SHORT COMMUNICATION

THE GEOMETRIC MEAN MODEL IN FINANCE

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The financial manager is increasingly involved in global product and financial markets. The globalization movement means that investment and financing decisions must be made in an international arena. International capital budgeting for example, embraces estimates of future rates of exchange between two or more currencies (James & John, 2005).

The geometric and Harmonic means are not commonly used in advanced statistical analysis because they are less tractable and do not conform to some sampling properties. In most advanced statistical work the arithmetic mean is more important because of its superior mathematical tractability and its application in sampling theory. However, Zameeruddin *et al.,* (1980) observed that in global banking, more efficient and consistent exchange rate average computation requires applications of the geometric mean. The reciprocity property of the geometric mean makes it superior to arithmetic and harmonic means in finance. The basic reciprocity property of a mean is discussed with some applications relating to the computation of the average exchange rate of a currency over a given period and an effective exchange rate index for a currency relative to a given referenced period.

Formulation and derivation of reciprocity property

The reciprocity property desired of a mean for exchange rate computation may be stated as follows. If one unit of a currency B equals on average x units of currency N, then one unit of currency N should equal on average $1/x$ units of currency B. According to Wentzel & Ovcharov, (1986), if $x_1, x_2, \ldots, x_n$ are positive real numbers, with associated weights (probabilities) $w_1, w_2, \ldots, w_n$ then

$$\sum_{i=1}^{n} w_i = 1. \qquad \ldots \qquad (1)$$

Suppose the mean of the data (x- variate) is given by M_x and that

$$y_i = 1/x_i, i = 1,2,\ldots,n, \qquad \ldots \qquad (2)$$

is the y- variate such that M_y represent its corresponding mean.

Then M_x and, or (M_y) has the reciprocity property if

$$M_x M_y = 1 \qquad \ldots \qquad (3)$$

Let A_x, G_x and H_x denotes, the arithmetic, geometric and harmonic means respectively of the x-variate such that;

$$A_x = \sum_{i=1}^{n} w_i x_i \qquad \ldots \qquad (4)$$

$$G_x = \prod_{i=1}^{n} x_i^{w_i} \qquad \ldots \qquad (5)$$

$$H_x = \left(\sum_{i=1}^{n} \frac{w_i}{x_i} \right)^{-1} \qquad \ldots \qquad (6)$$

If $w_i = 1/n$ for each i then A_x, G_x and H_x are the usual unweighted means. Subsequently let the y- variate of the arithmetic, geometric and harmonic be given respectively by A_y, G_y and H_y. It is easily shown that only the geometric mean possesses the reciprocity property, i.e.

$$G_x G_y = 1 \qquad \ldots \qquad (7)$$

The relationships for the arithmetic geometric and harmonic are given in the form of lower and upper bounds, such that

$$H \le G \le A \qquad \ldots \qquad (8)$$

for any non-negative variate (Francis, 1995). Hence,

$$A_y = H_x^{-1} \ge G_x^{-1} \ge A_x^{-1} \qquad \ldots \qquad (9)$$

$$H_y = A_x^{-1} \le G_x^{-1} \le H_x^{-1} \qquad \ldots \qquad (10)$$

equality is attained only when the $x's = y's$.

Daily average exchange rates

Relevant information about the daily fluctuations of the official exchange rate of a home currency with respect to the standard currencies, such as US dollar, Euro and or the Pound Sterling, is important to the global banking community, researchers, potential investors, businessmen and traders.

Such information can be provided by the central bank of the country under consideration at the end of a given period (daily, weekly, etc.) rate or as average rate for a period (week, month, quarter, e.t.c.).

Suppose there are n working days in the period and x_i is the exchange rate of the standard currency in terms of the home currency, i.e. one unit of the standard currency equals x_i units of the home currency in day *i*. The geometric mean ensures that on the average a unit of the foreign currency equals G_x units of the domestic currency and also equals G_x^{-1}, units of the foreign currency.

Nominal effective exchange rate index

The price of a unit of the foreign currency in term of the domestic currency is regarded as the exchange rate of the domestic currency. In the exchange rate policy, a country might wish to determine whether its currency is overvalued or undervalued. If it is overvalued, the price of a unit of the foreign currency is relatively low and this could lead to excessive imports and have an adverse effect on the foreign exchange reserves of the country. On the other hand if the currency is undervalued, the price of a unit of the foreign currency is relatively high, and this may tend to discourage imports of needed raw materials and machinery and thus slows down the growth of the economy of the country. The extend of appreciation or depreciation of a domestic currency with respect to the foreign currencies , various exchange rates indices can be computed depending on the monetary systems of the country (Pandey**,** 1999).

According to Jean (1976), a commonly used nominal exchange rate index is one that measures the movements of the currencies of all the trading partners of a country relative to a based period, taking into account the weight of each currency as reflected by the volume of trade conducted in each standard currency.

Suppose that a given country trades with n foreign currencies. Let $x_{i,t}$ represent the price of one unit of currency of the i^{th} trading partner at time *t* in terms of the home currency. Let $y_{i,t}$ be the price of the home currency at time *t* in term of the currency *i.* The two exchange rates are joined by the equation

$$x_{i,t} = 1/y_{i,t} \quad \ldots \quad (11)$$

We shall express exchange rates relative to an initial period (t=0) in terms of $x_{i,t}$ and $y_{i,t}$ as

$$\bar{x}_{i,t} = x_{i,t} / x_{i,0} \quad \ldots \quad (12)$$

$$\bar{y}_{i,t} = y_{i,t} / y_{i,0} \quad \ldots \quad (13)$$

The effective exchange rate indices of a given set of weights w_i are defined either by the x-variate or the y- variate. The geometric effective exchange rate and the arithmetic effective rate indices can be defined as shown below:

Geometric	**Arithmetic**	**Harmonic**
$GR_x = \prod_{i=1}^{n} \bar{x}_{i,t}^{\,w_i}$	$AR_x = \sum_{i=1}^{n} w_i \bar{x}_{i,t}$	$HR_x = \left(\sum_{i=1}^{n} \frac{w_i}{x_{i,t}} \right)^{-1}$
$GR_y = \prod_{i=1}^{n} \bar{y}_{i,t}^{\,w_i}$	$AR_y = \sum_{i=1}^{n} w_i \bar{y}_{i,t}$	$HR_y = \left(\sum_{i=1}^{n} \frac{w_i}{y_{i,t}} \right)^{-1}$

(14)

It can easily be verified that:

$$GR_x / GR_y = 1 \quad \ldots \quad (15)$$

It is observed that only the geometric nominal effective exchange rate index possesses the reciprocity property given by equation (3). From equations (8) and (9) it is observed that

$$AR_y \geq 1/ AR_x \quad \ldots \quad (16)$$

equality being attained when $\bar{x}_{i,t}$ and $\bar{y}_{i,t}$ are equal. The indices in real situation are computed by using the representative period's average exchange rates for $\bar{x}_{i,t}$ (or $\bar{y}_{i,t}$) and comparing these with the corresponding base period averages.

Numerical example

Consider the computation of a nominal effective exchange rate index for the domestic currency **N** of a given country in 2007 relative to 2006.

We shall assume for simplicity that the country trades in only three foreign currencies B1, B2 and B3 and that the weights of these two currencies in 2006 are given by $w_1 = 1/6,\ \ w_2 = 1/3$ and $w_3 = 1/2$ respectively (assuming there is no variation in 2007), based on the share attributable to each currency in 2007 trade transactions. We shall also assume that exchange rates averaged for 2006 and 2007 are as given next page:

2006	**2007**
1 unit of B_1= 2 units of N	1 unit of B_1=3 units of N
1 unit of B_2=6 units of N	1 unit of B_2=3 units of N
1 unit of B_3=5 units of N	1 unit of B_3= 4 units of N

The computation of the arithmetic, geometric and the harmonic exchange rates indices gives the following values with the corresponding reciprocal of each.

TABLE 1. EXCHANGE RATE INDICES AND THEIR RECIPROCAL.

Computed index	Reciprocal
$GR_x = 0.7595$	$GR_y = 1.3166$
$AR_x = 0.8367$	$AR_y = 1.4028$
$HR_x = 0.6698$	$HR_y = 1.8947$

The GR_x value of 0.7595 is interpreted to indicate a depreciation of 0.2405 in 2007 of foreign currencies with respect to the domestic currency. GR_y shows an appreciation of 0.3166 in 2007 of the domestic currency with respect to the foreign currencies. The geometric indices (unlike the arithmetic and harmonic indices) are consistent because of their reciprocity property. This implies that GR_x, is on average, the domestic currency value of 1 unit of the foreign currency in 2007 equals 0.7595 times the domestic currency value of 1 unit of the foreign currency in 2006.

Consistently the value of GR_y implies that, on the average, the foreign currency value of 1 unit of the domestic currency in 2007 equals 1.7595 times the foreign currency value of 1 unit of the domestic currency in 2006.

Exchange rate index

The nominal effective exchange rate is a measure of the changes in a country's exchange rate with respect to its trading partners relative to a based period.

These changes do not have any connection with the purchasing power of the currency, nor do they indicate the level of changes in the competition on tradable goods of the country over the periods. The real effective exchange rate index can be used to measure the changes in the purchasing power of a currency over time by determining relative index movements (Howard & Gareth, 1991). The real effective exchange rate index for a domestic currency known as geometric real effective exchange rate

$$GRR_y = \prod_{i=1}^{n} \left(\bar{y}_{i,t} / R_{i,t} \right)^{w_i} \quad \ldots \quad (17)$$

where $\bar{y}_{i,t}$ is given in (12), and $R_{i,t}$ is price index ratio of the first trading partner to the price index of the domestic currency in period t.

The applications of geometric mean discussed in this paper, could prove useful in the financial industry for international trade and business. It can also be used in monitoring the stability of the economy of a country with respect to its trading partners. It is a tool for making very useful discoveries.

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