FULL LENGTH RESEARCH ARTICLE

THE APPLICATION OF THE SPLIT-PLOT DESIGN IN THE ANALYSIS OF EXPERIMENTAL RABBIT FEEDS DATA

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ABSTRACT

The Split-plot design model was used to analyze rabbit feeds data obtained from the Department of Agricultural Science, Federal College of Education Pankshin in order to determine whether there is significant variation in the categories of feeds given. The result shows that there was no significant effect in the different categories of feed used in feeding the rabbits. A significant difference was observed between the quantity of feeds given to the rabbits and the feed conversion efficiency on the rabbits. It determined that there was no significant interaction between factor A (feeds) and factor B (application method) of the analysis.
Key words: Experimental Units Factors, Levels, Randomized Block Design, Split-plot Design, Subplots

INTRODUCTION

Usually the split-plot design is an analysis of variance technique where the levels of one factor are assigned at random to large experimental units within blocks of such units. The large units are then divided into smaller units, and the levels of the second factor are assigned at random to the small units within the larger units. In the terminology of agricultural research, where these designs were developed, the large units are called whole plots or main plots, while the small units are called split-plots or subplots (Petersen 1985; Montgomery 1991).

According to (Montgomery 1991), the linear statistical model for the split-plot design is given by
$$Y_{ijk} = \mu + \tau_i + \beta_j + (\tau\beta)_{ij} + \gamma_k + (\tau\gamma)_{ik} + (\beta\gamma)_{jk} + (\tau\beta\gamma)_{ijk}$$
, for i =1,2,..., a;
j = 1,2,..., b and k = 1,2,..., c, where τ_i, β_j and $(\tau\beta)_{ij}$ represent the whole plot and correspond respectively to blocks (factor A), main treatments (factor B), and whole plot error (AB); and $\gamma_k, (\tau\gamma)_{ik}, (\beta\gamma)_{jk}$, and $(\tau\beta\gamma)_{ijk}$ represent the subplot and correspond respectively to the subplot treatment (factor C), the AC and BC interactions, and the subplot error.

In the split-plot design the whole-plot factor effects are estimated from the large units, while the subplot effects and the interaction of the whole-plot and subplot factors are estimated from the small units. In view of the fact that there are two sizes of unit, there are two experimental errors, one for each type of unit. Generally the error associated with the subplots is smaller than that for the whole plots.

The reasons for this could be that small units within the large units tend to be positively correlated. This has the effect of reducing experimental error. Error degrees of freedom for the whole plots are usually less than those for the subplots. This has the effect of increasing the whole-plot error relative to that of the subplots (Satterthwaite 2000).

MATERIALS AND METHODS

According to Norman (1961), the split-plot design has a number of advantages and a few disadvantages. Some of the advantages include the following:

1. It permits the efficient use of some factors, which require large experimental units in combination with other factors, which require small experimental units.
2. It provides increased precision in the comparison of some factors.
3. It permits the introduction of new treatments into an experiment, which is already in progress.

While some of the disadvantages of the split-plot design include the following:

1. Statistical analysis is complicated because different comparisons have different error variances.
2. Low precision on the whole plots can result in large differences being non significant, while small differences on the subplots may be statistically significant even though they are of no practical significance.

The total corrected sum of squares for obtaining the various sum of squares for the split-plot design were obtained from
$$\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n}\left(y_{ijk}-\bar{y}_{...}\right)^2 = \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n}\left[\left(\bar{y}_{i..}-\bar{y}_{...}\right)+\left(\bar{y}_{.j.}-\bar{y}_{...}\right)+\left(\bar{y}_{ij.}-\bar{y}_{i..}-\bar{y}_{.j.}+\bar{y}_{...}\right)+\left(y_{ijk}-\bar{y}_{ij.}\right)\right]^2$$
$$= bn\sum_{i=1}^{a}\left(\bar{y}_{i..}-\bar{y}_{...}\right)^2 + an\sum_{j=1}^{b}\left(\bar{y}_{.j.}-\bar{y}_{...}\right)^2 + n\sum_{i=1}^{a}\sum_{j=1}^{b}\left(\bar{y}_{ij.}-\bar{y}_{i..}-\bar{y}_{.j.}+\bar{y}_{...}\right)^2 + \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n}\left(y_{ijk}-\bar{y}_{ij.}\right)^2$$

Since the six cross products on the right-hand side are zero. Notice that the total sum of squares has been partitioned into a sum of squares due to "rows" or factor A (SSA); a sum of squares due to "columns" or factor B (SSA); a sum of squares due to the interaction between A and B (SSAB); and a sum of squares due to error, (SSE). By extending the total corrected sum of squares above to the sum of squares in the split-plot design; the sum of squares in the split-plot design may be computed symbolically as

1. Correction term, $C = \frac{G^2}{rab}$
2. $SS_{Tot} = \sum_i \sum_j \sum_k y_{ijk}^2 - C$
3. $SS_R = \left(\frac{1}{ab}\right) \sum_i R_i^2 - C$
4. $SS_A = \left(\frac{1}{rb}\right) \sum_j A_j^2 - C$
5. $SS_{EA} = \left(\frac{1}{b}\right) \sum_i \sum_j y_{ij.}^2 - C - SS_R - SS_A$
6. $SS_B = \left(\frac{1}{ra}\right) \sum_k B_k^2 - C$
7. $SS_{AB} = \left(\frac{1}{r}\right) \sum_j \sum_k T_{.jk}^2 - C - SS_A - SS_B$
8. $SS_{EAB} = SS_{Tot} - SS_R - SS_A - SS_{EA} - SS_B - SS_{AB}$

Mean squares are computed by dividing the sums of squares by their associated degrees of freedom. The F ratios are computed somewhat differently, however, since there are two errors in a split – plot design. The proper ratios are shown in the ANOVA table, in Table 1. We have

$$F_R = \frac{MS_R}{MS_{EA}}$$

$$F_A = \frac{MS_A}{MS_{EA}}$$

$$F_B = \frac{MS_B}{MSE_{AB}}$$ and

$$F_{AB} = \frac{MS_{AB}}{MSE_{AB}}$$

The significance tests for the various factors in the split-plot design are given as follows:

1. F_A , with (a –1) and a(r-1)(a –1) degrees of freedom (d.f.), is used to test the significance of differences among A factor means (main effect of factor A).
2. F_B, with (b –1) and a(r –1)(b –1) d.f., is a test statistics for the significance of the main effect of factor B.
3. F_{AB}, with (a –1)(b –1) and a(r –1)(b –1) d.f., provides a test of the significance of the A X B interaction.
4. F_R, with (r – 1) and (r –1)(a –1) d.f., provides an approximate test of the effectiveness of blocking in reducing the whole-plot error.

ILLUSTRATION

For the data used in this work, the experiment involved ten replicates with the whole plots in a randomized block design. The data for this work was obtained from the department of Agricultural Science, Federal College of Education Pankshin, Nigeria. The data was generated from an experiment that run for ten weeks, in order to determine the feed conversion efficiency for rabbits based on the type of feed given to the rabbits; the quantity given to the rabbits as well as the quantity consumed by the rabbits. The quantity of feed was measured as well as the feed conversion efficiency of the rabbits which resulted in the data that form the basis for this analysis in a split-plot design. The data used for this analysis was run for ten weeks, thereby representing the number of replicates in a randomized block design where the weeks are represented by the blocks in the design. The feeds are in four categories T_1, T_2, T_3 and T_4 where T_1 represent pure industrial feed, T_2 represent industrial feed mixed with farm formulated feed of equal proportion, T_3 represent farm formulated feed mixed with industrial feed where farm formulated feed is in a larger proportion than the industrial feed, and T_4 represent pure farm formulated feed. The quantity given to the rabbits for each replicate was in kilogram and the feed conversion efficiency data was also given in kilogram. The data used for this analysis is given in Tables 2, 3 and 4.

In this design the feeds represent the whole plots and the application method, which constitutes the quantity of the feed as well as the feed conversion efficiency of the rabbits, represents the subplots within the whole plots. Here the whole plots represent factor A and subplots represents factor B. Since the experiment was run in a randomized block design, the weeks represent the blocks. Furthermore, factor A levels are termed feed category, while factor B levels are termed application methods. The format for the analysis of variance of a split-plot design is illustrated in Table 1.

TABLE 1. ANOVA TABLE OF A SPLIT-PLOT DESIGN

Source	Degree of freedom	Sum of squares	Mean square	F
Total	rab-1	SS_{Total}		
Block	r-1	SS_R	MS_R	F_R
Factor A	a-1	SSA	MS_A	F_A
Error (A)	(r-1)(a-1)	SS_{EA}	MS_{EA}	
Factor B	b-1	SS_B	MS_B	F_B
AB Interaction	(a-1)(b-1)	SS_{AB}	MS_{AB}	F_{AB}
Error (AB)	A(r-1)(b-1)	SS_{EAB}	MS_{EAB}	

TABLE 2: WEEKLY FEED CONVERSION EFFICIENCY

Week	T1			T2			T3			T4		
	Av intake	Av Gained	FCE	Av intake	Av gained	FCE	Av intake	Av gained	FCE	Av intake	Av gained	FCE
1	0.38	-	0.38	0.42	-	0.42	0.44	-	0.44	0.40	-	0.40
2	0.49	0.05	9.8	0.47	0.05	9.4	0.25	0.05	5	0.44	0	0.44
3	0.5	0.1	5	0.50	0.1	5	0.5	0.05	10	0.5	0	0.5
4	0.59	0.3	1.96	0.53	0.3	1.76	0.57	0.35	1.65	0.55	0.35	1.57
5	0.62	0.4	1.55	0.57	0.1	5.7	0.61	0.10	6.1	0.6	0.05	12
6	0.63	0.15	4.2	0.6	0.2	3	0.63	0.15	4.2	0.63	0.1	6.3
7	0.71	0.30	2.36	0.7	0.25	2.8	0.65	0.2	3.25	0.63	0.15	4.2
8	0.73	0.30	2.43	0.73	0.15	4.86	0.69	0.25	2.76	0.70	0.25	2.8
9	0.75	0.15	5	0.75	0.33	2.27	0.71	0.25	2.84	0.55	0.05	11
10	0.75	0.2	3.75	0.75	0.20	0.75	0.75	0.25	3.00	0.75	0.35	2.14
Total			36.43						39.24			41.35

TABLE 3: WEEKLY AVERAGE FEED INTAKE

Week	T_1			T_2			T_3			T_4		
	Feed supply	Feed intake	Aver. Feed intake	Feed supply	Feed intake	Aver. Feed intake	Feed supply	Feed intake	Aver. Feed intake	Feed supply	Feed intake	Aver. Feed intake
1	1	0.75	0.38	1	0.84	0.42	1	0.87	0.44	1	0.79	0.40
2	1	0.96	0.49	1	0.93	0.47	1	0.5	0.25	1	0.88	0.44
3	1	1	0.5	1	0.99	0.49	1	1	0.5	1	1	0.5
4	1.25	1.17	0.59	1.25	1.06	0.53	1.25	1.14	0.57	1.25	1.1	0.55
5	1.25	1.23	0.62	1.25	1.14	0.57	1.25	1.21	0.61	1.25	1.2	0.6
6	1.25	1.25	0.63	1.25	1.2	0.6	1.25	1.35	0.63	1.25	1.25	0.63
7	1.5	1.42	0.71	1.5	1.4	0.7	1.5	1.3	0.65	1.5	1.25	0.63
8	1.5	1.45	0.73	1.5	1.46	0.73	1.5	1.38	0.69	1.5	1.39	0.70
9	1.5	1.5	0.75	1.5	1.49	0.75	1.5	1.45	0.71	1.5	1.1	0.55
10	1.5	1.5	0.75	1.5	1.5	0.75	1.5	1.5	0.75	1.5	1.5	0.75
Total	13.00	-	-	13.00	-	-	13.00	-	-	13.00	-	-

TABLE 4: WEEKLY AVERAGE WEIGHT GAINED IN KILOGRAM

Week	T_1		T_2		T_3		T_4	
	Weight	Average Gained	Weight	Average Gained	Weight	Average Gained	Weight	Average Gained
1	1.05	-	1.05	-	1.0	-	1.1	-
2	1.1	0.05	1.1	0.05	1.05	0.05	1.1	0
3	1.2	0.1	1.2	0.1	1.1	0.05	1.1	0
4	1.5	0.3	1.5	0.3	1.45	0.35	1.45	0.35
5	1.9	0.4	1.6	0.1	1.55	0.10	1.5	0.05
6	2.05	0.15	1.8	0.2	1.7	0.15	1.6	0.1
7	2.35	0.30	2.05	0.25	1.9	0.2	1.75	0.15
8	2.65	0.30	2.2	0.15	2.15	0.25	2.0	0.25
9	2.8	0.15	2.35	0.33	2.4	0.25	2.05	0.05
10	3.0	0.2	2.55	0.20	2.65	0.25	2.4	0.35
Total	19.6	1.65	17.4	1.68	16.95	1.65	16.05	1.3

TABLE 5. BLOCK X FEED TOTALS

Block	T1	T2	T3	T4	Sum
1	1.38	1.42	1.44	1.4	5.64
2	1.49	1.47	1.25	1.44	5.65
3	1.5	1.49	1.5	1.5	5.99
4	1.84	1.78	1.82	1.8	7.24
5	1.87	1.82	1.86	1.85	7.4
6	1.88	1.85	1.88	1.88	7.49
7	2.21	2.2	2.15	1.88	8.44
8	2.23	2.23	2.19	2.2	8.85
9	2.25	2.25	2.21	2.05	8.76
10	2.25	2.25	2.25	2.25	9
Sum	18.9	18.76	18.55	18.25	74.46

TABLE 6. FEED X APPLICATION METHOD

	Feed				
Application Method	T1	T2	T3	T4	Sum
S1	12.75	12.75	12.75	12.5	50.75
S2	6.15	6.01	5.8	5.75	23.71
Sum	18.96	18.76	18.55	18.25	74.46

As usual, the feeds are considered to be factor A and the categories T_1, T_2, T_3 and T_4 are levels of factor A which all constitute the whole plots. Similarly, the sub-plots are considered to be application method, which is termed factor B with levels given as quantity of feed and feed conversion efficiency denoted by S_1 and S_2 respectively. The summary tables for the totals needed to compute the analysis of variance are given as Tables 5 and 6 respectively. The analysis of variance of the data is presented in Table 7.

The sum of squares for the analysis of variance may be computed as follows:

From equation 2 the sum of squares totals is computed as

$$SS_{Totals} = \sum_i \sum_j \sum_k y_{ijk}^2 - \frac{G^2}{rab}$$

$$= (1)^2 + (0.38)^2 + \ldots + (0.75)^2 - \frac{(74.46)^2}{10 \times 4 \times 2}$$

$= 80.7196 - 69.303645$
$= 11.42$

From equation 3 the sum of squares for blocks (weeks) is computed as

$$SS_R \text{ (Blocks)} = \frac{\sum R_i^2}{ab} - \frac{G^2}{rab}$$

$$= \frac{(5.64)^2 + \ldots + (9)^2}{4 \times 2} - \frac{(74.46)^2}{10 \times 4 \times 2}$$

$= 1.97$

The sum of squares for factor A (feeds) is computed from equation 4 as

$$SS_A \text{ (Feeds)} = \frac{\sum A_j^2}{rb} - \frac{G^2}{rab}$$

$$= \frac{(18.9)^2 + \ldots + (18.25)^2}{10 \times 2} - \frac{(74.46)^2}{10 \times 4 \times 2}$$

$= 0.011985$

The sum of square error for factor A is computed from equation 5 as

$$SS_{EA} = \frac{\sum_i \sum_j y_{ij}^2}{b} - \frac{G^2}{rab} - SS_R - SS_A$$

$$= \frac{(1.38)^2 + \ldots + (2.25)^2}{2} - \frac{(74.46)^2}{10 \times 4 \times 2} - 1.96 - 0.012$$

$= 0.059865$

The sum of squares for factor B (application method) is computed from equation 6 as

$$SS_B \text{(Application Method)} = \frac{\sum B_k^2}{ra} - \frac{G^2}{rab}$$

$$= \frac{(50.75)^2 + (23.71)^2}{10 \times 4} - \frac{(74.46)^2}{10 \times 4 \times 2}$$

$= 9.13952$

The sum of square interaction between factors A and B is computed from equation 7 as

$$SS_{AB} \text{ (Feed X Application)} = \frac{\sum_j \sum_k T_{.jk}}{r} - \frac{G^2}{rab} - SS_A - SS_B$$

$$= \frac{(12.75)^2 + \ldots + (5.75)^2}{10} - \frac{(74.46)^2}{10 \times 4 \times 2} - 0.01198 - 9.13952$$

$= 0.23622$

The sum of square error for interaction between factors A and B is obtained by subtraction from equation 8 as

$SS_{EAB} = SS_{Total} - SS_R - SS_A - SS_{EA} - SS_B - SS_{AB}$
$= 11.42 - 1.97 - 0.011985 - 0.059865 - 9.13952 - 0.00311$
$= 0.23622$

TABLE 7. ANOVA OF RABBIT FEED DATA

Source	Degree of freedom	Sum of squares	Mean square	F
Total	79	11.42		
Week (Block)	9	1.97	0.21881	1.8018
Feed (A)	3	0.012	0.004	
Error (A)	27	0.06	0.0022	1393.2
Appl. Method	1	9.14	9.14	0.158
Feed X Application method	3	0.003	0.001	
Error (AB)	36	0.236	0.00656	

The various sum of squares for all the components involved in the split-plot design as calculated above are fixed in the ANOVA Table 7.

CONCLUSION

From the analysis of variance table 4 one could deduce that at 5% and 1% significant levels, no significant effect in the different categories of feed used for feeding the rabbits. Also at the 5% and 1% significant levels, there is significant difference between the levels of factor B that is between the quantity of feed given to the rabbits and their feed conversion efficiency. Finally, at the 5% and 1% levels of significant, there is no significant interaction between factor A and factor B, that is there is no significant relationship between the categories of feeds given to the rabbits and the quantity as well as the feed conversion efficiency of the rabbits.

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