LB}{UB} \times 100\%$$

RESULTS

Computational Results and Best Bounds

The Kaduna Municipal Solid Waste Disposal Assignment Problem, formulated as a capacitated transportation-flow model, was evaluated using the supplied AMPL Lagrangian subgradient run. The run completed 28 subgradient iterations. The best lower bound obtained was ₦180,783,542.83, while the best feasible upper bound was ₦225,100,000.00, resulting in an optimality gap of 19.69%. The run terminated because the maximum number of iterations was reached; therefore, the lower bound is not reported as the proven optimal MILP objective value. The final reported violation was 960. The corresponding transportation flows, number of trips analysis and lower bound subgradient iteration and convergence analysis are presented in the following analysis.

DISCUSSIONS

Transportation Flow Analysis

Table 2A: Final positive waste flows E (tons)

Collection point	Disposal site	Compactor	Tipper4	Tipper6	Unit
C2	D1	400	150	250	tons
C4	D1	300	100	200	tons
C6	D1	550	220	320	tons
C1	D2	500	200	300	tons
C3	D2	600	250	350	tons
C5	D2	450	180	300	tons
C7	D2	350	130	240	tons

The transportation flow results (Table 3A) reveal how waste is optimally distributed from collection points (C1 - C7) to disposal sites (D1 and D2) using different conveyance types.

A key observation is that disposal site D2 receives a significantly higher proportion of waste compared to D1. Most collection points (C1, C2, C3, C5, and C7) transport their waste exclusively to D2, while only a few routes (notably from C2, C4 and C6) utilize D1. This indicates that D2 is the preferred disposal site, likely due to lower transportation costs, shorter distances, or better handling capacity.

Furthermore, several routes show zero flow values, which implies that the optimization model has effectively eliminated inefficient or non-economical routes. This is consistent with classical transportation theory, where only routes with minimum reduced cost are utilized in the optimal solution.

Another important feature is the structured allocation of waste across conveyance types. Larger volumes are assigned to routes and vehicles that can transport them more economically, demonstrating efficient utilization of vehicle capacity. This selective routing strategy reduces redundancy and ensures that resources are not wasted on suboptimal paths.

Overall, the transportation flow results highlight the importance of route optimization and selective allocation, which significantly enhance the efficiency of municipal solid waste logistics.

Number of Trips Analysis

The number of trips required to execute the selected transportation plan provides an additional operational measure of the solution. The AMPL computational output reports positive trip counts for the selected routes between the seven collection points and the two disposal sites. These trip counts are useful for translating the optimized waste flows into a practical vehicle-dispatch plan. For a route-specific flow, the theoretical number of trips can be estimated from the transported waste volume and the allowable load per trip as

$$N_{ijk} = \frac{E_{ijk}}{A_{ijk}}$$

Where N_{ijk} is the number of trips from collection point i to disposal site j using conveyance k , E_{ijk} is the quantity of waste transported on the route, and A_{ijk} is the maximum volume transportable per trip. The ceiling operator ensures that a partial load is counted as a complete trip. The reported AMPL output, however, should remain the authoritative source for the trip counts used in the computational analysis because the implemented model generates those values.

Table 2B: Reported number of trips by selected route and conveyance type

Collection point	Disposal site	Compactor	Tipper (4 tyres)	Tipper (6 tyres)
C2	D1	8	5	5
C4	D1	4	3	3
C6	D1	4	4	3

C1	D2	10	7	8
C3	D2	11	13	12
C5	D2	5	5	5
C7	D2	9	13	8
Total		51	50	44

Source: AMPL Lagrangian subgradient computational output

The AMPL output gives 51 compactor trips, 50 four-tyre tipper trips, and 44 six-tyre tipper trips across the selected routes, giving 145 reported trips in total. The largest number of compactor trips occurs on C3–D2 (11 trips), while the largest four-tyre tipper trip count occurs on C3–D2 and C7–D2 (13 trips each). For six-tyre tippers, C3–D2 has the highest reported number at 12 trips.

The trip distribution also shows that the operational burden is concentrated on the routes selected by the optimization procedure. Routes C2–D1, C4–D1 and C6–D1 serve disposal site D1, while C1–D2, C3–D2, C5–D2 and C7–D2 serve D2. Thus, the trip analysis complements the flow analysis by indicating not only how much waste is allocated, but also the number of vehicle movements required to implement the allocation.

For operational planning, the reported trip counts can be used to estimate vehicle utilization, dispatch frequency, fuel requirements and route workload. They should not, however, be interpreted as an independent proof of optimality. They are operational outputs associated with the reported computational solution, while the Lagrangian subgradient analysis establishes the lower-bound and upper-bound information reported in the study.

Lower-Bound Subgradient Iteration and Convergence Analysis

The supplied AMPL subgradient output shows that the Lagrangian objective increased from ₦177,100,000.00 at iteration 1 to ₦180,783,542.83 at iteration 28. The best lower bound therefore improved progressively during the reported run, reaching ₦180,783,542.83 at iteration 28. The best feasible upper bound remained ₦225,100,000.00, and the optimality gap decreased from 21.32% at iteration 1 to 19.69% at iteration 28. The run terminated with “Maximum iterations reached”; hence, the target gap was not achieved. The output also reports a final constraint violation of 960 and final Lagrange multipliers for the relaxed demand constraints. The detailed AMPL iteration output is reproduced below.

Table 2C: Lagrangian subgradient iteration results from the supplied AMPL output.

Iteration	Lagrangian objective (₦)	Best LB (₦)	UB (₦)	Gap (%)	Step
1	177,100,000.00	177,100,000.00	225,100,000.00	21.32%	0.7619
2	177,308,761.90	177,308,761.90	225,100,000.00	21.23%	0.7273
3	177,508,034.63	177,508,034.63	225,100,000.00	21.14%	0.6957
4	177,698,643.33	177,698,643.33	225,100,000.00	21.06%	0.6667
5	177,881,309.99	177,881,309.99	225,100,000.00	20.98%	0.6400
6	178,056,669.99	178,056,669.99	225,100,000.00	20.90%	0.6154
7	178,225,285.38	178,225,285.38	225,100,000.00	20.82%	0.5926
8	178,387,655.75	178,387,655.75	225,100,000.00	20.75%	0.5714
9	178,544,227.18	178,544,227.18	225,100,000.00	20.68%	0.5517
10	178,695,399.59	178,695,399.59	225,100,000.00	20.62%	0.5333
11	178,841,532.93	178,841,532.93	225,100,000.00	20.55%	0.5161
12	178,982,952.28	178,982,952.28	225,100,000.00	20.49%	0.5000

13	179,119,952.28	179,119,952.28	225,100,000.00	20.43%	0.4848
14	179,252,800.76	179,252,800.76	225,100,000.00	20.37%	0.4706
15	179,381,741.94	179,381,741.94	225,100,000.00	20.31%	0.4571
16	179,506,999.08	179,506,999.08	225,100,000.00	20.25%	0.4444
17	179,628,776.86	179,628,776.86	225,100,000.00	20.20%	0.4324
18	179,747,263.35	179,747,263.35	225,100,000.00	20.15%	0.4211
19	179,862,631.77	179,862,631.77	225,100,000.00	20.10%	0.4103
20	179,975,042.03	179,975,042.03	225,100,000.00	20.05%	0.4000
21	180,084,642.03	180,084,642.03	225,100,000.00	20.00%	0.3902
22	180,191,568.86	180,191,568.86	225,100,000.00	19.95%	0.3810
23	180,295,949.81	180,295,949.81	225,100,000.00	19.90%	0.3721
24	180,397,903.30	180,397,903.30	225,100,000.00	19.86%	0.3636
25	180,497,539.66	180,497,539.66	225,100,000.00	19.81%	0.3556
26	180,594,961.88	180,594,961.88	225,100,000.00	19.77%	0.3478
27	180,690,266.23	180,690,266.23	225,100,000.00	19.73%	0.3404
28	180,783,542.83	180,783,542.83	225,100,000.00	19.69%	0.3333

The AMPL iteration record shows a progressive improvement in the best lower bound from ₦177,100,000.00 at iteration 1 to ₦180,783,542.83 at iteration 28. The upper bound remained ₦225,100,000.00 throughout the reported iterations, while the gap decreased from 21.32% to 19.69%. The step size also changed across iterations, as shown in Table 4. The run stopped at the maximum of 28 iterations, so convergence to the specified target gap was not established.

Conclusion

The management of municipal solid waste remains one of the pressing challenges facing rapidly growing urban centres, particularly where increasing waste generation places pressure on existing disposal infrastructure. This study formulated the Kaduna Municipal Solid Waste Disposal Assignment Problem as a multi-conveyance capacitated transportation-flow model and applied Lagrangian relaxation with a subgradient procedure to generate lower bounds for the transportation-cost objective. The supplied AMPL computational run produced a best lower bound of ₦180,783,542.83 and a best feasible upper bound of ₦225,100,000.00 after 28 iterations, corresponding to a 19.69% optimality gap.

The computational results show that the model can identify feasible waste-allocation routes and conveyance assignments while providing quantitative bounds on the objective. The Lagrangian subgradient sequence improved the lower bound across the 28 reported iterations, although the target gap was not reached before termination. The selected flows show that waste is distributed between the two disposal sites and three conveyance types, with several routes receiving zero flow. These results illustrate the usefulness of Lagrangian relaxation as a bounding and decision-support approach for the capacitated transportation-flow formulation.

The study also demonstrates the relevance of operations research to public-sector waste management by converting route, conveyance, demand, capacity, and distance information into a structured optimization model. The AMPL subgradient implementation provides a reproducible computational procedure for evaluating the Lagrangian relaxation and monitoring lower-bound improvement and the optimality gap.

Baseline comparison: the present computational data set provides route costs, waste volumes, and the optimized solution, but it does not provide a documented pre-optimization total transportation cost for the existing operational allocation. Therefore, this study does

not claim a percentage cost saving relative to the current system. The reported lower bound, upper bound, and optimality gap are instead used to assess the performance of the Lagrangian relaxation procedure. A future field study can establish a baseline by recording the actual current routes, loads and transportation expenditure and comparing them with the optimized plan

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APPENDIX A: COMPUTATIONAL INPUT DATA

Table A1. Transportation cost by route and conveyance type

Collection point	Disposal site	Conveyance	Cost (₦)
C1	D1	Compactor	40,000

C1	D1	Tipper (six tyres)	30,000
C1	D1	Tipper (four tyres)	20,000
C1	D2	Compactor	35,000
C1	D2	Tipper (six tyres)	25,000
C1	D2	Tipper (four tyres)	15,000
C2	D1	Compactor	40,000
C2	D1	Tipper (six tyres)	30,000
C2	D1	Tipper (four tyres)	20,000
C2	D2	Compactor	45,000
C2	D2	Tipper (six tyres)	35,000
C2	D2	Tipper (four tyres)	25,000
C3	D1	Compactor	50,000
C3	D1	Tipper (six tyres)	35,000
C3	D1	Tipper (four tyres)	20,000
C3	D2	Compactor	40,000
C3	D2	Tipper (six tyres)	30,000
C3	D2	Tipper (four tyres)	15,000
C4	D1	Compactor	30,000
C4	D1	Tipper (six tyres)	15,000
C4	D1	Tipper (four tyres)	10,000
C4	D2	Compactor	40,000
C4	D2	Tipper (six tyres)	20,000
C4	D2	Tipper (four tyres)	15,000
C5	D1	Compactor	45,000
C5	D1	Tipper (six tyres)	35,000
C5	D1	Tipper (four tyres)	20,000
C5	D2	Compactor	40,000
C5	D2	Tipper (six tyres)	30,000
C5	D2	Tipper (four tyres)	15,000
C6	D1	Compactor	40,000
C6	D1	Tipper (six tyres)	25,000
C6	D1	Tipper (four tyres)	15,000
C6	D2	Compactor	30,000
C6	D2	Tipper (six tyres)	20,000
C6	D2	Tipper (four tyres)	10,000
C7	D1	Compactor	40,000
C7	D1	Tipper (six tyres)	35,000
C7	D1	Tipper (four tyres)	20,000
C7	D2	Compactor	30,000
C7	D2	Tipper (six tyres)	25,000
C7	D2	Tipper (four tyres)	10,000

Source: KEPA (2022), as reported in the study data.

Table A2. Volume of solid waste at collection points (tons)

Collection point	Compactor	Tipper (six tyres)	Tipper (four tyres)
C1	500	300	200
C2	400	250	150
C3	600	350	250
C4	300	200	100
C5	450	300	180
C6	550	320	220
C7	350	240	130

Source: KEPA (2022), as reported in the study data.

Table A3. Maximum volume transportable per trip (tons/trip)

Collection point	Disposal site	Compactor	Tipper (six tyres)	Tipper (four tyres)
C1	D1	70	60	40
C1	D2	50	40	30

C2	D1	55	60	30
C2	D2	70	50	25
C3	D1	60	45	25
C3	D2	55	30	20
C4	D1	90	70	45
C4	D2	80	50	30
C5	D1	120	80	50
C5	D2	100	60	40
C6	D1	150	110	70
C6	D2	130	100	40
C7	D1	60	40	20
C7	D2	40	30	10

Source: KEPA (2022), as reported in the study data.

Table A4. Demand at disposal sites by conveyance type (tons)

Disposal site	Compactor	Tipper (six tyres)	Tipper (four tyres)
D1	1200	800	500
D2	1500	900	600

Source: KEPA (2022), as reported in the study data.

Table A5. Distance from collection points to disposal sites (km)

Collection point	D1	D2
C1	8	6
C2	5	12
C3	15	3
C4	7	10
C5	10	8
C6	6	13
C7	12	5

Source: KEPA (2022), as reported in the study data.

APPENDIX B: AMPL LAGRANGIAN SUBGRADIENT COMPUTATIONAL OUTPUT

The following computational information is incorporated directly from the supplied AMPL subgradient run. The run was invoked with the model and data files followed by the subgradient run script.

ampl: model lagrangian.mod;
ampl: data KMSWDAP.dat;
ampl: include subgradient-3.run;

B1. Final computational summary

Item	AMPL result
Iterations completed	28
Best lower bound	₦180,783,542.83
Best feasible upper bound	₦225,100,000.00
Final optimality gap	19.69%
Final violation	960.0000
Termination status	MAXIMUM ITERATIONS REACHED

B2. Final Lagrangian multipliers

Conveyance	D1	D2
COMPACTOR	-688.846	-5510.77
TIPPER4	413.308	-2204.31
TIPPER6	413.308	-3995.31