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ON THE EXTINCTION PROBABILITY OF A FAMILY NAME

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ABSTRACT

This study investigated twenty different family trees to identify the factors that determine the extinction probability of a family name the most, using the birth-and-death process theory. It is only the male descendants that were considered since all the families studied are patrilinear, i.e., only the males propagate the family name. The study showed that, out of all the attributes of a family tree, the birth rate and the death rate are the most directly related to the extinction probability. Contrary to what one expects, the mean number of offspring per individual does not seem to have a direct effect on the probability of extinction.

***Keywords*:** Birth rate, Death rate, Branching process, Birth-and-death process, Extinction probability.

INTRODUCTION

Haccou *et al.*, (2005) observed that extinction of families, populations and species are frequent in real life. According to Scott (2006), most scientists agree that life on Earth is now faced with the most severe extinction episode since the event that drove the dinosaurs extinct. No one knows exactly how many species are being lost because no one knows exactly how many species exist on Earth. Estimates vary, but the most widely accepted figure lies between 10 and 13 million species (Scott, 2006). Of these species, as many as 27,000 species become extinct each year (Scott, 2006). Walsh (2006) reported that over 99.9% of all species that once existed have gone extinct.

According to Grimmett & Stirzaker (1992), accurate models for the evolution of populations are notoriously difficult to handle. However, there are tractable and mathematically interesting models, one of which is the branching process. Many methods used in population biology have a branching process background or interpretation (Haccou *et al.,* 2005).

To understand branching processes, we note that population behavior is deduced from individuals within it. Haccou *et al.,* (2005) explain that an individual is understood in a broad sense. It might be an animal or a plant, a cell or an elementary particle – the defining property is that it gives birth, splits into, or somehow generates new individuals. The name branching process refers to the stochastic process that records the population sizes at different times, the sizes of subsequent generations, or less often the family tree thus arising. Gale (1990) pointed out that often, the term is used when the individuals being considered propagate individually. Branching processes are thus individual-based models for the growth of populations.

The problem of survival of a family name was first posed in 1874 by Rev. H. W. Watson. This laid the foundation for the study of branching processes. We consider a situation where the family name is inherited by only the male descendants. Each male has probability p_k of fathering k male offspring. This leads one, to wonder whether such a family will keep reproducing indefinitely or will sometime later, go extinct.

The continued overcrowding of our cities and increase in social vices is not far from being a result of overpopulation. The excessive number of offspring that our people produce is, among others, to ensure the continued existence of their family names. The motivation for this work is borne out of the hope that, with an understanding of the phenomenon of extinction of family names, families will not necessarily need to be very large in order to ensure continued existence.

Like many other people, Nigerian males are concerned about the continued existence of their family names, especially their own names. In some cultures, people believe their names will continue to exist forever, mostly because of what they have been able to achieve in their lifetimes. However, most people believe that their names will continue to exist through their offspring. In this paper, we will report attempt to build a birth-and-death process model (whose parameters can be empirically calculated) to compute the probabilities of extinction of family names. The resultant probabilities will then be compared with other attributes of the respective family trees to see which ones affect it the most using families in Benue State, Nigeria.

FORMULATION OF MODEL EQUATIONS

According to Doucet & Sloep (1992), the probability of extinction is an important phenomenon in population dynamics and the study of evolution. Consider a population (with birth and death probability densities λ and μ) that starts out with a single individual. As far as ultimate extinction is concerned, the only quantity associated with an individual that we need consider is the total number, *N,* of offsprings born to him (Cox & Miller, 1997). The process (birth-and-death process) is governed by two events: a birth and a death, where the probability of a birth is $\lambda = \frac{b}{(b+d)}$ and that of a death is

$\mu=\frac{d}{(b+d)}$. The interval between events is exponentially distributed with parameter $(b+d)$.

Let $p_i(t)=\text{prob}\{N(t)=i\}$, where $p_i(t)$ represent the probability that there are i individuals in the population at time t, and $N(t)$ the actual number of individuals in the population at time t. Also, suppose that $N(0)=n_0$. Then for $i=0,1,2,\ldots,$ we have that in the interval $(t,t+\Delta t)$, $p_i(t,t+\Delta t)=$

prob
$\{N(t)=i$ and a birth and a death occur in the interval $(t,t+\Delta t)$
or
$N(t)=i\text{-}1$ and a birth occurs and no death occurs in the interval $(t,t+\Delta t)$
or
$N(t)=i+1$ and a death occurs and no birth occurs in the interval $(t,t+\Delta t)\}$
That is,

$$p_i(t,t+\Delta t)=p_i(t)\{1\text{-}i(\lambda+\mu)\Delta t\}+p_{i\text{-}1}(t)(i\text{-}1)\lambda\Delta t+p_{i+1}(t)(i+1)\mu\Delta t+o(\Delta t)$$

Now, letting $\Delta t\longrightarrow 0$, we have that

$$\lim_{\Delta t\to o} p_i(t,t+\Delta t)=p'_i(t)$$

$$p'_i(t)=\text{-}i(\lambda+\mu)p_i(t)+(i\text{-}1)\lambda p_{i\text{-}1}(t)+(i+1)\mu p_{i+1}(t),$$

$$p_i(0)=\delta_{in_0}.$$

where n_0 is the number of individuals in the initial generation, and

δ_{in_0} is the Kronecker delta.

If the generating function is

$$G(z,t)=\sum_{i=0}^{\infty} z^i p_i(t),$$

Then we have that

$$\frac{\partial G(z,t)}{\partial t}=\sum_{i=0}^{\infty} z^i \frac{d}{dt}p_i(t)$$

$$=\sum_{i=0}^{\infty} z^i p'_i(t)$$

$$=\sum_{i=1}^{\infty} z^i\left[\text{-}i(\lambda+\mu)p_i(t)+(i\text{-}1)\lambda p_{i\text{-}1}(t)+(i+1)\mu p_{i\text{-}1}(t)\right]$$

Therefore,

$$\frac{\partial G(z,t)}{\partial t}=(\lambda z\text{-}\mu)(z\text{-}1)\frac{\partial G(z,t)}{\partial z}$$

The auxiliary equations are

$$\frac{dt}{1}=\frac{dz}{(\lambda z\text{-}\mu)(z\text{-}1)}.$$

The solution satisfying the initial condition is

$$G(z,t)=\left\{\frac{\mu(1\text{-}z)\text{-}(\mu\text{-}\lambda z)e^{\text{-}(\lambda\text{-}\mu)t}}{\lambda(1\text{-}z)\text{-}(\mu\text{-}\lambda z)e^{\text{-}(\lambda\text{-}\mu)t}}\right\}^{n_0} \quad \ldots\ (1)$$

The probability of this population going extinct at or before time t is given by the coefficient of z^0 in (1), which is, when $\lambda\neq\mu$,

$$G(0,t)=\left\{\frac{\mu\text{-}\mu e^{\text{-}(\lambda\text{-}\mu)t}}{\lambda\text{-}\mu e^{\text{-}(\lambda\text{-}\mu)t}}\right\}^{n_0}.$$

This is a geometric model of the probability of extinction at time t. Now, under the assumption of independence, the fate of a population of n_0 initial individuals is equivalent to the fate of n_0 populations, each with one initial individual.
When $\lambda=\mu$, then

$$G(0,t)=\left\{\frac{\lambda t}{1+\lambda t}\right\}^{n_0}$$

The probability of extinction is the value of $G(z,t)$ when t tends to infinity. If we denote that probability by p_E, then

$$p_E=p_0(\infty)=\lim_{t\to\infty}\left\{\frac{\mu\text{-}\mu e^{\text{-}(\lambda\text{-}\mu)t}}{\lambda\text{-}\mu e^{\text{-}(\lambda\text{-}\mu)t}}\right\}^{n_0}$$

To evaluate this limit, we apply L'Hospital's rule,

And we have that

$$p_0(\infty)=\left\{\frac{\mu(\lambda\text{-}\mu)\text{-}\mu e^{(\lambda\text{-}\mu)\infty}}{\lambda(\lambda\text{-}\mu)\text{-}\mu e^{(\lambda\text{-}\mu)\infty}}\right\}^{n_0}$$

$$=\left\{\frac{\mu}{\lambda}\right\}^{n_0}$$

If $\lambda<\mu$, then clearly p_E tends to 1, and in the case where $\lambda=\mu$, $p_E=1$, and the population will therefore be certain to disappear at one time or the other.

Therefore, we have

$$p_E=\begin{cases}1, & \text{for } \lambda\leq\mu\\ \left\{\frac{\mu}{\lambda}\right\}^{n_0}, & \text{for } \lambda>\mu\end{cases}$$

Assumptions

We shall make the following assumptions, bearing in mind that in branching processes, the population attributes are obtained from those of its constituent members.

a) all family sizes have the same probability mass function *f* and generating function *G*;
b) each individual in the population reproduces independently of others;
c) the family name is passed on to the next generation only through the male descendants, and therefore only the male descendants are counted (Feller, 1968). This is obtained in cultures where patrilinear lineage is practiced.

RESULTS

The data was collected through the use of questionnaire. Due to the nature of the data required for the analysis and the nature of expected respondents, the questionnaire was administered orally. The information required is the historical record of a family, going several generations back. The respondents were expected to be the elderly members of a family, since they are the ones that are expected to have that information. Since most elderly people are usually illiterate, and even the literate ones may have difficulty reading and writing, the researcher had to personally administer the questionnaire.

In presenting the data, we bear the following in mind:

1. The death of an individual who has produced male offspring does not pose a threat to the continued existence of the family genealogy. However, the death of an individual who has not yet produced a male offspring is a threat, since the continuation of such a trend will certainly result in extinction of the family line.

2. No individual is expected to live on after more than three or four generations, since under usual circumstances, the maximum life span of a human is 120 years.

3. The time of death of an individual is not considered in this work. An individual that dies at 80 years with no offspring is considered the same as one who dies at 5 or 30 years with no offspring.

Fig. 1 shows a family tree that goes three generations back. The fourth generation came through the female offspring, and therefore is not included in the analysis. The following should be noted here;

b = number of births in the family tree
d = number of deaths in the family tree
λ = birth rate, and is given by $\lambda=\frac{b}{b+d}\times 1000$
μ = death rate, and is given by $\mu=\frac{d}{b+d}\times 1000$

In this family,

$$b=14,\text{ and } d=2$$

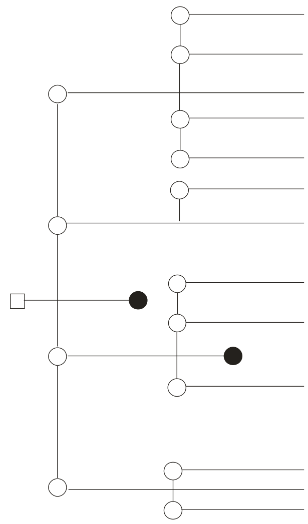

FIG. 1. A FAMILY TREE. WE DENOTE THE INITIAL ANCESTOR BY A SQUARE (□), THE BIRTH OF AN INDIVIDUAL WITH A CLEAR CIRCLE (○) AND THE DEATH OF AN INDIVIDUAL WITH A SHADED CIRCLE (●).

Therefore,

$$\lambda=\frac{b}{b+d}\times 1000=\frac{14}{16}\times 1000=875$$

And

$$\mu=\frac{d}{b+d}\times 1000=\frac{2}{16}\times 1000=125$$

The probability of extinction for this family is given by

$$P_E=\left(\frac{\mu}{\lambda}\right)^{n_0}=\left(\frac{125}{875}\right)^{1}=0.143.$$

Further analysis reveals the following about the family;
Number of individuals that died without a male offspring = 0
Number of individuals that are alive but without a male offspring = 10

Mean number of sons per father =

$$\frac{\text{number of births}}{\text{number of fathers}}=\frac{14}{5}=2.8$$

Total number of individuals at present = 13

This analysis was carried out on twenty different families of different backgrounds. The main features of the twenty family trees analysed are summarized in the Table 1.

TABLE 1. SUMMARY OF FEATURES OF THE FAMILY TREES STUDIED

S/ N	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
I	0	10	2.800	13	16	14	2	0.143	875	125
II	2	14	3.140	18	27	22	5	0.227	815	185
III	4	31	2.830	44	60	51	9	0.176	850	150
IV	8	51	3.407	68	112	92	20	0.217	821	179
V	3	75	3.516	92	126	109	17	0.156	865	135
VI	2	14	2.250	22	34	27	7	0.259	794	206
VII	0	20	3.110	27	30	28	2	0.071	933	67
VIII	0	11	3.000	15	16	15	1	0.067	938	63
IX	2	19	3.220	24	35	29	6	0.207	829	171
X	3	21	2.440	31	48	39	9	0.231	813	188
XI	0	18	3.830	22	25	23	2	0.087	920	80
XII	3	96	3.066	123	154	138	16	0.116	896	104
XIII	0	14	2.860	21	20	20	0	0.000	1000	0
XIV	0	11	2.430	17	18	17	1	0.059	944	56
XV	3	11	2.850	15	26	20	6	0.300	769	231
XVI	7	21	3.167	29	48	38	10	0.263	792	208
XVII	2	27	4.110	32	43	37	6	0.162	860	140
XVIII	3	28	3.140	41	48	44	4	0.091	917	83
XIX	4	37	3.000	45	64	54	10	0.185	844	156
XX	4	22	3.780	28	41	34	7	0.206	829	171

Key:
C1 – Number of individuals that died without producing offspring
C2 – Number of individuals that are alive but without a male offspring
C3 – Mean number of sons per father
C4 – Total number of individuals alive
C5 – Total number of events
C6 – Total number of births
C7 – Total number of deaths
C8 – Extinction probability
C9 – Birth rate per thousand
C10 – Death rate per thousand

DISCUSSION

This work aimed to find the probability of a family name going extinct at a point in time. The probabilities of extinction for 20 families have been computed using the birth rate and the death rate. There are 9 attributes that have been identified as factors that could affect extinction probability of a family. These are;

The number of individuals that died without producing male offspring – they are assumed to pose a threat to the continued existence of the family name.

The number of people alive who have not yet produced male offspring – they still have the potential to reproduce, and a form of promise of continued existence of the family name.

The mean number of sons per individual – this shows what to expect should the individuals who have no male offspring yet start producing offspring, and it determines whether the population under study is sub-critical, critical or super-critical. As it turns out, all the families studied are supercritical, since in each case the mean number of sons per father is greater than unity.

The total number of individuals alive at present – this gives us an idea of the family population sizes.

Number of events (both births and deaths) – this shows how active or inactive the population has been.

The total number of births – this indicates the number of individuals that have entered the family tree at one point or the other.

The total number of deaths – this indicates the number of individuals that have left the family tree at one point or the other.

The Birth rate – this shows us at what rate the population increased, expressed as number of individuals per thousand.

The Death rate – this shows us at what rate the population decreased, expressed as number of individuals per thousand.

It is clear that the death rate and the birth rate are the chief contributing factors to the probability of extinction of a family name. The other factors considered, such as number of births, number of deaths, all contribute to the probability of extinction, but not in isolation. The mean number of sons per individual, which seems (to many in the society) to be the chief determining factor of the probability of extinction, has proven otherwise. According to the analysis in this work, a higher mean number of sons does not necessarily imply a lower extinction probability.

To ascertain the probability that one's family name will go extinct, a proper record needs to be made of all the generations stemming from the initial ancestor of the family. Then the family tree can be drawn, as shown in section three. From the tree, the necessary requirements for computing the probability of extinction can be obtained. They are the birth rate and the death rate. This work leaves out other important aspects of family extinction.

It has been observed during the data collection that some families have lost all their departed members to a particular disease (Diabetes, for instance). A study that goes into the causes of each of the deaths recorded in the families being examined for extinction probability will be useful, but is left for further research.

Based on the findings of this work, the following recommendations are hereby made.

- Families that wish to consciously avoid extinction should not focus on producing more offspring. They should rather focus on maintaining such a birthrate that will ensure non-extinction, but at the same time produce offspring that they can sustain financially, socially, spiritually and intellectually.

- Government and non-governmental bodies should ensure that the populace is well educated on what it takes to ensure the continued existence of a family name. The level of awareness of HIV/AIDS and related issues, for instance, has been achieved through great effort. Such effort is recommended regarding family extinction.

- In order to ensure that family members live longer, good medical care, environmental hygiene and diet must be provided and encouraged among our people. This will ensure increased productivity and longer life spans, which will in turn reduce the death rate. This will result in a reduction of the probability of extinction in families.

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